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THE

COMMON-SCHOOL ARITHMETIC,

DESIGNED FOR

LEARNERS OF EVERY CLASS;

AND PARTICULARLY FOR THOSE WHO ARE DESIROUS OF
ACQUIRING A THOROUGH KNOWLEDGE OF

PRACTICAL MATHEMATICS.

By PLINY E. CHASE, A. M.
AUTHOR OF "THE ELEMENTS OF ARITHMETIC."

WORCESTER, MASS:
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(8.) Among the original features of the work, in addition to the peculiarities already alluded to, the attention of teachers is invited to the explanation of Subtraction, § 31,—to the method pursued in Alligation Alternate, § 170,—to the explanations of the rules for extracting Square and Cube Roots, § 178—181,—to the Problems in § 231,—and to the rules for determining the greatest Common Divisor, § 66,—for reduction of Sterling Money, § 136,—and for Complex Analysis, § 140.

In conclusion, the author submits his work with confidence to those teachers who are already acquainted with his "Elements," as containing the result of four years additional experience. And of all who are interested in the subject of education, and the improvement of text books, he solicits an impartial examination. If the book is such as he believes it to be, he would gladly see it introduced into general use; but he has no wish that it should be employed in the place of *better* works, or that it should be retained a moment to the disadvantage of the pupils in any school.

ARITHMETIC.

1. ARITHMETIC is the art of computing by numbers.

A NUMBER, in Arithmetic, is an expression that denotes how many things are spoken of.

Written numbers may be expressed by words, by letters, or by combining the ten Arabic figures invented for this purpose:

One,	Two,	Three,	Four,	Five,	Six,	Seven,	Eight,	Nine,	Naught.
1,	2,	3,	4,	5,	6,	7,	8,	9,	0.

The first nine of these characters are called *digits*, from the Latin *digitus*, a finger: because the numbers they express are often counted on the fingers.

The tenth character, 0, expresses no value by itself, and is, therefore, commonly called *Naught*. It is also called *Zero*, or *Cipher*.

Arithmetic consists of five fundamental operations, in each of which something is given, and something is required; namely, Numeration, Addition, Subtraction, Multiplication, and Division.

In Numeration, numbers are either given in figures to be read in words, or given in words to be written in figures.

In Addition, two or more numbers are given, and their sum or amount is required.

In Subtraction, two numbers are given, and their difference is required.

In Multiplication, two numbers are given, and their product is required.

In Division, two numbers are given, and their quotient is required.

I. NUMERATION.

DIRECTIONS TO THE PUPILS.

Never pass over any explanation until you fully understand it. If your teacher does not render it perfectly plain at first, he will wish you to ask him again, and you can desire some one at home to repeat the explanation, and make it fuller, if necessary.

When you are performing any example, explain every step of it to yourself, as if you were showing some one else *how* to do it, and *why* it is done in that way.

The heavy figures enclosed in parentheses, that you will frequently observe in the explanations and examples, as (§ **29**), refer to some preceding section. Whenever you see one of these references, turn to the proper section, and find how it will assist you in understanding the subject before you.

If you adhere strictly to these directions, your progress will be comparatively easy, pleasant, and satisfactory. If you neglect them, your difficulties will increase as you advance, and you will require almost constant assistance.

2. Numeration is the process by which we express numbers in figures, and read them when so expressed.

3. The Arabic method is now generally adopted in Numeration. By this method, *the value of any figure is made ten times as large by being removed one place to the left* Thus, 1 represents simply ONE, but 10 represents one ten and nothing, or TEN.

4. The number one is called a UNIT, and all numbers are formed by a collection of units, or parts of units. Thus, four is the same as four ones, or units; fifteen is the same as fifteen units.

5. Every figure is said to occupy a certain PLACE, *and a name is* given to each place.

In naming the places in whole numbers, we always commence at units, and numerate from the right to the left. The first figure occupies units' place; the second, tens' place; the third, hundreds' place; as below.

Hundreds.	Tens,	Units,
4	0	2

The figure Naught, 0, is used only when there is no digit to occupy its place, (§ **1**,)

6. Units are read by simply giving the name of the figure that stands in units' place; as, 7, *seven;* 5, *five.*

7. In reading tens,

10	one ten is	called Ten.
20	two tens are	called Twenty.
30	three tens are	called Thirty.
40	four tens are	called Forty.
50	five tens are	called Fifty.
60	six tens are	called Sixty.
70	seven tens are	called Seventy.
80	eight tens are	called Eighty.
90	nine tens are	called Ninety.

The Naught in each of the above numbers is used so that the other figure may occupy the second place, which is the place of tens, (§ **5**,) there being no units to occupy the first, or units' place.

8. The numbers between ten and twenty are read as follows:

11	or ten-one	is read Eleven.
12	or ten-two	is read Twelve.
13	or ten-three	is read Thirteen.
14	or ten-four	is read Fourteen.
15	or ten-five	is read Fifteen.
16	or ten-six	is read Sixteen.
17	or ten-seven	is read Seventeen.

18 or ten-eight is read Eighteen.
19 or ten-nine is read Nineteen.

9. The numbers between twenty and one hundred, are read by first naming the tens, as in (§ 7,) and then the units. Thus:

27 two tens and seven units is read Twenty-seven.
65 six tens and five units is read Sixty-five.
98 nine tens and eight units is read Ninety-eight.

10. The hundreds are read by adding the word *hundred* to the name of the figure which occupies hundreds' place; as,

400 four hundreds, is read Four hundred.

360 three hundreds and six tens, is read Three hundred and sixty.

582 five hundreds, eight tens, and two units, is read Five hundred and eighty-two.

207 two hundreds and seven units, is read Two hundred and seven.

EXERCISES FOR BEGINNERS.

1—30. Read the following numbers: 10; 19; 13; 18; 12; 17; 11; 16; 14; 20; 13; 21; 23; 25; 27; 29; 22; 24; 26; 28; 30; 32; 34; 36; 31; 33; 35; 38; 37; 39.

31—60. Read the following numbers: 40; 44; 48; 42; 46; 41; 45; 43; 47; 49; 50; 62; 73; 84; 95; 56; 67; 78; 89; 91; 52: 63; 74; 85; 96; 53; 64; 72; 85; 99.

61—90. Read: 100; 200; 300; 400; 500; 600; 700; 800; 900; 101; 202; 303; 104; 205; 306; 407; 508; 609: 701; 802; 903; 404; 105; 206: 307; 408; 509; 610; 711; 812.

91—120. Read: 913; 114; 215; 316; 417; 518; 619; 720; 821; 932; 143; 254; 365; 476; 587; 698; 700; 809; 908; 890; 980; 407; 470; 704; 740; 873; 925; 592; 439; 797.

121—150. Read: 1000; 2000; 3000; 4000; 5000;

6000; 7001; 8002; 9003; 1004; 2005; 3006; 4007; 5008; 6009; 7010; 8011; 9015; 2020; 3025; 4030; 5037; 8043; 9080; 1069; 2008; 3094; 4700; 5108; 6234.*

11. Examples to be read by the Pupil.

1—25. 15; 11; 19; 17; 14; 25; 36; 13; 95; 80; 12; 47; 74; 16; 18; 20; 50; 83; 38; 49; 94; 77; 65; 10; 99.

26—50. 33; 61; 40; 27; 29; 72; 92; 100; 101; 113; 120; 137; 103; 301; 310; 130; 85; 58; 66; 566; 33; 91; 70: 207; 590.

51—75. 127; 400; 212; 560; 309; 653; 356; 365; 635; 536; 563; 801; 118; 180; 870; 357; 429; 560; 608; 742; 897; 930; 215; 419; 711.

12. Large numbers are divided into periods of three places each. Units, tens, and hundreds, occupy units' period; units of thousands, tens of thousands, and hundreds of thousands, constitute Thousands' period; units, tens, and hundreds of millions, constitute Millions' period. The names of a few periods are given in the table below.

PERIODS.	Sextillions.		Quintillions.			Quadrillions.			Trillions.			Billions.			Millions.			Thousands.			Units.		
PLACES.	Tens of	Units of	Hundreds of	Tens of	Units of	Hundreds of	Tens of	Units of	Hundreds of	Tens of	Units of	Hundreds of	Tens of	Units of	Hundreds of	Tens of	Units of	Hundreds of	Tens of	Units of	Hundreds of	Tens of	Units of
	1	5,	3	4	8,	0	0	0,	0	1	3,	0	9	0,	5	0	0,	4	0	8,	0	6	3,

To read whole numbers consisting of more than three figures, *commence at units, and divide the number by commas, into periods of three figures. Then begin at the left hand, and read the numbers standing in each period, adding the name of every period except units.* The number in the table is read thus: Fifteen sextillion, three hundred and forty-eight quintillion, thirteen trillion, ninety billion, five hundred million, four hundred and eight thousand, and sixty-three.

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* It may be well for beginners, before proceeding any further, to perform the Examples in § 18, 24, 29, 35, 47, 70; the teacher giving such oral explanation to each pupil, as may be found necessary.

13. EXAMPLES FOR THE PUPIL.

76—100. 1000; 1200; 4308; 1007; 2080; 5033; 904; 840; 716; 5090; 6072; 15130; 20009; 9050; 87020; 3387; 4062; 18307; 95728; 7805; 8191; 4700; 1084; 6215; 1106.

101—118. 4516; 60060; 200002; 15384; 17009; 284391; 100901; 72053; 40080; 900900; 33015; 284007; 197040; 30603; 804007; 1015002; 19000070; 40040.

119—130. 40117020; 8040003; 100000900; 1009; 3807000; 500000000; 400000016; 7003000605; 10; 28400127; 4015090600000; 7000200000050007; 19; 5649380075109; 5003400070000000000.

14. DECIMALS. We have already seen (§ 3), that the value of any figure is increased ten times by being removed one place to the left. Hence, 10 *of any place are equivalent to* 1 *of the next higher place, and* 1 *of any place is equivalent to* 10 *of the next lower place.* Thus:

10 hundreds are 1 thousand; 1 hundred is 10 tens.
40 hundreds are 4 thousands; 4 hundreds are 40 tens.
70 tens are 7 hundreds; 7 tens are 70 units.
85 tens are equal to 8 hundreds and 5 tens.
93 hundreds are equal to 9 thousands and 3 hundreds.

15. The principle stated in the preceding paragraph, may be extended as far as we please, both to the left and to the right of units. In order to mark the place of units, a point (.), called the *decimal point*, is written at the right hand of units, as in the example below.

Millions.	Hund. Thousands.	Ten Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.	Hund. Thousandths.	Millionths.
5	0	7	0	8	3	2.	0	0	9	4	1	6

The figures at the right hand of units, are called Decimals. The names of the places below and above units, correspond to each other with the exception of the decimal termination *ths;* as, *tens, tenths; hundreds, hundredths.* The name *tenth, hundredth,* or *thousandth,* denotes that the unit is divided into 10, 100, or 1000 equal parts. When decimals are written by themselves, they are always preceded by the decimal point; as, .76; .084.

Note.—We have seen that any figure represents a number ten times as large by being removed one place to the left, or one tenth as large by being removed one place to the right, (§ 14, 15.) If removed two places to the left, it will represent a number ten times ten, or one hundred times as large; or, if two places to the right, it will be only one tenth of one tenth, or one hundredth as large. In like manner,

if removed to the right				if removed to the left			
3 places	it will represent a number	1000	times as large.	3 places	it will represent a number	.001	as large.
4 "		10000		4 "		.0001	
5 "		100000		5 "		.00001	
6 "		1000000		6 "		.000001	
7 "		10000000		7 "		.0000001	
&c.		&c.		&c.		&c.	

16. To read Decimals. *Read the figures as if they represented whole numbers, and add the name of the right hand decimal place.*

.8 eight *tenths;* .015 fifteen *thousandths.*

.08 eight *hundredths;* .0103 one hundred and three *ten thousandths.*

.008 eight *thousandths;* .009009 nine thousand and nine *millionths.*

If whole numbers and decimals are combined, we first read the whole number, and then the decimal. Thus, the Example in § **15**, is read: *Five million, seventy thousand, eight hundred and thirty-two,* and nine thousand four hundred and sixteen *millionths.*

17. EXAMPLES FOR THE PUPIL.

131—160. .7; .07; .09; .19; .97; .007; .001; .046; .013; .104; .267; .85; .085; .0085; .0163; .8742; .5009; .6; .728; .44; .0805; .0004; .78; .0078; .078; .04; .107; .0609; 10.4; 1.04.

161—180. 14.3; 78.02; 195.12; 2.004; 5.017; 68.309; 40.072; 300.03; .305; 300.005; .217; 200.017; .0604; 600.0004; .0905; .00905; 900.00005; .04007; .30002; 30000.00002.

181—190. 120007095; 12000709.5; 1200070.95; 120007.095; 12000.7095; 1200.07095; 120.007095; 12.0007095; 1.20007095; .120007095.

18. TO WRITE WHOLE NUMBERS, *Commence with the highest denomination, and write all the periods in their proper order, allowing three places for each period after the first. The vacant places and periods must be filled by naughts.*

Thus, in writing three sextillion, fifteen quintillion, seventy trillion, five hundred and nine billion, and seven thousand, we must place 3 in sextillions' period, 015 in quintillions' period, 000 in quadrillions' period, 070 in trillions' period, 509 in billions' period, 000 in millions' period, 007 in thousands' period, and 000 in units' period. 3,015,000,070,509,000,007,000.

That part of Numeration which teaches us how to express numbers by figures, is sometimes called NOTATION.

EXERCISES FOR BEGINNERS.

1. Write on the slate all the numbers from 1 to 100

2. Write on the slate all the numbers from 100 to 200.

3. Write on the slate all the numbers from 900 to 1000.

4. Write all the numbers from 1000 to 1100.

5. Write all the numbers from 1600 to 1700.

6. Write all the numbers from 2000 to 2200.

19. EXAMPLES FOR THE PUPIL.

Write the following numbers in figures.

191—202. Eleven; fifteen; nineteen; twenty-seven; seventy-two; fifty-one; ninety-one; forty; thirty-three; eighty-six; sixty-eight; one hundred.

203—210. One hundred and nine; nine hundred and one; two hundred and ten; six hundred and forty; five hundred and thirty-seven; seven hundred and thirty-five; eight hundred and three; four hundred and eleven.

211—216. One thousand and three; two thousand and seventeen; five thousand and twenty; eleven thousand four hundred; forty thousand one hundred and eighty; two hundred thousand and one.

217—222. Five million; nine hundred million; two million and two; ninety million, sixty thousand, and four; two hundred million and sixty; eight hundred and seven million and nine thousand.

223—228. Eight billion; two billion, seven thousand and fifteen; ten billion and twenty; one billion and seventy million; two hundred billion and forty-one thousand; one billion, one thousand, and ten.

229—233. Seven trillion; sixty trillion and three billion; two trillion and two hundred thousand; five hundred and eight trillion, twenty-three million and fifty; one trillion, ten billion and fifteen thousand.

234—240. One quadrillion and one hundred; two billion and forty thousand; six quintillion and six; seven million and fifty-nine; three thousand and four; two hundred million and ten; ten sextillion thirty quadrillion two hundred and nine trillion and five thousa

20. To write Decimals, *First determine the number of places the decimal must occupy. Then write the decimal as a whole number, and prefix enough naughts, (if any are needed,) to make the requisite number of places, placing the decimal point in its proper position*, (§ **15**.)

EXAMPLES.

Write fifty thousand and ninety-five *hundred-billionths.*

Hundred-billionths occupy eleven decimal places, and as there are but five figures in 50095, we must prefix the decimal point and six naughts, thus: .00000050095.

Eleven, and sixty-three *ten-thousandths*, 11.0063.

Five hundred and one, and nineteen *millionths*, 501.000019.

Seven hundred and three *tenths*, 70.3.

Six million and fifteen *thousandths*, 6000.015.

21. Examples for the Pupil.

Write the following numbers in figures.

241—250. Eight *tenths ;* two *hundredths ;* eleven *hundredths ;* forty-five *hundredths ;* seven *tenths ;* fifty-two *tenths ;* fifty-two *hundredths ;* fifty-two *thousandths ;* five hundred *tenths ;* nine *thousandths.*

251—255. Seventy, and eight *tenths ;* seven, and eight *hundredths ;* seven hundred and eight *thousandths ;* seven hundred, and eight *thousandths ,* seven hundred, and eight *tenths.*

256—260. Four *thousandths ;* one, and sixteen *thousandths ;* nine, and seven *ten-thousandths ;* ninety-five *ten-thousandths ;* eleven thousand four hundred and two *hundredths.*

261—268. Twenty thousand and nine *tens ;* write the same number of *units ; tenths ; hundredths ; thousandths ; ten-thousandths ; hundred-thousandths ; millionths.*

269, 270. Seven hund four, and one thousand and five *millionths;* thirteen million and sixty-three *ten-billionths.*

271. In 63 thousandths, how many hundredths and how many thousandths? In 27 thousandths? In 45 thousandths? (See § **14**.)

272. In 63 hundredths, how many tenths and how many hundredths? In 27 hundredths? In 45 hundredths?

273. In 63 tenths, how many units and how many tenths? In 27 tenths? In 45 tenths? In 19 tenths?

274. In 63 units, how many tens and how many units? In 17 units? In 94 units? In 37 units?

275. In 63 tens, how many hundreds and how many tens? In 14 tens? In 87 tens? In 22 tens?

276. In 63 hundreds, how many thousands and how many hundreds? In 49 hundreds? In 275 hundreds?

277. In 5 hundreds, how many tens? how many units? how many tenths? how many hundredths?

278. In 15 hundreds, how many tens? how many units? tenths? hundredths? thousandths?

279. In 28 tens, how many units? tenths? hundredths? thousandths? ten-thousandths? hundred-thousandths?

280. In 24.07, how many hundreds? how many tens? how many units? how many tenths? how many hundredths? how many thousandths? how many ten-thousandths?

NOTE.—We may place as many naughts as we please, at the right hand of Decimals, without altering their value. Thus, in the preceding example, as there is no figure to occupy thousandths' and ten-thousandths' places, we may fill them with naughts. Then 24.07 is equivalent to 24.070, or to 24.0700.

22. FEDERAL MONEY.

The currency of the United States is called Federal Money. The value of the several coins is such that they can be expressed decimally. The dollar is regarded as the unit, and dollars are, therefore, written as whole numbers, cents as hundredths, and mills as thousandths.

The sign $ is usually prefixed to dollars. This sign is supposed, by some, to be a contraction for U. S.($). denoting United States' currency; others regard it as derived from the figure 8, because the Spanish dollar is equivalent to 8 rials, and is sometimes called "the piece of eight."

EXAMPLES.

$19. Nineteen dollars.
$4.50 Four dollars and fifty cents.
$7.625 Seven dollars sixty-two cents and five mills.

In 19 dollars, how many cents? how many mills?

We must reduce 19 to hundredths to represent cents, and to thousandths to represent mills. 19 is equivalent (NOTE to Ex. 280,) to 19.00 or 19.000. There are, therefore, 1900 cents, or 19000 mills.

In 625 cents, how many dollars and cents?

As cents are hundredths of dollars, we place the decimal point so as to represent hundredths; thus, $6.25.

In 6250 mills, how many dollars, cents and mills?

Pointing off three figures for thousandths, the answer is $6.250.

23. EXAMPLES FOR THE PUPIL.

Read the following numbers:

281—290. $17; $40.00; $35.50; $107.375; $62.00; $0.18; $.125; .50; .94; .625.

291—298. How many cents in $25? in $12.50? in $608? in $1.25? in $5.375? in $900? in $5.125? in $0.475?

299—304. How many dollars in 600 cents? in 575 cts.? in 4120 cts.? in 65300 cts.? in 11000 cts.? in 47805 cts?

305—310. How many dollars in 1000 mills? in 16000 mills? in 5980 mills? in 4725 mills? in 2009 mills? in 10075 mills?

311—320. How many mills in $5? in $10? in $275? in $2.75? in $4.50? in $1.234? in $12.34? in $1234? in $7.10? in $.16?

321—325. Write in figures, twelve dollars; nine dollars and ten cents; one dollar and four cents; six dollars and seventeen cents; ten dollars and nine cents.

326—330. One dollar and one mill; two dollars five cents and five mills; one hundred dollars and one hundred mills; six dollars twelve cents and five mills; one thousand dollars nine cents and two mills.

If there are any decimals below mills, they may be read as tenths, hundredths, thousandths, &c., of mills; thus,

$1.627849 may be read, one dollar, sixty-two cents, seven mills, and 849 thousandths of a mill.

331 — 340. Read: $5.0975; .08325; $5.62875; $15.009125; $4.6918; $0.1875; $9.09375; $4.046875; .0034375; $7.28059.

Write the following numbers, and read them as Federal Money:

341—345. 14 million and 3, and 47 *ten thousandths;* 165 *hundredths;* 165 *hundreds;* 28 thousand and 1, and 5 *thousandths;* 105 thousand and 26, and 206 *hundred-thousandths.*

346 — 350. 165 *ten-thousands;* 165 *ten-thousandths;* 5 million 5 thousand and 5, and 5 *millionths;* 35 thousand and 19, and 1001 *hundred-thousandths;* 23 billion 60 thousand and one, and 408 *ten-millionths.*

* * *

II. ADDITION.

24. ADDITION is the process by which we join two or more numbers together, to find their *sum* or *amount.*

To represent the operations of Addition, the following *signs* are often used.

The sign + (plus,) signifies that two or more numbers are to be added together. The sign = (equal to,) denotes that one number or series of numbers is equal to another. Thus, $2 + 3 + 5 = 10$, signifies that the sum of 2, 3, and 5 is equal to 10.

EXERCISES FOR BEGINNERS.

1—10. Add 111 to each of the following numbers: 111; 211; 221; 222; 322; 433; 544; 655; 877; 988.

11—30. Add 222 to each of the following numbers: 111; 211; 221; 222; 321; 322; 332; 333; 432; 421; 422; 433; 443; 444; 544; 554; 555; 655; 877; 976.

31—50. Add 333 to each of the following numbers: 111; 211; 221; 222; 321; 322; 332; 333; 432; 421; 422; 433; 443; 444; 544; 554; 555; 655; 701; 810.

51—70. Add 444 to each of the following numbers: 111; 211; 221; 222; 321; 322; 332; 432; 433; 443; 444; 544; 555; 751; 800; 805; 850; 912; 834; 953.

71—90. Add 555 to each of the following numbers: 111; 211; 221; 222; 321; 322; 332; 432; 433; 443; 444; 544; 643; 732; 821; 910; 901; 840; 932; 721.

91—110. Add 666 to each of the following numbers: 111; 211; 221; 222; 321; 322; 332; 432; 433; 532; 621; 721; 733; 700; 803; 830; 920; 921; 833; 621.

111—130. Add 777 to each of the following numbers: 111; 211; 221; 222; 321; 322; 400; 501; 502; 620; 621; 622; 202; 811; 821; 812; 900; 921; 910; 912.

131—150. Add 888 to each of the following numbers: 111; 211; 210; 310; 301; 401; 411; 511; 500; 601; 610; 710; 711; 700; 801; 810; 811; 911; 910; 901.

25. EXAMPLES FOR THE PUPIL.

Add the following numbers.

1—9. 1 and 1; 1 and 2; 1 and 3; 1 and 4; 1 and 5; 1 and 6; 1 and 7; 1 and 8; 1 and 9.

10—18. 2 and 1; 2 and 2; 2 and 3; 2 and 4; 2 and 5; 2 and 6; 2 and 7; 2 and 8; 2 and 9.

19—27. 3 and 1; 3 and 2; 3 and 3; 3 and 4; 3 and 5; 3 and 6; 3 and 7; 3 and 8; 3 and 9.

28.—36. 4 and 1; 4 and 2; 4 and 3; 4 and 4; 4 and 5; 4 and 6; 4 and 7; 4 and 8; 4 and 9.

37—45. 5 and 1; 5 and 2; 5 and 3; 5 and 4; 5 and 5; 5 and 6; 5 and 7; 5 and 8; 5 and 9.

46. Add 6 to all the numbers from 1 to 9.

47. Add 7 to all the numbers from 1 to 9.

48. Add 8 to all the numbers from 1 to 9.

49. Add 9 to all the numbers from 1 to 9.

50. Add 10 to all the numbers from 1 to 10.

51. Count 100 by adding ones, thus: 1 and 1 are 2; 2 and 1 are 3; 3 and 1 are 4; 4 and 1 are 5; &c.

52. Count 100 by adding twos, thus: 2 and 2 are 4; 4 and 2 are 6; 6 and 2 are 8; 8 and 2 are 10; &c.

53. Count 101 by commencing at 1 and adding twos, thus; 1 and 2 are 3; 3 and 2 are 5; 5 and 2 are 7; &c.

54—75. How many are 2 + 3; 5 + 2; 6 + 1; 8 + 3; 2 + 4; 1 + 5; 9 + 1; 2 + 5; 7 + 2; 8 + 2; 2 + 6; 8 + 4; 5 + 3; 7 + 1; 2 + 7; 5 + 1; 9 + 6; 7 + 8; 10 + 5; 3 + 9; 7 + 7; 9 + 8?

76. Count 102 by adding threes, thus: 3 and 3 are 6; 6 and 3 are 9; &c.

77. Count 100 by commencing at 1 and adding threes, thus: 1 and 3 are 4; 4 and 3 are 7; 7 and 3 are 10; &c.

78. Count 101 by commencing at 2 and adding threes.

79. Count 101 by commencing at 1 and adding fours.

80. Count 102 by commencing at 2 and adding fours.

81. Count 100 by adding fives.

82. Count 101 by commencing at 1 and adding fives.

83. Count 102 by commencing at 2 and adding fives.

84. Count 104 by commencing at 4 and adding fives.

85. Count 103 by commencing at 1 and adding sixes.

86. Count 104 by commencing at 2 and adding sixes.

87. Count 100 by commencing at 2 and adding sevens.

88. Count 100 by commencing at 4 and adding eighths.

89. Count 105 by commencing at 1 and adding eighths.

90. Count 100 by commencing at 1 and adding nines.

91. Count 1001 by commencing at 1 and adding tens.

92. Add 7, 8, 4, 5, 7, 7, 3, 2, 10, 6, 9, 9, 1, 7, 7, and 8.

93. Add 5, 4, 4, 6, 6, 5, 7, 9, 8, 8, 9, 4, 4, 3, 1, and 10.

94. Add 10, 3, 3, 8, 2, 7, 1, 5, 5, 9, 4, 6, 8, 8, 5, 1, and 9.

95. Add 4, 2, 4, 10, 9, 8, 7, 6, 5, 1, 2, 3, 4, 7, 7, and 6.

96. Add 9, 9, 2, 2, 7, 3, 6, 6, 4, 8, 8, 5, 1, 10, 10, and 1.

97. Add 2, 6, 8, 4, 10, 1, 3, 5, 7, 7, 10, 10, 9, 9, and 7.

98. Add 6, 5, 4, 3, 2, 1, 7, 8, 9, 10, 9, 10, 9, 10, and 2.

99. Add 3, 6, 6, 7, 7, 8, 9, 5, 4, 2, 1, 10, 8, 9, and 3.

100. Add 8, 1, 9, 10, 7, 3, 4, 2, 5, 6, 8, 8, 7, and 4.

26. EXAMPLE FOR THE ADDITION OF LARGE NUMBERS.

A man owns a house worth 2750 dollars, a barn worth 975 dollars, a farm worth 2625 dollars, furniture worth 893 dollars, and he has 3967 dollars in bank. *How much* is he worth?

$$\begin{array}{r}2750\\975\\2625\\893\\3967\\\hline 11210\end{array}$$

We first arrange the numbers so that the figures in each place may stand in a perpendicular column, units under units, tens under tens, hundreds under hundreds, and thousands under thousands. Then commencing at the right hand, we find the sum of units' column is 20 units, equivalent (§ **14**, and § **21**, Ex. 271—280,) to 2 tens and 0 units. We write 0 in units' place, and add or *carry* 2 to the column of tens. 2 *to carry* + 6 + 9 + 2 + 7 + 5 tens, are 31 tens, or (§ **14**, and § **21**,) 3 hundreds and 1 ten. Write 1 in tens' place and *carry* 3 hundreds. 3 *to carry* + 9 + 8 + 6 + 9 + 7 hundreds, are 42 hundreds, or 4 thousands and 2 hundreds. Write 2 in hundreds' place, and *carry* the 4 thousands. 4 *to carry* + 3 + 2 + 2 thousands, are 11 thousands, or 1 ten-thousand and 1 thousand. Write 1 in thousands' place, and as there are no more ten-thousands to be added, we also write the 1 in ten-thousands' place. The whole amount is 11210 dollars. Hence the following

RULE FOR ADDITION.

Arrange the numbers to be added, so that all the figures which occupy the same place may stand in the same perpendicular column, units under units, tenths under tenths, &c.

Commence at the right hand and add the columns in order from right to left. If the sum of any column is less than 10, *write it underneath. If it is* 10 *or more, place the right hand figure underneath, and add the remaining figure or figures to the next column.*

PROOF. *Perform the addition twice, adding the columns downwards at one time, and upwards at the other. The two results should be alike.*

27. EXAMPLES FOR THE PUPIL.

101—105. Add 10, 11, 22, 33, and 23; 14, 31, 20, 22, and 31; 111, 222, and 333; 500, 67, 31, and 201; 112, 231, and 756.

106—110. Add 27, 32, 63, 11, and 15; 84, 15, 23, 19, and 7; 106, 95, 9, 81, and 250; 220, 119, 364, and 58; 401, 657, 94, and 500.

111—115. Add 2408, 657, and 5090; 93, 722, and 9904; 1203, 4567, 87, and 940; 95614, and 2895; 17306, and 44195.

116—120. Add 84723, and 91457; 44285, and 19391; 8720, and 15640; 9999, and 9876; 84320, and 159642.

121—125. How many are 684 + 507 + 69 + 1720; 118 + 2604 + 59 + 970; 4762 + 9543 + 1609; 1180 + 66 + 730 + 59 + 117; 404 + 873 + 95 + 1144?

126—135. How many must be added to 5 to make 6; 7; 8; 9; 10; 11; 12; 13; 14; 15; 16?

136—145. How many must be added to 4 to make 8; 10; 5; 12; 14; 13; 7; 9; 11; 6?

146—155. How many must be added to 3 to make 12; 4; 8; 7; 5; 13; 9; 11; 6; 10?

156—165. What single figure when added to 2, will make a number terminating in 3; 2; 4; 6; 8; 0; 7; 5; 1; 9?

166—175. What single figure when added to 6, will make a number terminating in 3; 2; 4; 6; 8; 0; 7; 5; 1; 9?

176—185. What single figure when added to 7, will make a number terminating in 7; 9; 0; 8; 1; 3; 6; 4; 2; 5?

186—195. What single figure added to 8, will make a number terminating in 8; 0; 2; 9; 1; 3; 7; 4; 6; 5?

196—205. What single figure added to 9, will make a number terminating in 9; 4; 1; 6; 0; 2; 5; 8; 3; 7?

206—209. What single figure added to 3, will make a number terminating in 3; 0; 2; 1?

210—214. What single figure must be added to 4, to make a number terminating in 4; 2; 0; 3; 1?

215—220. What single figure must be added to 5, to make a number terminating in 5; 3; 1; 4; 0; 2?

221—225. Add the following columns of numbers.

284.37	.0019	156.7	1.59	19.87
195.63	.0675	29.875	22.8	25.
271.44	1.38	3.004	4.72	49.94
18.39	5.724	945.	3.04	8.76

226—230. Add 19.847 and 504.03 ; .2759 and 7.094 ; .00872 and .00359 ; 564.7 and 2837.85 ; 2.6753 and 198.8.

231—235. How many are 45.094 + 7.7356 ; 1.9908 + .0572 ; 279.91 + 720.09 ; 468.3 + 7254.75 ; 28.7 + 539.04 + .075 + 842.68 + 19 ?

236. Add $15.75 ; $284.30 ; $149 ; $1.625 ; and 375.

237. Add $870 ; $44.19 ; $2.50 ; $59.80 ; and .51.

238. Add $15.04 ; $7.382 ; $4.018 ; $1070 ; and $3.56.

239. $9.29 + .47 + $11.07 + .082 + $25 + $62.50 = ?

240. $140 + $28.63 + $15.09 + .842 + $276.30 + $69 = ?

28. Questions to be solved by Addition.

241. George Washington was born A. D. 1732. He was elected President when 57 years old, and died 10 years afterwards. In what year did he die?

242. A gardener has 218 apple trees, 195 peach trees, 94 plum trees, 368 pear trees, 207 quince trees, and 1009 ornamental trees. How many trees are there in the nursery?

243. Charles is worth 489 dollars, James is worth 107 dollars more than Charles, and John is worth 256 dollars more than James. How much is John worth?

244. What number is 240785 greater than 199306?

245. The difference of two numbers is 960843, and the smaller number is 229317. What is the larger number?

246. The distance from Washington to Baltimore being 38 miles, thence to Philadelphia 99 miles, thence

to New York 90 miles, thence to Worcester 175 miles, thence to Boston 44 miles, how far is Boston from Washington?

247. What is the price of four barrels of flour, at 575 cents a-piece?

248. Four men entered into partnership, each contributing 3875 dollars. What was their whole stock?

249. How many days in a leap year, there being 7 months of 31 days each, 1 month of 29 days, and 4 months of 30 days each?

250. How many strokes does a common clock strike in 24 hours?

251. How many acres in a farm, which has 85 acres of woodland, 97 acres of pasture, 179 acres under cultivation, and 23 acres covered with water?

252. A man pays 2875 dollars for a farm, 1625 dollars for a house, 837 dollars for a barn, and 519 dollars for farm stock. What is the whole cost?

253. What will five yards of broadcloth cost, at $5.875 per yard?

254. When coal is $4.625 per ton, what will be the cost of six tons?

255. The stock of a grocer cost $7963.375. For how much must he sell it, to gain $2059.50?

256. How many yards in six pieces of cloth, No. 1 measuring 31.75 yds.; No. 2 29 yds.; No. 3 30.5 yds.; No. 4 32.25 yds.; No. 5 34 yds.; and No. 6 31.875 yds.?

257. A merchant bought 169 barrels of flour for $887.25; 97 barrels for $485; 217 barrels for $1057.875; and 418 barrels for $1776.50. How many barrels did he buy, and how much did he pay for the whole?

258. The whole property of a bankrupt is worth $13079.50, which is only one-half of what he owes. How much does he owe?

259. Bought two barrels of flour at $5.875 per barrel, a keg of butter for $6.625, and a barrel of molasses for $11.50. What was the cost of the whole?

260. A draper purchased 2 pieces of broadcloth,

containing 56.5 yards, for $282.50; 3 pieces, containing 96.25 yards, for $385; 1 piece containing 19.5 yards, for $87.625; and 4 pieces, containing 80 yards for $430. How many pieces did he buy? How many yards? What did he give for the whole?

Add all the following examples without using the slate.

261—280. 18 and 11; 21 and 15; 36 and 43; 72 and 87; 40 and 69; 82 and 83; 91 and 65; 81 and 77; 58 and 51; 63 and 44; 57 and 30; 21 and 96; 12 and 94; 35 and 42; 56 and 52; 23 and 35; 73 and 16; 44 and 54; 35 and 24; 78 and 52.

281—300. 82 and 49; 98 and 93; 66 and 54; 85 and 95; 37 and 27; 83 and 59; 16 and 95; 24 and 77; 56 and 66; 48 and 84; 68 and 98; 35 and 79; 49 and 54; 66 and 77; 99 and 99; 99 and 88; 79 and 97; 38 and 86; 78 and 45; 55 and 97.

* * *

III. SUBTRACTION.

29. SUBTRACTION is the process by which we take one number from another, to find their *difference.*

The difference found by Subtraction, is sometimes called the *remainder.*

The termination *nd,* in many arithmetical terms, signifies *to be.* Thus, the *Subtrahend* is the number *to be subtracted.* The *Minuend* is the number *to be diminished.*

If the *Difference* of two numbers is added to the less number, (§ **28**, Ex. 245,) it will give the greater number. The *Remainder* is, therefore, the number which must be added to the *Subtrahend* to give the *Minuend.* Thus, if 3 be taken from 9, the remainder will be 6. And if the remainder 6, be added to the subtrahend 3, it will give the minuend 9.

9 minuend.
3 subtrahend.
—
6 remainder.

The sign — (minus,) written between two numbe shows that the latter is to be taken from the former; as, 9 — 3 = 6, 9 *minus* 3 *equals* 6.

EXERCISES FOR BEGINNERS.

1—20. Take 110 from each of the following numbers: 111; 211; 221; 222; 322; 321; 331; 332; 333; 330; 430; 431; 432; 433; 443; 444; 765; 876; 987; 988.

21—40. Take 221 from each of the following numbers: 222; 322; 332; 333; 433; 432; 442; 443; 444; 544; 554; 555; 655; 665; 666; 766; 776; 777; 877; 987.

41—60. Take 333 from each of the following numbers: 333; 433; 443; 444; 544; 554; 555; 655; 665; 666; 766; 776; 777; 877; 887; 888; 988; 998; 999; 987.

61—80. Take 444 from each of the following numbers: 444; 544; 554; 555; 655; 665; 666; 766; 776; 777; 877; 887; 888; 988; 998; 999; 1099; 1188; 1277; 1366.

81—100. Take 555 from each of the following numbers: 555; 655; 665; 666; 766; 776; 777; 877; 887; 888; 988; 998; 999; 1099; 1188; 1277; 1366; 1455; 1566; 1677.

101—120. From 999 take each of the following numbers: 111; 222; 333; 444; 555; 666; 777; 888; 999; 321; 433; 543; 654; 766; 876; 987; 500; 610; 702; 951.

30. EXAMPLES FOR THE PUPIL.

1. Take 1 from all the numbers between 1 and 11: thus, 1 from 1 = 0; 1 from 2 = 1; 1 from 3 = 2, &c.
2. Take 2 from all numbers between 2 and 12.
3. Take 0 from all numbers between 1 and 9.
4. Take 3 from all numbers between 3 and 13.
5. Take 4 from all numbers between 4 and 14.
6. Take 5 from all numbers between 5 and 15.
7. Take 6 from all numbers between 6 and 16.
8. Take 7 from all numbers between 7 and 17.
9. Take 8 from all numbers between 8 and 18.

10. Take 9 from all numbers between 9 and 19.

11. Take 10 from all numbers between 10 and 50.

12—20. Take 2 from 5; 7 from 9; 8 from 8; 6 from 9; 5 from 8; 1 from 7; 3 from 10; 9 from 14; 8 from 15.

21—30. Take 3 from 11; 2 from 11; 4 from 13; 5 from 13; 8 from 16; 7 from 16; 9 from 18; 1 from 10; 0 from 9; 7 from 7.

31—40. Take 9 from 16; 7 from 13; 8 from 14; 8 from 17; 6 from 15; 5 from 12; 5 from 14; 7 from 14; 9 from 15; 9 from 17.

41—50. Take 9 from 12; 8 from 13; 9 from 13; 8 from 12; 7 from 12; 7 from 15; 7 from 11; 6 from 13; 6 from 11; 5 from 11.

51—60. Take 5 from 10; 6 from 12; 6 from 14; 10 from 19; 8 from 10; 7 from 10; 3 from 12; 9 from 16; 8 from 14; 8 from 17.

61—70. Find the remainders in the following examples, by taking units from units, tens from tens, and hundreds from hundreds.

Minuends.	456	602	999	978	657
Subtrahends.	123	501	275	108	352
Remainders.					

1043	1190	1587	1799	1432
343	750	930	963	812

71—80. Find the remainders in the following Examples.

2579	48[illegible]	1957	9004	9875
1379	1121	1052	3002	4345

8099	8735	9999	9989	7579
4076	2031	7509	6421	7532

81—90. Find the difference between 108 and 279; 500 and 1065; 201 and 984; 370 and 1491; 2804 and 2874; 390 and 1390; 4072 and 1032; 8795 and 163; 49 and 1399; 172805 and 92401.

91—100. How many are 643—640; 487—281; 1034—702; 7485—7025; 9537—9502; 187320—94120; 7589—589; 19583—13523; 4701—600; 99889—92215.

31. EXAMPLES OF DIFFICULT SUBTRACTION.

1. If I pay 3850 dollars for a farm, and sell it for 3275 dollars, how much shall I lose by the operation?

```
3850
3275
————
 575
```

I shall lose the *difference* between what I pay and what I receive, and the difference between any two numbers, (§ **29**,) is the number which must be added to the less to make the greater. The only number of units which can be added to 5 units to make a number terminating in 0 units (§ **27**, Exs. 215—220,) is 5 units, 5 must, therefore, be written as the units' figure of the remainder. The 5 units of the subtrahend added to the 5 units of the remainder, make the 0 units of the minuend, and 1 ten to be carried to the tens. 7 tens and 1 *to carry*, are 8 tens, and the number to be added to 8 tens to make a number terminating in 5 tens, (§ **27**, Exs. 186—195,) is 7 tens, which we write in the tens' place of the remainder. 8 tens of the subtrahend, and seven tens of the remainder, make the 5 tens of the minuend, and 1 hundred to be carried to the hundreds. 2 hundreds and 1 *to carry*, are 3 hundreds, and 5 hundreds are 8 hundreds. 5 is, therefore, the hundreds' figure of the remainder. To 3 thousands of the subtrahend, 0 thousands need be added to make 3 thousands of the minuend. There are, therefore, 0 thousands in the remainder; but as the 0 is not needed to preserve the proper places of the other figures, we do not write it down.

2. *From* 2805 take 1978.

2805
1973
——
——

3 units and 2 units are 5 units. 2 is therefore the units' figure of the answer. 7 tens and three tens are 10 tens, equivalent to 0 tens and 1 hundred. 3 is therefore the tens' figure of the answer, and we have 1 *to carry* to hundreds. 9 hundreds and 1 *to carry* are 10 hundreds, and 8 hundreds are 18 hundreds, equivalent to 8 hundreds and 1 thousand. 8 is therefore the hundreds' figure of the answer, and we have 1 *to carry* to thousands. 1 thousand and 1 *to carry* are 2 thousands and 0 thousands are 2 thousands. There are, therefore, no thousands in the answer. The entire remainder is 832.

From these examples we derive the following

RULE FOR SUBTRACTION.

Write the less number under the greater, placing them as in Addition, so that units may fall under units, tenths under tenths, &c.

Commence at the right hand, and subtract each figure, if possible, from the one above it. When this cannot be done, add 10 *to the upper figure, then subtract, and add* 1 *to the next figure of the subtrahend.*

Proof. *The sum of the remainder and subtrahend will be equal to the minuend.*[*]

---

[*] For the convenience of teachers who may prefer the ordinary method of explaining subtraction, the following is inserted.

From 2805 take 1973.

2805
1973
——
832

3 units from 5 units leave 2 units. 7 tens cannot be taken from 0 tens, but if we unite 1 of the 8 hundreds to the 0 tens we shall have 10 tens, and 7 tens from 10 tens leave 3 tens. Having taken 1 of the 8 hundreds, there are but 7 hundreds left in the minuend, and 9 hundreds cannot be taken from 7 hundreds. But uniting 1 of the 2 thousands to the 7 hundreds, we obtain 17 hundreds, and 9 hundreds from 17 hundreds leave 8 hundreds. As we have taken 1 of the 2 thousands, there is but one thousand left in the minuend, and 1 thousand from 1 thousand leaves 0 thousand.

32. EXAMPLES FOR THE PUPIL.

Insert the proper figures in the blanks of the following examples.

101. From 2843 take 1772. 2 and — are 3. What must be done with the —? 7 and — are 14. What must be done with the —? 7 and — to carry are — and — are 8. What is the hundreds' figure of the remainder? 1 and — are 2. What must be done with the —?

2843
1772
———
———

102. Find the difference between 1973 and 4891. 3 and — are —. 7 and — to carry are — and — are —. 9 and — are —. 1 and — to carry are — and — are —.

4891
1973
———
———

103—110. Take 69 from 94; 87 from 113; 25 from 70; 30 from 115; 44 from 51; 73 from 111; 19 from 106; 59 from 95.

111—120. From 300 take 27; 72; 189; 240; 153; 108; 9; 11; 284; 137.

121—130. From 974 take 890; 436; 368; 779; 106; 585; 293; 968; 607; 589.

131—140. Subtract each of the following numbers from 8054. 280; 905; 1056; 633; 7959; 6372; 555; 2038; 1760; 5907.

141—150. Find the difference between 497 and 1109; 2830 and 927; 4700 and 803; 477 and 1169; 325 and 8333; 999 and 2600; 4037 and 3599; 726 and 1520; 3900 and 3604; 678 and 8764.

151—160. How many are 1079 — 690; 2203 — 1304; 1850 — 1732; 2772 — 906; 4715 — 4696; 16403 — 7761; 28340 — 18627; 30000 — 19001; 43518 — 33519; 81160 — 74063?

161—170. By how much does 118609 exceed 4720; 3680; 15529; 87362; 109576; 18610; 19575; 38673; 100881; 49997?

171—180. What is the remainder in the following examples? 2440 — 1601; 8075 — 7980; 4321 — *1234; 11000 — 9075*; 33085 — 9092; 7563 — 3824;

5159 — 2875; 10178 — 6192; 23800 — 15706; 150926 — 149028.

181—190. Find the remainder in the following examples.

84.975	476.28	.01963	5578.2	2.4907
28.483	179.19	.00975	1969.3	.8659

.57957	246.04	68.702	904.62	2.6001
.0988	195.95	19.8	807.	1.904

191—200. Solve the following examples. REMEMBER that *when the subtrahend has more places than the minuend, naughts may be annexed to the minuend,* (§ **21**, Ex. 280, Note,) as in the first example below.

27.300	.153	4.	$108.	462.5
19.264	.09288	3.9625	$ 95.875	319.75

$900.	$28.704	$44.40	6.872	$56.25
$899.19	$19.67	$39.29	1.943	$29.068

201—205. What is the difference between $48 and $38.63; $72 and $101.50; 357 and 162.94; 63 and 50.6544; $283.375 and $310?

206—215. From $1009 take $28.625; $940; $388.10; $950.875; $39.33; $480.05; $76.009; $11.05625; $9.001; $842.025.

216—220. Take $9.005 from $17; $8.625 from $11.50; $44.87 from $135; $956 from $1050.10; $.03875 from $10.

221. What is the difference between 640 and 48.0173?

222. The difference of two numbers is 84.628, and the larger number is 100.5. What is the smaller number?

223. 847.25 — 68.1499 = ?

224. $5798603 — $4878705.0625 = ?

225. $7005001.50 — $383148.2753 = ?

33. QUESTIONS TO BE SOLVED BY SUBTRACTION.

226. Subtract 338 repeatedly from 1690, until nothing remains.

227. Subtract 1907 from 11442 until nothing remains.

228. Subtract 4720 from 28335 until 15 remains.

229. How many times can 3095 be taken from 21665?

230. How many times can 5732 be taken from 46363, and how many will be left after the last subtraction?

231. John was born in 1817, and Charles in 1836. How much older than Charles is John?

232. Benjamin Franklin died in 1790, aged 84 years. In what year was he born?

233. How many years have elapsed since the discovery of America by Columbus, A. D. 1492?

234. A. is worth 14750 dollars, and B. is worth 10975 dollars. How much less is B. worth than A.?

235. If William is 23 years old, in what year was he born?

236. If I buy a farm for 1375 dollars, and sell it again for 1650 dollars, how much shall I gain by the transaction?

237. Henry has 287 cents. How many more must he get to have 500?

238. What must be added to 6873 to make 17881?

239. The difference of two numbers is 19602, and the larger number is 73807. What is the smaller number?

240. How many bushels of potatoes at 45 cents a bushel, can be bought for 360 cents? (See Ex. 229.)

241. How many barrels of flour at $5.625 per barrel, can be bought for $45?

242. How many bushels of wheat at .875 per bushel, can be bought for $9, and how much money will be left after the purchase?

243. A cargo of fruit cost $530, but it was so damaged by a storm that the owner lost $298.375 by the sale of it. For how much did he sell it?

244. A man went a journey of 195 miles in two days, travelling 97.25 miles the first day. How far did he go on the second day?

245. The annual salary of a clerk was $700, and his expenses $475.625. How much did he save?

246. If I sell a house lot for $889.41, for which I gave $797.22, how much do I make by the sale?

247. A store valued at $9050, was destroyed by fire. What was the owner's loss, there being an insurance on it, of $6887.375?

248. A farmer bought a house for $1064, paying $973.25 in produce, and the rest in money. How much money did he pay?

249. If I possess property worth $9309, and owe $3892.875, how much shall I be worth after paying my debts?

250. A merchant sold oil for $962, gaining $109.36 by the sale. How much did the oil cost?

Solve the following examples, without using the slate.

251—265. 37—17; 71—20; 62—41; 86—56; 91—11; 43—32; 70—50; 89—60; 88—38; 73—13; 99—49; 55—25; 102—72; 194—94; 84—33.

266—280. 63—21; 31—15; 70—27; 86—29; 80—32; 41—26; 92—75; 94—35; 80—16; 76—28; 71—27; 91—39; 62—19; 80—14; 58—29.

281—300. 78—34; 91—78; 82—66; 90—15; 80—18; 87—18; 73—37; 75—57; 64—29; 92—48; 43—19; 152—87; 173—98; 154—77; 113—85; 135—78; 183—96; 124—86; 197—99; 165—66.

34. MISCELLANEOUS EXAMPLES
IN ADDITION AND SUBTRACTION.

301. From the sum of 4936 *and* 7208, *take the sum of 1137, 2065, and* 6820.

302. Add 1600, 250087, 664, and 18330, and subtract 263000 from their sum.

303. Add the difference between 7963 and 4875, to the difference between 175815 and 184030.

304. From the difference between 295004 and 976003, subtract the difference between 1100908 and 637265.

305. Charles had 175 marbles. How many had he left, after giving 25 to William, 19 to George, 38 to Samuel, 49 to Edward, 41 to John, and losing 2?

306. A flour dealer had 2864 barrels in store; how many were left after selling one man 95 barrels, another 160, another 479, and another 1192?

307. A vessel starting on a voyage of 2987 miles, sailed 729 miles the first week, 697 the second, and 753 the third; how far had she still to go?

308. How many days in the first six months of a Leap Year?

309. How old was a person in 1848, who was born in 1819?

310. The distance from Boston to Albany by railroad, is 200 miles. Suppose one locomotive to have gone 68 miles from Boston towards Albany, and another 95 miles from Albany towards Boston, how far are they apart?

311. To what number must 597 be added 4 times to make 2500?

312. From what number must 306 be subtracted 5 times, to leave 173?

313. From what number must you take 5972 to leave 3840?

314. To what number must you add 89650 to make 108731?

315. A farmer paid 30 dollars for a cow, and her keeping cost him 27 dollars. From her milk he made butter worth 47 dollars, and cheese worth 19 dollars, and then sold her for 32 dollars; how much did he gain by her?

316. Received from A. $418.50, from B. $362.75,

and from C. $1198.875. Paid D. $597.63, and E. $940; how much was left?

317. A tailor charged $29.75 for a coat, $48.375 for a cloak, and $5.875 for a vest; but for defects in making, he deducted $9.28 from the bill. What remained to be paid?

318. The smaller of two numbers is 9.87564321, and their difference is 14.097738; what is the larger number?

319. The larger of two numbers is 26.0084, and their difference is 17.9742986; what is the smaller number?

320. A laborer receives $1.125 for each working day, and spends for the support of his family $.64 per day; how much can he save in a week?

321. How many yards of broad cloth at $4.25 per yard, can be bought for $34?

322. Bought a farm for $2775, and paid for buildings and stock $1938.45; for how much must I sell the whole to gain $1350?

323. My father was 29 years old when I was born; how old was he when I was 18, and how old was I when he was 57?

324. How many gallons does a cistern hold, which is filled in 11 hours, at the rate of 79.25 gallons per hour?

325. In how many hours will a cistern of 404 gallons be emptied, at the rate of 50.5 gallons per hour?

IV. MULTIPLICATION.

35. Multiplication is the process by which we find the amount of any number, when repeated a given number of times. It is a short method of performing addition, when all the numbers to be added are alike.

52	52
52	4
52	——
52	208
——	
208	

Thus, if we wish to find the cost of 4 bushels of potatoes, at 52 cents a bushel, we may either set down 52 four times, and add; or, setting down simply 52, we may multiply it by 4; 4 times 2 units is 8 units, and 4 times 5 tens is 20 tens. The answer in both cases is the same.

The numbers that are to be multiplied together are called *factors* of the result. The factor *to be multiplied*, is called the *Multiplicand*, (§ 29.) The factor by which we multiply, is called the *Multiplier*. The result obtained by multiplication, is called the *Product*. Any number that can be produced by multiplying together two other numbers, (as 16, which is equal to 2 times 8,) is called a *Composite Number*. In the example above, 52 is the multiplicand, 4 is the multiplier, and 208 the product. 4 and 52 are factors of 208· 208 is a composite number.

The sign × (multiplied by,) is used to show that two or more numbers are to be multiplied together; as, 2 × 5 = 10, 2 *multiplied by* 5 *equals* 10; 2 × 3 × 4 = 24.

• • • •
• • • •
• • • •

In forming the product of two numbers, the result will be the same whichever is employed as the multiplier. Three rows of trees with four in a row, will give the same number as four rows with three in a row, and so with any other numbers.

EXERCISES FOR BEGINNERS.

1—15. Multiply each of the following examples by 1 and by 2: 111; 121; 221; 222; 321; 322; 333; 444; 343; 432; 400; 530; 301; 420; 402.

16—30. Multiply each of the following numbers by 3: 111; 211; 221; 222; 321; 322; 323; 313; 231; 213; 303; 330; 300; 432; 531.

31—45. Multiply each of the following numbers by 4: 111; 211; 221; 222; 311; 321; 322; 212; 312, 122; 400; 402; 520; 621; 712.

46—60. Multiply each of the following numbers by 5: 100; 110; 111; 200; 210; 211; 311; 301; 401; 511; 501; 510; 611; 700; 811.

61—75. Multiply each of the following numbers by 6: 100; 101; 110; 111; 200; 201; 210; 211; 300; 401; 501; 510; 610; 611; 701.

76—90. Multiply each of the following numbers by 7: 111; 110; 201; 311; 410; 500; 601; 610; 611; 511; 711; 810; 801; 900; 911.

91—105. Multiply each of the following numbers by 8: 111; 201; 300; 410; 511; 601; 700; 810; 911; 901; 801; 811; 711; 710; 600.

106—120. Multiply each of the following numbers by 9: 111; 201; 300; 401; 511; 610; 700; 810; 911; 901; 811; 801; 701; 710; 600.

36. EXAMPLES FOR THE PUPIL.

1. Multiply 2 by all the numbers from 1 to 9; thus, 2 times 1 are 2; 2 times 2 are 4; 2 times 3 are 6, &c.

2—10. Multiply by all numbers from 1 to 9, each of the following numbers: 1; 3; 4; 5; 6; 7; 8; 9; 0.

11—20. How many are 5 times 4; 4 times 5; 3 times 7; 7 times 3; 6 times 6; 9 times 2; 2 times 9; 8 times 8; 8 times 6; 6 times 8?

21—30. Multiply 9 by 6; 6 by 9; 8 by 7; 7 by 8; 5 by 7; 7 by 5; 6 by 8; 8 by 6; 9 by 7; 7 by 9.

31—40. Multiply 8 by 4; 4 by 8; 9 by 3; 3 by 9; 7 by 7; 6 by 5; 5 by 6; 6 by 7; 7 by 6; 9 by 9.

41—50. How many are 6 times 3 tens; 6 times 30; 30 times 6; 3 times 6 tens; 3 times 60; 60 times 3; 5 times 2 tens; 5 times 20; 20 times 5; 2 times 50?

51—60. How many are 4 times 7 tens; 4 times 70; 70 times 4; 8 times 3 tens; 8 times 30; 30 times 8; 7 times 20; 2 times 70; 5 times 40; 30 times 3?

61—70. How many are 4 times 60; 60 times 4; 7 times 30; 80 times 3; 90 times 6; 4 times 90; 8 times 70; 60 times 5; 3 times 70; 80 times 8?

71—80. Multiply 6 hundreds by 2; 600 by 2; 2 by

600; 700 by 3; 4 by 500; 800 by 7; 6 by 900; 8 by 600; 900 by 9; 7 by 900.

81—90. Multiply 3 thousands by 2; 300 by 2; 4 by 2 ten-thousands; 20000 by 4; 5000 by 7; 6 by 80000; 100000 by 9; 7000000 by 8; 800 by 7; 60 by 9.

91—100. Multiply 40 by 8; 40 by 8 tens; 40 by 80; 30 by 7; 30 by 7 tens; 30 by 70; 600 by 2; 600 by 2 hundreds; 600 by 200; 500 by 400.

101—110. Multiply 200 by 3 thousands; 200 by 3000; 5000 by 100; 600 by 30; 50 by 900; 4000 by 20; 30000 by 300; 90000 by 70; 800 by 8000; 700 by 80.

111—120. Find the products of the following numbers, by multiplying the units, tens, and hundreds of the multiplicand, by the multiplier, (§ 35.)

402	513	301	723	912
2	2	3	3	3

222	801	912	510	801
4	4	4	5	5

121—130. Multiply 111 by 1; 2; 3; 4; 5; 6; 7; 8; 9; 0.

131—140. Multiply 70321 by 0; 1; 2; 3; 4;—92103 by 1; 4; 2; 0; 3.

141. What numbers are factors of 4; 10; 6; 9; 8; 20; 14; 12; 15; 18; 30; 22; 27; 24; 28; 35; 32; 33; 36; 40? (See Multiplication Table.)

142. What are factors of 50; 49; 42; 45; 48; 44; 60; 54; 55; 56; 63; 70; 64; 66; 72; 80; 77; 81?

143. What are factors of 90; 100; 99; 84; 96; 88; 108; 120; 110; 132; 144?

37. Nine square feet make one square yard. How many square feet in 5708 square yards?

$$\begin{array}{r}5708\\ 9\\ \hline 51372\end{array}$$

There will be 9 times as many feet as yards, and we therefore multiply 5708 by 9. 9 times 8 units are 72 units, or 7 tens and 2 units. Write down the 2 units, and carry the 7 tens to the remaining tens. 9 times 0 tens are 0 tens, and 7 tens to carry are 7 tens, which we set down in tens' place. 9 × 7 hundreds, are 63 hundreds = 6 thousands and 3 hundreds. Set down the 3 hundreds, and carry the 6 to thousands. 9 × 5 thousands, are 45 thousands, and 6 thousands to carry are 51 thousands, which we set down, as there are no ten-thousands to be multiplied. The answer is therefore 51372 square feet.

38. Examples for the Pupil.

144—152. Multiply 206 by 2; 3; 4; 5; 6; 7; 8; 9; 0, and add all the products together.

153—160. Multiply 492 by 2; 3; 4; 5; 6; 7; 8; 9, and add the products.

161—170. Multiply 1705 by 0; 4; 2; 7; 1; 3; 6; 8; 5; 9, and add the products.

171—180. Multiply 9180 by 2; 5; 0; 1; 8; 3; 7; 6; 9; 4, and add the products.

181—190. Multiply 80473 by 3; 1; 6; 9; 4; 0; 2; 8; 7; 5, and add the products.

191—200. Multiply 61508 by 7; 2; 5; 9; 3; 1; 0; 4; 8; 6, and add the products.

201—205. Multiply each of the following numbers by 1 ten; and by 10. 90859; 76320; 44876; 30059; 27000. (See Ex. 41—110.)

206—210. Multiply each of the following numbers by 2 tens; and by 20. 17059; 32804; 79630; 125000; 87154.

211—215. Multiply each of the following numbers by 3 tens; by 30. 12700; 30541; 16829; 22435; 99800.

216—220. Multiply each of the following numbers by 4 tens; by 40. **11930; 22435; 19071; 36642; 59876.**

221—225. Multiply each of the following numbers by 50. 20615; 94121; 38745; 68300; 90587.

226—240. Multiply each of the following numbers by 60. 30115; 23140; 47493; 90082; 57689. Multiply the same numbers by 70; by 80.

241—250. Multiply each of the following numbers by 90. 407; 670; 831; 915; 6044; 3709; 56018; 94321; 81799; 22975.

251—254. Multiply 4798 by 1 hundred; by 100; by 1 thousand; by 1000; by 1 ten-thousand; by 10000; by 1 hundred-thousand; by 100000.

255—259. Multiply 3275 by 200; by 2000; by 20000; by 200000; by 2000000.

260—264. Multiply 9180 by 300; by 3000; by 30000; by 300000; by 3000000.

265—269. Multiply 8074 by 400; 4000; 40000; 400000; 4000000.

270—274. Multiply 9317 by 500; 5000; 50000; 500000; 5000000.

275—279. Multiply 18200 by 600; 6000; 60000; 600000; 6000000.

280—284. Multiply 73520 by 700; 7000; 70000 700000; 7000000.

285—289. Multiply 105109 by 800; 8000; 80000; 800000; 8000000.

290—294. Multiply 99780 by 90000; 900; 900000; 9000; 9000000.

295—300. Multiply 26703000 by 80; by 7000; by 900; by 60000; by 300000; by 50.

301—310. Multiply 609510000 by 30; by 20000; by 4000000; by 800; by 6000; by 50; by 700000; by 900; by 80; by 5000.

39. Twenty-four hours make 1 day. How many *hours in 365 days?*

$$\begin{array}{rl} 365 & \\ 8 & \\ \hline 2920 & = 8 \times 365 \\ 3 & \\ \hline 8760 & = 3 \times 8 \times 365 \\ & \quad \text{or } 24 \times 365 \end{array}$$

As there are 24 hours in every day, in any number of days there are 24 times as many hours as there are days. In 365 days there are, therefore, 24 times 365 hours, and we obtain the answer by multiplying 365 by 24.

The multiplier in this instance is a composite number, (§ **35**,) equivalent to 3 × 8. Therefore, 24 × 365 = 3 × 8 × 365. Multiplying first by 8 we obtain 8 × 365 = 2920, and multiplying 2920 by 3 will give us 3 × 8 × 365, or 24 × 365 = 8760, the number of hours required.

Whenever the multiplier is a composite number, we may *separate the multiplier into its factors, and multiply first by one factor, then multiply the product by another factor, and so proceed until all the factors have been employed.* The final product will be the product required.

40. Exercises with Composite Multipliers.

311—320. Multiply 4027 by 14; 16; 18; 22; 24; 28; 32; 35; 15; 21.

321—330. Multiply 5903 by 27; 33; 36; 42; 44; 45; 48; 49; 54; 55.

331—340. Multiply 9418 by 56; 63; 64; 66; 72; 77; 81; 84; 88; 96.

341—350. Multiply 77.63 by 35; 54; 64; 99; 108; 110; 81; 121; 132; 144.

351—360. Multiply .0847 by 27; 42; 28; 56; 32; 48; 72; 54; 33; 88.

361—370. Multiply 1.0925 by 56; 14; 27; 16; 63; 35; 64; 42; 28; 108.

371—380. Multiply each of the following numbers by 168 = 4 × 6 × 7. 95.7; 84.3; 1.57; .909; .0881; 1.763; 28.47; .0099; 432.5; .872.

381—390. Multiply each of the following numbers by 175 = 5 × 5 × 7. .428; 390; 7.27; 45.6; 9.78; .0499; 2.081; 4993; .0947; 1108.

391—400. Multiply each of the following numbers by 945 = 3 × 5 × 7 × 9. 53.7; 68.4; 392; 4.75; .0841; .6873; 55.9; 7481; 2.049; 35.62.

41. Find the product of 47893 by 9057.

47893
9057

335251 product by 7 units
239465 product by 5 tens
431037 product by 9 thousands

433766901 product by 9057.

The product by 9057 is equivalent to the sum of the products by 7 units, 5 tens, and 9 thousands. The product by 7 *units* is 335251 *units*. The right hand figure is therefore written in units' place. The product by 5 *tens* is 239465 *tens*, which is represented by putting the right hand figure in tens' place. The 0 which would occupy units' place if the number stood by itself, may be omitted, as every figure already stands in its appropriate place. The product by 0 hundreds is 0. The product by 9 *thousands* is 431037 *thousands*, which is represented by placing the right hand figure in thousands' place. Adding the several partial products we obtain 433766901 for the entire product sought.

From the several preceding examples we may deduce the following

RULE FOR MULTIPLICATION.

If the multiplier contains but one figure. *Commence at the right hand of the multiplicand, and multiply each figure successively. Place the right hand figure of each product in the result, and add the remaining figures to the succeeding product.*

If the multiplier contains more than one figure. *Multiply the multiplicand by each figure of the* *multiplier, setting the right hand figure of each product under the figure by which you are multiplying. The sum of the several partial products will be the entire product.*

The product will have as many decimals as both its factors. (§ **43**.)

Contractions. *If there are naughts at the right hand of either factor, we may omit them in multiplying, and annex as many naughts to the product as we have omitted from both factors.* (§ **38**, Ex. 201—310.)

If the multiplier is a composite number, we may multiply by its several factors in succession. (§ **39**.)

Proof. *Multiply the multiplier by the multiplicand, and the result must be the same as the product of the multiplicand by the multiplier.* (§ **35**.)

42. Examples for the Pupil.

401—410. Multiply 321 by 17; 13; 19; 23; 29; 31; 37; 47; 59; 61.

411—420. Multiply 1507 by 53; 74; 39; 82; 61; 90; 87; 95; 39; 111.

421—430. Multiply 4826 by 101; 203; 507; 904; 608; 123; 645; 879; 400; 702.

431—440. Multiply 9570 by 236; 414; 708; 569; 958; 432; 805; 889; 357; 999.

441—450. Multiply 7489 by 7311; 4208; 3009; 8755; 1109; 2757; 3800; 9988; 7676; 4395.

451—460. Multiply 3862000 by 7940; 86020; 95300; 801400; 89000; 76030; 50902; 80089; 2738; 999000.

461—470. Multiply 59040 by 800098; 620074; 408030; 750630; 98760; 590480; 275195; 123456; 700890; 6688000.

Multiply the following examples without using the slate.

471—495. 19 by 2; 17 by 3; 18 by 4; 18 by 5; 16 by *6; 19 by 7;* 19 by 8; 19 by 9; 40 by 10; 21 *by 6; 32 by 5;* 43 by 5; 46 by 4; 65 by 5; 28 by 6;

17 by 7; 60 by 9; 140 by 8; 91 by 10; 98 by 2; 93 by 3; 44 by 4; 25 by 9; 37 by 6; 130 by 7.

496—520. 38 by 8; 68 by 7; 36 by 20; 64 by 60; 21 by 11; 70 by 25; 83 by 40; 92 by 20; 63 by 12; 84 by 11; 11 by 78; 12 by 32; 62 by 11; 16 by 15; 29 by 16; 44 by 13; 17 by 14; 14 by 35; 19 by 24; 13 by 41; 14 by 22; 33 by 19; 48 by 13; 15 by 21; 51 by 17.

43. At **$4.875 per barrel, what would 1.111 barrels of flour cost?**

It would cost **1.111 times as much as one barrel. We therefore multiply 4.875 by 1.111.**

4.875	
1.111	
4875	product by .001
4875	product by .01
4875	product by .1
4.875	product by 1
5.416125	

If we multiply in the usual way, there will be no difficulty except in determining the proper place for the decimal point. As .001 × 4 = .004, the left hand figure of the first product must be in thousandths' place. As .01 × 4 = .04, the left hand figure of the second product is in hundredths' place. For a similar reason the left hand figure of the third product is in tenths' place, and the left hand figure of the fourth product in units' place. We must, therefore, to obtain the entire product of 4.875 by 1.111, add together

4.875 × .001 = .004875
4.875 × .01 = .04875
4.875 × .1 = .4875
4.875 × 1. = 4.875 which gives

4.875 × 1.111 = 5.416125

It will be observed that in each of the partial products, as well as in the entire product, *there are as many decimals as in both factors*, as stated in the *Rule, (§ **41**.)* By recurring to the principles of Numeration, (§ **15**, Note,) it will be seen that this must always be the case. *If there are not enough figures in the product to make the requisite number of decimal places, naughts must be prefixed to supply the deficiency*, as in the following examples.

.135 × .2 = .0270 ; 64.9 × .00005 = .003245
6400 × .015 = 96.000; 28.07 × .0025 = .070175

44. EXAMPLES FOR THE PUPIL.

521—530. Multiply .27 by .6; .49 by .08; .78 by .78; .093 by .093; 85.1 by .89; 274 by .62; 930 by 1.7; .602 by .25; 99 by .056; 700 by .07.

531—540. How much are 84.7 × .018; 2.39 × .015; .687 × .09; .372 × .087; .439 × .0042; .0368 × .0592; .00961 × .0088; 735 × .00014; .36 × .0036; .872 × 1.07?

541—550. Multiply $27.95 by .06; .015; .075; .045; .18; .24; .0375; .0001875; .942; 1.763.

551—560. Multiply 69000 by .069; 870 by .043; $75 by .075; $68 by .042; $59.625 by .125; $62.50 by .0625; 920000 by .045; .091 by .00878; $630 by 4.71; $.1875 by .01875.

561—570. Multiply $5.55 by .084; $230 by .097; $64.20 by 1.43; $98 by .089; $575 by .0625; $1.666 by 274; $9.31 by 14.5; $683 by .000491; 38.97 by 1252; 91.92 by 1.47.

45. QUESTIONS TO BE SOLVED BY MULTIPLICATION.

When the value of one thing is known, and the value of any number of things is required, the answer is obtained by Multiplication. (§ **35**.)

571. How many days are there in 637 weeks?

572. How many days in 429 years?

573. A mile is 320 rods. How many rods in 730 miles?

574. There are 16 drams in one ounce, and 16 ounces in 1 pound. How many drams in 1 pound? In 490 pounds?

575. How many minutes in 8760 hours?

576. One ton is 20 hundred-weight, and 1 hundred-weight is 112 pounds. How many pounds in 19 tons?

577. If 12 men can do a piece of work in 28 days, how many men would do it in 1 day?

578. How long would it take 1 man to build a wall that can be finished in 37 days by 49 men?

579. How many yards of muslin in 93 pieces, each piece containing 37 yards?

580. Any thing that is 1 foot long and 1 foot broad, contains a square foot. How many square feet in a board 19 feet long and 1 foot broad? 19 feet long and 17 feet broad?

581. How many square feet in a room 37 feet long and 24 feet wide?

582. How many square miles in a tract of land 1897 miles long and 134 miles broad?

583. If a clock strikes 156 times in a day, how often will it strike in 59 weeks?

584. What is the price of 49 bales of sheeting, each bale containing 17 pieces, and each piece 31 yards, at 13 cents a yard?

585. 1672 × 90700 × 8041 = ?

586. If a man spends $1.125 per day, how much will he spend in a year?

587. J. H. Mason,
Bought of John Wilson & Co.,

5 yds. of Broadcloth,	at	$4.75
7 yds. of Cassimere,	at	$1.125
9 yds. of Satinett,	at	$0.875

588. Thompson & Allen,
To Westcott & Co., Dr.

To 19 oz. Quinine,	at	$3.75
" 14 lb. Opium,	at	$5.50
" 29 lb. Red Bark,	at	$1.56
" 145 lb. Gum Arabic,	at	.67

589. Geo. Norton & Co.,
To Thos. J. Smith, Dr.

To	5 reams paper,	at	$2.625
"	16000 quills,	at	$1.375 pr. 1000
"	68 gallons of ink,	at	$1.25
"	5 dozen Arithmetics,	at	.375 apiece

590. Charles Harris,
Bought of William Eaton,

39 tons of hay,	at	$16.75
163 bundles of straw	at	.08
95 bushels of potatoes,	at	.34
348 lbs. of beef,	at	.115

591. If 2.5 lbs. of wool make 1 yd. of broadcloth, how many pounds will make 48 pieces, there being 21.75 yards in each piece?

592. What will 19 tons of sugar cost at 7 cents a pound? (See Ex. 576.)

593. .06 of 1 is .06. How much is .06 of 284. .06 of 19.53?

594. How much is .575 of $284.50?

595. If the interest of $1 for a certain time is $.0875, what is the interest of $431.25 for the same time?

596. If the interest of $1 is $.06 per annum, what is the interest of $205.40 for 3.5 years?

597. If the interest of $1 is $.05 per annum, what is the interest of $199 for 4.25 years?

598. How many months in 15 years? In .25 years? In .584 years? In 1.75 years?

599. What is the amount of the following bill?

6 chests of tea, each 45 lbs.,	at	$0.625 per pound.
9 bags of coffee, each 28 lbs.,	at	$0.115 per pound.
5 barrels of sugar, each 184 lbs.,	at	$0.0975 per pound.
19 barrels of flour,	at	$5.75 per barrel.

600. What is the amount of the following bill?

4	pieces of muslin,	each 31.5	yards, at	$.125 per yard.
6	do	each 30	yards, at	$.115 " "
5	*do*	each 31	yards, at	$.11 " "
7	*do*	each 29	yards, at	$.13 " "

46. Miscellaneous Examples,

IN ADDITION, SUBTRACTION, AND MULTIPLICATION.

601. How many days in 37 weeks and 3 days?

602. How many hours are there in 3 years?

603. How many hundred-weight in 35 tons? In 49 tons and 13 hundred-weight? (See Ex. 576.)

604. How many rods in 11 miles and 149 rods? (Ex. 573.)

605. How many ounces in 14 pounds and 13 ounces? (Ex. 574.)

606. In 7 pounds, 11 ounces, and 9 drams, how many ounces? How many drams?

607. In 43 hours, 57 minutes, how many minutes? How many seconds?

608. How many square feet in two rooms, one of which is 29 feet long, and 24 feet wide, the other 27 feet long, and 19 feet wide? (Ex. 580.)

609. Multiply the difference between 49400 and 38675, by the sum of 97, 235, and 609.

610. From the sum of 1683, 574, 3097, and 5115, take the difference between 9165 and 14320, and multiply the remainder by 167 × 2009.

611. How many men will accomplish as much work in 1 day, as 16 men could in 24 days?

612. How much provision will 29 men eat in 4 months and 17 days, if each man eats 3 pounds a day?

613. Find the cost of 10 pounds of tea, at 50 cents a pound, 12 pounds at 75 cents, and 16 pounds at 62 cents.

614. What is the cost of 18 gallons of molasses at 25 cents a gallon, 20 gallons at 30 cents, 24 gallons at 35 cents, and 5 gallons of water?

615. What are a clerk's wages for 2 years and 7 months, at 29 dollars a month?

616. A. and B. are travelling in the same direction, but A. is 142 miles in advance of B. A. travels 40 miles a day, and B. 55 miles a day. How far apart will they be in a week?

617. If 9103 were set down 572 times, and the

whole were added together, what would be the amount?

618. Bought 45 hogsheads of molasses, each of which originally contained 117 gallons. How much remains, 17 hogsheads having been sold, and 59 gallons having leaked out of the remainder?

619. If the subtrahend is 5894, and the remainder 64403, what is the minuend?

620. If the minuend is 90087, and the remainder 26089, what is the subtrahend?

621. What is the difference between six dozen dozen, and half-a-dozen dozen?

622. The difference of two numbers is 431, and the greater number is 918. What is their product?

623. The sum of two numbers is 1162, and one of them is 790. What is their product?

624. From what number can 874 be taken 23 times, and leave a remainder of 275?

625. To what number must 516 be added 19 times, to make 10387?

626. Bought 16 bags of coffee, at $12.75 per bag. For how much must the whole be sold, to gain $27.25?

627. What is the duty on 3 bales of broadcloth, each bale containing 265 yards,—the duty being $1.125 per yard?

628. A man offered 14 tons of hay at $10.50 a ton, and 45 bushels of oats at $0.30 a bushel, in payment of a debt of $242.75. How much of the debts remained unpaid?

629. Four men contributed $7988.375 in partnership. A.'s contribution was $956.75; B.'s, $1872.50; C.'s, $2440.125. How much did D. contribute?

630. A grocer who had 421 kegs of butter, sold 56 kegs at $12.50 per keg; 91 kegs at $11.75; 100 kegs at $12.25; and the remainder at $11.50 per keg. How much did he receive for the whole? *Ans.* $4995.25

631. An estate was divided among 4 heirs. The *first received $125*; the second, three times as much *as the first; the third*, as much as the first and second;

and the fourth, twice as much as the third. What was the whole estate, and each man's share ?

632. A bankrupt can pay only $0.4375 on every dollar of his debts. What is the worth of a debt of $160?

633. The salary of a clerk was $95 per month, and his family expenses $2.75 per day. How much did he save in a year ?

634. If I buy goods to the amount of $1176.06, pay for storage $23.74, for commission $54.16, and for interest $49.11, and sell the whole for $1293.97, how much do I gain or lose ?

635. What is the cost of 87.25 yards of broadcloth, at $4.75 per yard ?

636. Find the cost of 15 hogsheads of molasses, at 9 cents a quart, there being 4 quarts in a gallon, and 63 gallons in a hogshead.

637. What are the annual expenses of a man who pays for house-rent $25 a month, provisions $1.25 a day, fuel $37.50 a year, and for other purposes $ .625 a day ?

638. The product of two equal factors is called the *second power* of the repeated factor. Thus, 2 × 2 = 4 is the second power of 2. What is the second power of 1860 ?

639. In the *third power* of a number there are three equal factors, in the *fourth power* four equal factors, and so on. What is the third power of 709 ?

640—645. What is the fourth power of 6.04 ? the 3d power of 72.5 ? the 5th power of 14 ? the 6th power of 19 ? the 7th power of .21 ? the 2d power of 9.706 ?

646—649. What is the 10th power of 9 ? the 15th power of 10 ? the 5th power of .57 ? the 4th power of 1.09 ?

650. Multiply the sum of the 3d power of 10, and the 4th power of 17, by the difference between the 5th power of 10, and the 3d power of 46.

651. How much will 562 men eat in 2 years and 11 days, if each man eats 2 pounds per day ?

V. DIVISION.

47. Division is the process by which we find how many times one number is contained in another. It is a short method of performing many subtractions, when all the numbers to be subtracted are alike.

$$\begin{array}{r} 15\\ \underline{5}\\ 10\\ \underline{5}\\ 5\\ \underline{5}\\ 0\end{array}\qquad \begin{array}{r} 5)\underline{15}\\ 3\end{array}$$

Thus, if we wish to find how many barrels of flour at 5 dollars a barrel, can be bought for 15 dollars, we may either set down 15, and subtract 5 from it as often as we can, (supposing ourselves to pay for 1 barrel at each subtraction,) or set down 15, and placing 5 at the left hand, say at once, 5 is contained in 15, 3 times. The answer is the same in both cases.

The number *to be divided*, is called the *Dividend* (§ **29**). The number by which we divide, is called the *Divisor*. The number of times the divisor is contained in the dividend, is called the *Quotient*. The number left after division, (if any,) is called the *Remainder*. As the quotient shows *how many times* the divisor the dividend is, the divisor and quotient are factors of the dividend. In the example above, the dividend is 15, the divisor is 5, the quotient is 3, and the remainder is 0.

The sign ÷ (divided by) signifies that the former of two numbers is to be divided by the latter, as, $4 \div 2 = 2$; $16 \div 4 = 8 \div 2$.

Division is also expressed by writing the divisor under the dividend, as $\frac{4}{2}$, or by placing a colon between the dividend and the divisor, as 4 : 2. Thus, $\frac{7}{8}$, $7 \div 8$, 7 : 8, may each be read 7 *divided by* 8. Whenever such expressions as $\frac{3}{4}$ or 3 : 4, $\frac{7}{9}$ or 7 : 9, occur in the present chapter, the pupil should always say 3 *divided by* 4,—not 3 *fourths*; 7 *divided by* 9,—not 7 *ninths*. For other modes of reading these expressions, see ***§ 70, 130.***

Since (§ **35**) $5 \times 3 = 3 \times 5$, 5 can be subtracted 3 times from 15, and 3 can be subtracted 5 times. In other words, 5 is contained 3 times in 5×3, and 3 is contained 5 times in the same number. Therefore, when the product of two factors, and one of the factors are given, to find the other factor, *divide the product of the two factors by the given factor.*

When the product of more than two factors, and all of the factors but one are given, to find the remaining factor, *divide the given product by the product of all the given factors.* Thus, $4 \times 2 \times 3 \div 4 \times 3 = 2$; $4 \times 2 \times 3 \div 4 \times 2 = 3$; $4 \times 2 \times 3 \div 2 \times 3 = 4$.

EXERCISES FOR BEGINNERS.

1—10. Divide each of the following examples by 1, and by 2. 222; 422; 442; 444; 446; 466; 666; 688; 888; 1088.

11—20. Divide each of the following examples by 3. 333; 336; 366; 666; 669; 999; 369; 1233; 1566; 1899.

21—30. Divide each of the following numbers by 4. 444; 448; 488; 888; 1244; 1600; 2008; 2488; 2844; 3280.

31—40. Divide each of the following numbers by 5. 555; 550; 500; 1050; 1005; 1555; 2050; 2005; 2550; 3055.

41—50. Divide each of the following numbers by 6. 666; 1260; 1200; 1806; 1866; 2406; 3060; 3066; 3600; 4206.

51—60. Divide each of the following numbers by 7. 777; 770; 1407; 1400; 2100; 2877; 2870; 3507; 3500; 4277.

61—70. Divide each of the following numbers by 8. 888; 808; 1680; 1688; 2400; 2480; 3288; 4000; 4808; 5688.

71—80. Divide each of the following numbers by 9. 999; 909; 1890; 1800; 2799; 3609; 4590; 5400; 5499; 6309.

48. EXAMPLES FOR THE PUPIL.

1. Repeat the Division Table.

2—10. Divide 14 by 7; 18 by 6; 12 by 3; 15 by 5; 16 by 4; 6 by 2; 18 by 9; 27 by 3; 36 by 9.

11—20. Divide 21 by 3; 49 by 7; 64 by 8; 36 by 6; 56 by 7; 63 by 9; 81 by 9; 25 by 5; 72 by 9; 63 by 7.

21—30. Divide 45 by 5; 36 by 4; 48 by 6; 56 by 8; 54 by 9; 72 by 8; 32 by 4; 54 by 6; 48 by 8; 40 by 5.

31—40. How many tens are 4 tens divided by 2; 8 tens by 4; 9 tens by 3; 16 tens by 4; 27 tens by 3; 45 tens by 9; 56 tens by 7; 56 tens by 8; 25 tens by 5; 32 tens by 4?

41—50. (See Ex. 31—40.) How many are 40 divided by 2; 80 by 4; 90 by 3; 160 by 4; 270 by 3; 450 by 9; 560 by 7; 560 by 8; 250 by 5; 320 by 4?

51—60. How many times are 2 tens contained in 4 tens; 3 tens in 9 tens; 7 tens in 14 tens; 5 tens in 15 tens; 3 tens in 18 tens; 7 tens in 63 tens; 8 tens in 40 tens; 9 tens in 27 tens; 8 tens in 24 tens; 3 tens in 24 tens?

61—70. (See Ex. 51—60.) Divide 40 by 20; 90 by 30; 140 by 70; 150 by 50; 180 by 30; 630 by 70; 400 by 80; 270 by 90; 240 by 80; 240 by 30.

71—80. Divide 6 hundreds by 3; 12 hundreds by 4; 18 hundreds by 2; 27 hundreds by 9; 16 hundreds by 4; 20 hundreds by 5; 36 hundreds by 4; 36 hundreds by 6; 36 hundreds by 9; 49 hundreds by 7.

81—90. Divide 8 thousands by 4; 12 thousands by 2; 15 thousands by 5; 18 thousands by 6; 21 thousands by 3; 24 thousands by 4; 24 thousands by 6; 24 thousands by 8; 54 thousands by 9; 54 thousands by 6.

91—100. Divide 400 by 2; 400 tens by 2 tens; 4000 by 20; 400 hundreds by 2 hundreds; 40000 by 200; 400 thousands by 2 thousands; 400000 by 2000; 27 tens *by 3: 270* tens by 3 tens; 2700 by 30.

101—110. (See Exs. 31—100.) Divide 180 by 3:

180 by 30; 180 by 6; 180 by 60; 810 by 9; 810 by 90: 540 by 6; 540 by 9; 540 by 60; 540 by 90.

111—120. Divide 4200 by 7; 560 by 8; 63000 by 7; 6300 by 9; 56000 by 7; 720 by 9; 4900 by 7; 810 by 9; 54000 by 6; 6400 by 8.

121—140. Solve each of the following examples by dividing the hundreds, tens, and units of the dividend by the divisor.

2)462	2)408	2)246	2)884	2)1000
3)300	3)603	3)963	3)969	3)2100
4)444	4)408	4)848	4)2840	5)505
6)666	6)1806	5)455	4)328	3)279

141—150. Divide each of the following numbers by 2. 4268; 8040; 6688; 10804; 1462; 1646; 1804; 1444; 1426; 18008.

151—160. Divide each of the following numbers by 3. 3663; 9630; 6099; 1233; 2109; 1593; 2799; 1869; 2406; 180396.

161—170. Divide each of the following numbers by 4. 4004; 8008; 2808; 2448; 1284; 1644; 2088; 3640; 2404; 3248.

171—180. Divide each of the following numbers by 5. 5055; 1055; 2050; 3005; 1550; 3500; 2555; 4000; 4505.

181—190. Divide each of the following numbers by *6, by determining* how many hundreds, how many *tens, and how many* units there must be in the quotient.

1266; 1806; 3624; 3042; 4860; 5448; 6042; 1230; 4848; 5436.

191—200. Divide each of the following numbers by 7. 2107; 1477; 2814; 735; 4263; 5670; 4928; 6356; 3577; 5614.

201—210. Divide each of the following numbers by 8. 1688; 3208; 824; 4024; 7256; 6432; 872; 5680; 3264; 8048.

211—220. Divide each of the following numbers by 9. 2718; 963; 3690; 5499; 1845; 6381; 972; 9963; 4554; 7236.

49. How many barrels of flour at 7 dollars a barrel, can be bought for 2440 dollars, and how much will be left after the purchase?

7)2440
———
348+4

We can buy as many barrels as there are times 7 dollars in 2440 dollars. We therefore divide 2440 by 7. The 2 in thousands' place will not contain 7, we therefore join it to the 4 hundreds. 24 hundreds divided by 7, give 3 hundreds, and there are 3 hundreds left undivided, which we join to the 4 tens, making 34 tens. 34 tens divided by 7, give 4 tens, and there are 6 tens over, which we join to the 0 units, making 60 units. 60 units divided by 7, give 8 units, and there are 4 units left after completing the division. We can therefore buy 348 barrels, and have 4 dollars left.

The above method is called SHORT DIVISION.

50. EXAMPLES FOR THE PUPIL.

221—230. Divide each of the following numbers by 2. 1708; 8114; 7218; 1356; 9750; 3374; 5936; 2790; 4538; 9978.

231—240. Divide each of the following numbers by 3. 2259; 1425; 7137; 4134; 5022; 8325; 7017; 2232; 8808; 2955.

241—250. Divide each of the following numbers by 4. 1036; 5108; 7024; 9332; 2740; 9996; 8768; 5432; 3456; 7892.

251—260. Divide each of the following numbers by

5. 1790; 8215; 9445; 3780; 19320; 4685; 32705; 88955; 9870; 6795.

261—270. Divide each of the following numbers by 6, and find the remainders. 8729; 1435; 5901; 42080; 73519; 44750; 70770; 89750; 9990; 43125.

271—280. Divide each of the following numbers by 7. 5235; 8590; 9763; 4325; 11990; 25786; 18494; 6789; 6044; 53072.

281—290. Divide each of the following numbers by 8. 25109; 63000; 9007; 3725; 5227; 4659; 7090; 73195; 33333; 70709.

291—300. Divide each of the following numbers by 9. 2468; 11350; 8749; 19599; 84000; 87654 32101; 48757; 6087; 7015.

51. Divide an estate of 47500 dollars equally among 29 heirs, and determine the share of each.

```
29 × 1 = 29        29)47500(1637 27/29.
29 × 2 = 58           29
29 × 3 = 87           ——
29 × 4 = 116          185
29 × 5 = 145          174
29 × 6 = 174          ———
29 × 7 = 203           110
29 × 8 = 232            87
29 × 9 = 261           ———
                        230
                        203
                        ———
                         27
```

It would require 29 dollars to give each of the heirs 1 dollar. We can therefore give as many dollars to each, as there are times 29 in 47500, which is found by dividing 47500 by 29. Before commencing the division, it will often be found convenient to multiply the divisor by each of the 9 digits and set down the products, as in the example above.

Where the divisor is large, we usually *write down* all the multiplications and subtractions, instead of *performing them mentally*, as in § **50**. We always commence by taking as many figures of the dividend as will contain the divisor one or more times. By the table we see that 47 will contain 29 only once. We then say, 29 in 47 *thousands*, 1 *thousand* times. Write 1 as the thousands' figure of the quotient, and subtract 29 × 1 *thousand* from 47 *thousands*, leaving 18 thousands undivided. To the 18 *thousands* annex the 5 *hundreds* of the dividend, making 185 *hundreds*. Referring to the table, we see that 29 is contained in 185 *hundreds*, 6 *hundreds* times. Write 6 as the hundreds' figure of the quotient, and subtract 29 × 6 *hundreds* from 185 *hundreds*, leaving 11 hundreds, undivided. To the 11 *hundreds* annex the 0 *tens* of the dividend, making 110 *tens*. 29 in 110 *tens*, 3 *tens* times. Write 3 as the tens' figure of the dividend, and subtract 29 × 3 *tens* from 110 *tens*, leaving 23 tens still undivided. To the 23 *tens* annex the 0 *units* of the dividend, making 230 *units*. 29 in 230 *units*, 7 *units* times. 7 is the units' figure of the dividend, and subtracting 29 × 7 *units* from 230 *units*, we have 27 units still undivided. But as the whole property is to be divided among the heirs, instead of leaving 27 as a remainder, we divide 27 by 29, representing the division as in § **47**; thus, $\frac{27}{29}$. Each share is, therefore, $1637\frac{27}{29}$ dollars.

The foregoing method is called LONG DIVISION.

From the explained examples we may deduce the following

RULE FOR DIVISION.

Write the divisor at the left of the dividend. Take as many figures of the dividend as will contain the divisor one or more times, for a first partial dividend. Find how many times the partial dividend will contain the divisor, and write the result as the first quotient figure. Multiply the divisor by this figure, and subtract the product from the first partial dividend.

To the remainder annex the next figure of the dividend for a second partial dividend, and proceed *as with the first partial dividend.* Thus continue *until the division is complete, being careful to place either* 0 *or one of the* 9 *digits in the quotient, for each figure of the dividend brought down.*

As the divisor and quotient are factors of the dividend, (§ **47**,) *there must be as many decimals in the dividend as in both the divisor and the quotient.* (§ **43**, **53**.)

If there are naughts at the right hand of the divisor, *we may cut them off, and divide the tens, hundreds, or thousands, &c., of the dividend, by the tens, hundreds, or thousands, &c., of the divisor.* (Exs. 51—110.)

Proof. *Add the remainder to the product of the divisor by the quotient, and you will obtain the dividend.*

52. Examples for the Pupil.*

301—320. Divide 87040 by all the numbers from 2 to 21. [In all the remaining exercises under this rule, express the division of the remainder, as in the Example § **51**, unless otherwise directed.]

321—330. Divide 194085 by all the numbers from 22 to 31.

331—340. Divide 904700 by all the numbers from 91 to 100.

341—350. Divide 487256 by all the numbers from 49 to 58.

---

* If the pupil has not prepared a table of the products of the divisor, as in § 51, he may be taught to employ the following rule whenever he is at a loss to determine the true quotient figure. *Add* 1 *to the first figure of the divisor, and find how many times the sum is contained in the figures of the* TRIAL *dividend which are of the same denomination as the first divisor figure. The result is either the true quotient figure, or some number smaller than the true figure.* If, for example, the first quotient figure were required in dividing 399402 by 4398, our trial dividend would be 39940. Adding 1 to the first divisor figure, would give us 5 *thousands*, which would be contained in 39 *thousands* 7 times. The quotient figure would, therefore, be *at least* 7, and we need try no number less than 7. At the third trial *it will be found that* 9 is the true quotient figure.

```
4398)399402(9
      39582
     ------
```

351—360. Divide 364720 by all the numbers from 360 to 369.

361—365. Divide 247093 by 158; 372; 609; 243; 247.

366—370. Divide 1593628 by 159; 951; 591; 863; 430.

371—375. Divide 83009101 by 1640; 3702; 8297; 7533; 4009.

376—380. Divide 55960372 by 2931; 5608; 5584; 9370; 8151.

381—385. Divide 70908536 by 1190; 4387; 8438; 9295; 7536.

386—390. Divide 66998877 by 6701; 8408; 7956; 4329; 9448.

391—395. Divide 30047912 by 4008; 1964; 2999; 8765; 9312.

396. Divide 725489090 by 180327.

397. Divide 4087736069 by 527590.

398. Divide 93615904372 by 9998764.

399. Divide 142073659980 by 2631972.

400. Divide 8594726880593 by 49078323.

53. Divide 7 by .013.

```
.013)7.000(538.46 +
     6 5..
     ----
      50.
      39.
      ----
      110
      104
      ----
        60
        52
        ----
         80
         78
         --
          2
```

As division teaches us how many times one number can be subtracted from another, (§ **47**,) the quotient in this instance will show how many times .013 can be subtracted from 7. But we can continue to subtract until we obtain a remainder less than .013; we must therefore continue to divide until we obtain a remainder less than .013, to do which, we must set down naughts for the tenths', hundredths', and thousandths' figures of the

[illegible]

54. [illegible]

[illegible] .18 : [illegible]

[illegible] .03 ; .007 ; [illegible]

[illegible] .035 : .154 ; [illegible]

431—440. Divide .97 by [illegible] .004 ; 8 ; .016 : .32 ; .125 ; .675 : .019 ; .0017 : .153.

441—450. Divide .021 by 3 ; 70 : 500 : 12.5 : 320 ; .16 ; 80 ; 3.75 ; 875 ; 1680.

451—460. Divide $2.40 by .04 ; .8 ; 300 ; 32 ; 4.8 ; $4.80 ; $ .01 ; 192 ; 8000 ; $38.40.

461—470. Divide \$13.75 by 125; \$12.50; \$25; \$275; 880; 17.6; .0704; \$.025; 16000; \$19800.

471—490. Solve the following examples without using the slate. 6.9 ÷ 3; 7.4 ÷ .2; 64 ÷ .2; .84 ÷ .4; .66 ÷ .02; 82 ÷ .02; 9.3 ÷ .03; .77 ÷ .7; .039 ÷ .3; 40 ÷ .002; .028 ÷ .7; .55 ÷ 50; 90 ÷ 5; 8.4 ÷ 7; 560 ÷ .8; .91 ÷ .7; 7.8 ÷ 6; 42 ÷ .03; .85 ÷ 5; 96 ÷ .02.

491—520. .76 ÷ 40; 1.26 ÷ 7; .387 ÷ 9; 26.4 ÷ .8; 175 ÷ .7; 5.34 ÷ .6; .192 ÷ .8; .644 ÷ .07; 594 ÷ .09; .324 ÷ 6; 5.28 ÷ 60; 792 ÷ .08; 45 ÷ .005; 483 ÷ 7; 459 ÷ .9; 6.72 ÷ .8; 88.3 ÷ .9; 770 ÷ 1.1; 770 ÷ .7; 7.7 ÷ 70; 440 ÷ 11; 5.94 ÷ 11; 6.03 ÷ .09; 630 ÷ .9; 306 ÷ .09; .171 ÷ .9; 752 ÷ .8; 5.95 ÷ .7; 4.28 ÷ 10; 65.1 ÷ 100.

55. QUESTIONS TO BE SOLVED BY DIVISION.

The value of any number of things, (§ **35**,) is equivalent to the value of one of the things multiplied by the number of things. The value of a number of things may therefore be considered as the product of two factors, one of which is *the value of one of the things*, and the other is *the number of things.*

When the product of two factors and one of the factors are given, (§ **47**,) to find the other factor, we divide the given product by the given factor. Hence,—

When the value of any number of things is given, to find the value of one of the things, *divide the given value by the number of things.*

When the value of a number of things is given, to find the number of things, *divide the given value by the value of one of the things.*

521. If a locomotive can go 513 miles in 19 hours, how far can it go in 1 hour?

522. If 13 barrels of flour cost 6175 cents, what is the cost of 1 barrel?

523. How many acres of land, at 19 dollars an acre, can be bought for 3610 dollars?

524. How many hogsheads in a cistern holding 7875 *gallons, a hogshead* being 63 gallons?

525. How long will it take a ship to sail 3857 miles, at the rate of 133 miles per day?

526. How many poor people will 7733 pounds of flour supply, allowing 19 pounds to each?

527. How many pounds in 14000 ounces, there being 16 ounces in a pound?

528. How many days in 4632 hours?

529. If 49 horses eat 1813 bushels of oats in a year, how much will 1 horse eat in the same time?

530. If a cart wheel turns 4693 times in going 13 miles, how many times will it turn in 1 mile?

531. What number multiplied by 109 will give 106602?

532. How many times can 124 be subtracted from 74167, and how much will be left?

533. What number can be subtracted 235 times from 257795?

534. If the product of two factors is 46320, and one of those factors is 24, what is the other?

535. Four farthings make a penny. How many pence and farthings in 79051 farthings?

536. Twelve pence make a shilling. How many shillings and pence in 558093 pence?

537. Twenty shillings make a pound. How many pounds and shillings in 443567 shillings?

538. In 4732580 farthings, how many pence? how many shillings? how many pounds?

539. In 884932 seconds, how many minutes? how many hours? how many days?

540. In 564009 inches, how many feet? how many yards?

541. In how many days can I travel 3393 miles, if I travel 39 miles per day?

542. How many acres of land would yield 8019 bushels of wheat, at 27 bushels per acre?

543. How many years in 15097 days?

544. A barrel of flour is 196 pounds. How many barrels will hold 640920 pounds?

545. The product of two factors is 113643. One of the factors is 207, what is the other?

546. 379014 = 543 × — ?

547. Two pints make a quart, and four quarts make a gallon. How many gallons in 647128 pints?

548. How many pounds, shillings, and pence, in 79084 pence?

549. Paid $28 for 16 barrels of apples. What was the price per barrel?

550. A farmer sold wheat at $1.13 per bushel, receiving for the whole $89.27. How many bushels did he sell?

551. There are 37 heirs to an estate of $16250. What is the share of each?

552. How many sheep at $4.75 a piece, can be bought for $921.50?

553. What is the price of broadcloth per yard, if 138 yards cost $155.25?

554. A journeyman charged $19.74 for making 21 pairs of boots. How much did he charge per pair?

555. How long will it take 27 men to do a piece of work, that 1 man can do in 729 days?

556. If I pay $34.65 for 33 bushels of wheat, how much must I pay for 1 bushel?

557. A merchant sold 45 pounds of indigo for $49.05. How much did he receive per pound?

558. How many kegs of butter can I buy for $198.875, if I pay $5.375 per keg?

559. When potatoes are 25 cents a bushel, how many bushels can be bought for $66.39?

560. What is the price of a yard of calico, if 4.091 yards cost $ .511375?

561. How much molasses at $ .375 per gallon, can be bought for $23?

562. A piece of satin measuring 29.5 yards, cost $47.50. What was it worth per yard?

56. MISCELLANEOUS EXAMPLES

INVOLVING ALL THE FOREGOING RULES.

563. Add 491, 1615, 380, and 907, subtract 2533 from the sum, multiply the remainder by 97, and divide the product by 113.

564. Add 45 × 25, 13 × 193, 864 ÷ 27, and 396 − 99.

565. Nine boys had 153 apples. If they were equally divided, how many would each have?

566. How many hours in six years?

567. If there is one death every second, how many persons die in a week?

568. If 76 hogsheads of molasses cost 2850 dollars, what is the price of 1 hogshead?

569. How many bales in 42253 yards, if each bale contains 29 pieces, and each piece measures 31 yards?

570. How many dozen in 21735?

571. A banker received 1378 dollars from one man, 2609 from another, and 1987 from another, and divided the whole equally among his five children. How much did he give to each?

572. A laborer receives 46 dollars a month, and expends 1 dollar a day. How much can he save in a year?

573. The smaller of two numbers is contained 15 times in 60, and the larger number is 19 times the smaller. What are the numbers?

574. Bought 4 barrels of flour at 675 cents a barrel, and 17 bushels of wheat at 127 cents a bushel. How much was gained, or lost, by selling the whole for 3875 cents?

575. If I pay 4 dollars a barrel for 39 barrels of flour, and 7 dollars for porterage, for how much must I sell the whole to gain 45 dollars?

576. If I give 153 barrels of flour worth 6 dollars a barrel, in exchange for 54 acres of land, how much is the land worth per acre?

577. What is the difference between 19 × 387 and 138159 ÷ 17?

578. Bought 537 pounds of sugar at 6 cents a pound, and sold the whole for 4296 cents. What was the gain per pound?

579. A. earns 50 cents a day, and B. 75 cents. In how many days will they both earn 4875 cents?

580. On the face of a brick wall there are 10882

bricks. How many are there in each row, there being 49 rows?

581. When eggs are 24 cents a dozen, how many can I buy for 1238 cents?

582. How many cows worth 28 dollars apiece, must be given in exchange for 84 tons of hay at 19 dollars a ton?

583. The quotient is 73, and the divisor 25. What is the dividend?

584. The dividend is 35250, and the quotient 375. What is the divisor?

585. Of what number is 37 both divisor and quotient?

586. If I exchange a watch worth 28 dollars, and 5 tons of hay worth 16 dollars a ton, for a horse, and 23 dollars in money, how much does the horse cost?

587. If I buy two pieces of broadcloth of equal length, for 5 dollars a yard, and gain 45 dollars by selling one piece for 240 dollars, what is the length of each piece?

588. At what price per yard can I sell the remaining piece, so as neither to gain or lose by the whole transaction?

589. A drover bought an equal number of cows and calves, for 703 dollars, giving 24 dollars apiece for the cows, and 13 dollars apiece for the calves. How many did he buy of each?

590. A man distributed 108 apples to some boys and girls, giving one to each boy, and two to each girl, and there were twice as many boys as girls. How many were there of each?

591. There are three farms, the first of which has 79 acres, the second 16 acres less than the first, and the third is as large as the other two. What is the value per acre, if the whole are worth 6532 dollars?

592. How many times $\frac{72}{4}$ in 36 × 28?

593. If my expenses are $1.50 per day, and my income $13 a week, how long will it take me to save $125.

594. Bought 294 yards of muslin, and sold 23 yards

to one man, 79 yards to another, and 37 yards to a third. What is the rest worth at $.125 per yard?

595. Bought 287 pounds of cheese for $29.375. For how much must it be sold per pound, to gain $6.50?

596. In 4340 farthings, how many pounds, shillings, and pence?

597. What is the nearest number to 591 that can be divided by 28 without a remainder?

598. What number besides 37 will divide 2923 without a remainder?

599. How many pence in 24 pounds, 7 shillings, and 6 pence?

600. If 2 skeins of silk cost 10 cents, what will 7 skeins cost?

601. If 3 yards of sheeting cost 30 cents, what will 8 yards cost?

602. If 5 yards of broadcloth cost $18.75, what will 7 yards cost?

603. If 3 men eat 12 pounds of meat in a week, how much would 8 men eat in the same time?

604. How much would 7 men eat in 2 weeks?

605. If a stock of provisions would last 5 men 3 weeks, how long would it last 1 man? 3 men?

606. How long will it take 2 men to do as much work as 3 men can do in 6 days?

607. If a bushel of grain will last 6 horses a week, how long will it last 2 horses?

608. How long will it take a man to save $1000, if he saves $6.25 a week?

609. A merchant bought 153 barrels of flour at $4.75 per barrel. How much did it cost him?

610. He paid for it 6 shares of stock worth $113.50 a share, and the rest in money. How much money did he pay?

611. He paid for storage $13.75, and shipped the flour to England at a freight of 75 cents a barrel. What were the expenses?

612. The flour was sold, 49 barrels at $6.50, and the remainder at $6.25 per barrel. What were the entire receipts?

613. Deducting $13.75 for commissions, and $29.30 for interest, how much did the merchant receive.

614. How much did he make by the transaction?

615. If 4 yards of cloth cost $12, how much will 6 yards cost?

616. If 3 tons of coal cost $15, how many tons can be bought for $35?

617. If 11 days' work cost $13.75, how many days' work can be obtained for $25?

618. If 7 men can build a road in 14 days, in what time could 2 men build it?

619. How many potatoes at 35 cents a bushel, will pay for 7 weeks' board, at $2.50 a week?

620. If 3 men earn $24 in 2 weeks, how much would 4 men earn in 3 weeks?

621. How much will 33 men eat in a year, if each man eats 5 pounds a day?

622. Mix 17 gallons of vinegar at 25 cents a gallon, 39 gallons at 22 cents, 21 gallons at 27 cents, and 18 gallons of water, and find the cost of the whole?

623. For how much per gallon must I sell the mixture, in the last example, in order to gain $4?

624. A grocer sold 421 kegs of butter for $4995.25, 56 kegs of which were sold at $12.50 per keg, 91 kegs at $11.75, and 100 kegs at $12.25. At how much per keg did he sell the remainder?

625. A bankrupt pays at one time .29, at another .31, and at another .15 of his debts. How much does he pay in all, on a debt of $1137.50?

626. Divide the 8th power of 6 by the 7th power of 3.

627—650. Perform the division which is expressed by the following numbers, by adding decimal naughts to the dividend. $1 \div 2$; $\frac{1}{4}$; $\frac{2}{4}$; $\frac{3}{4}$; $1:5$; $2 \div 5$; $\frac{3}{5}$; $\frac{4}{5}$; $\frac{3}{6}$; $\frac{1}{5}$; $\frac{3}{8}$; $\frac{7}{16}$; $9:32$; $11:25$; $2:3$; $\frac{7}{11}$; $8:9$; $1:99$; $\frac{33}{64}$; $\frac{7}{125}$; $\frac{9}{128}$; $\frac{5}{7}$; $\frac{10}{11}$; $\frac{2}{13}$.

651—675. Perform the division expressed by the following examples. $\frac{126}{5}$; $126:7$; $\frac{64}{8}$; $229 \div 13$; $85:11$; $\frac{5}{6}$; $8:9$; $\frac{97}{5}$; $\frac{37}{8}$; $\frac{4}{11}$; $7:12$; $24:9$; [illegible]; [illegible]; $\frac{7}{8}$; $\frac{95}{16}$; $\frac{11}{16}$; $\frac{19}{4}$; $\frac{4}{19}$; $\frac{163}{17}$; $\frac{48}{13}$; $\frac{9}{16}$; $\frac{5208}{19}$; $\frac{7524}{11}$; [illegible].

QUESTIONS ON THE FUNDAMENTAL RULES.

What is ARITHMETIC? What is a number? In what different ways may numbers be expressed? What are the digits, and why are they so called? What different names has the character 0? Can you give any reason for calling naught the figure of place? What are the fundamental operations of Arithmetic? What is given, and what is required in each?

What is NUMERATION? What method of numeration is generally adopted? What is the peculiarity of that method? What is a unit? What is the difference between a *place* and a *period?* What names are given to the places in each period? How do you read units?—tens?—tens and units?—hundreds?—hundreds, tens, and units? Repeat the names of the first eight periods.* How do we read whole numbers? Can the Arabic method represent numbers less than a unit? What are such numbers called? How are they distinguished from whole numbers? How do we read decimals?

What is the effect of removing a figure two places to the left?—two places to the right?—one *period* to the left?—to the right?—two periods to the left?—to the right? What is meant by Notation? How do you write whole numbers? In writing five sextillion, thirty billion, and seven thousand, what would you place in each period? If you were to omit the naughts in any one period, what effect would it have on all the figures above it? How do you write decimals? How would you make 5702 represent hundredths?—millionths?—thousands?—thousandths? What is the effect of placing naughts at the right hand of decimals? —at the left hand?—at the right hand of integers?*—at the left hand?

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* The Numeration Table may be continued to any extent we please. The periods above Sextillions, to Vigintillions, are: Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, Tredecillions, Quatuordecillions, Quindecillions, Sexdecillions, Septendecillions, Octodecillions, Novemdecillions, Vigintillions. The following illustration will show how difficult it is for us to form any distinct idea of the value of very large numbers.

If every man, woman, and child, on the face of the globe, were to count at the rate of four every second, without ceasing day or night, *the whole amount of all that they could count in 5000 years, would be less than one sextillion.*

If the sun, all the planets, and all the fixed stars, that are visible through the most powerful telescope, were reduced to a powder so fine that 100000 grains would be less in bulk than a drop of water, and if one of these grains were destroyed in every million years, *the whole visible universe would probably be annihilated in less than one vigintillion years.*

What is Federal Money? How may it be written? What is the probable origin of the sign that is usually prefixed to dollars? How may dollars be reduced to cents?—to mills? How may cents be reduced to dollars?—to mills? How may mills be reduced to cents? —to dollars? If there are decimals below mills, how may they be read?

Since removing the decimal point in any number changes the place of each figure, what is the effect of removing the decimal point two places to the left?—to the right?—three places to the left?—to the right?—seven places to the left?—to the right? Then how can you most readily find, 10, 100, or 1000 times any number?—.1, .01, .001, or .0001 of any number?

What is ADDITION? What sign is used to denote addition? What is the sign of equality? How do you arrange numbers that are to be added together? Where do you commence the addition? What do you do with the sum of each column? Why do you not always place the whole sum of a column underneath? How do you find whether your answer is correct? Can any one of a series of numbers be greater than the sum of the whole series?

What is SUBTRACTION? What is the remainder?—the subtrahend?—the minuend? What is the signification of the termination *nd* in many arithmetical terms? If you add the difference of two numbers to the less number, what will you obtain? If you take the difference from the greater number, what will you obtain? How do you find the minuend, if the remainder and subtrahend are given?—the subtrahend, if the remainder and minuend are given? What is the sign of subtraction? Of what operation is subtraction the opposite? How do you write numbers in subtraction? Where do you begin to subtract? If any figure is greater than the one above it, what may be done? Explain the reason of this. What may be done when the subtrahend has more decimal places than the minuend? How do you prove subtraction? In finding the difference between two numbers, which must be the minuend? How can you tell which is the larger number? Can you think of any method of proving addition by subtraction?

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* *An integer is* a whole number, as seven, forty-nine.

What is MULTIPLICATION? Of what is it an abbreviation? What is meant by the multiplicand?—the product?—the multiplier?—the factors?—a composite number? Name some composite numbers? What is the sign of multiplication? Can the multiplier and multiplicand exchange places? Give an illustration in proof of your answer. How do you multiply by a single figure? How may you multiply by a composite number? How do you proceed when the multiplier consists of a number of figures? How many decimals must there be in the product? What do you do if there are not decimals enough? What may be done, if there are naughts at the right hand of either factor? How can you most readily multiply by 10, 100, 1000, &c.? Explain the reason for the several modes of multiplication. How do you prove multiplication? What must you know before you can find the value of any number of things?

What is DIVISION? Of what is it an abbreviation? What is meant by the dividend?—the quotient?—the divisor?—the remainder? Of what are the divisor and quotient factors? What are the different modes of expressing division? Give, in your own words, a *rule for division,* and explain the reason for each step of the process. Of what operation is division the opposite? How many decimals must the quotient contain? Why? How do you find the dividend, when the divisor and quotient are given?—the divisor, when the dividend and quotient are given? What is the difference between short and long division? What may be done, if there are naughts at the right hand of the divisor? How can you most readily divide by 10, 100, 1000, &c.? How do you prove division? How can you avoid frequent trials, in finding the true quotient figure? When the value of any number of things is given, how do you find the value of one? When the value of a number of things is given, how can you find the number of things? Can you think of any method for proving multiplication by division?

When the sum of two numbers and one of the numbers are given, how do you find the other? When the greater of two numbers and their difference are given, how do you find the less? When the less of two numbers and their difference are given, how do you find the greater? When the factors are given, how do you find the product? When the product and one of the factors are given, how do you find the other factor? When the value of any number of things is given, how do you find the value of any *other* number? By which of the fundamental rules do you increase a number? By which do you diminish a number? In what cases would you employ addition?—subtraction? — multiplication? — division? What is meant by the 3d,—7th,—15th power of a number?

VI. PRIME AND COMPOSITE NUMBERS.

57. A PRIME NUMBER is a number that has no factors greater than 1, and less than itself; as, 2; 7; 11; 13.

A COMPOSITE NUMBER (§ **35**) is a number that hatwo or more factors, each greater than 1, and less than itself; as, $8 = 2 \times 2 \times 2$; $42 = 2 \times 3 \times 7$.

The following is a list of all the prime numbers less than 223, viz: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211.

2 is a factor of all numbers ending in 0, 2, 4, 6, or 8. For, as 2 will divide 10, it will divide *any number* of tens, or any number of tens *plus* 2, 4, 6, or 8 units. Numbers divisible by 2 are called EVEN,—all others, ODD numbers.

5 is a factor of all numbers ending in 0, or 5. For, as 5 will divide 10, it will divide *any number* of tens, or any number of tens *plus* 5.

3 is a factor of all numbers in which the sum of the digits is divisible by 3. For, if from 1 standing in any place, (as 10, 100, 1000,) we subtract 1 unit, the remainder will either be 0, or will consist entirely of nines. Then *any number* of tens, hundreds, thousands, &c., diminished by *as many* units, will be divisible by 3, and if the sum of the numbers subtracted, or, in other words, the sum of the digits, is similarly divisible, the number itself will be so.

9 is a factor of all numbers in which the sum of the digits is divisible by 9, for the same reason that 3 is a factor when the sum is divisible by 3.

58. Find all the prime factors of 5940.

2	5940
2	2970
3	1485
3	495
3	165
5	55
11	11
	1

We try to divide by the several prime numbers in order, commencing with 2, *and dividing as often as possible by* EACH. We thus find that $5940 = 2 \times 2970$; $2970 = 2 \times 1485$; $1485 = 3 \times 495$; $495 = 3 \times 165$; $165 = 3 \times 55$; $55 = 5 \times 11$. Therefore $5940 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 11$, which may be represented by employing a small figure as an *index* to show the number of times each factor is employed; thus, 5940 $= 2^2 \times 3^3 \times 5 \times 11$.

59. EXAMPLES FOR THE PUPIL.

1—20. Find the prime factors of 60; 32; 54; 42; 63; 36; 56; 45; 81; 91; 38; 44; 66; 72; 64; 48; 95; 90; 88; 92.

21—40. Find the prime factors of 102; 510; 816; 308; 408; 513; 343; 414; 712; 928; 648; 999; 576; 564; 289; 294; 772; 154; 184; 969.

41—60. Find the prime factors of 598; 276; 462; 663; 962; 840; 822; 924; 247; 540; 510; 915; 1001; 1848; 1862; 1180; 1332; 1632; 1976; 6804.

60. Determine the prime factors of 1327.

Trying successively 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, and 31, we find that neither of them will divide without a remainder. Therefore, if 1327 is not a prime number, its least factor must be greater than 31. But its least factor *cannot* be greater than 31, because the next prime number above 31 is 37, and 37 × 37 = 1369, which is greater than 1327. Then 1327 is a prime number.

Any number is called the *square root* of its product by itself, and the product is called the *square* of the root. Thus, 37 is the *square root* of 1369; 1369 is the *square* of 37. As the product of two factors, each greater than the square root of a number, would be greater than the number itself, in attempting to discover the factors of any number, *we need not try any prime divisor greater than its square root.* In the following table the columns headed R., contain all the prime square roots of all numbers less than 49729.

R.	Squ.	R.	Squ.	R.	Squ.	R.	Squa.	R.	Squa.	R.	Squa.
1	1	9	361	53	2809	89	7921	131	17161	173	29929
2	4	23	529	59	3481	97	9409	137	18769	179	32041
3	9	29	841	61	3721	101	10201	139	19321	181	32761
5	25	3	961	67	4489	103	10609	149	22201	191	36481
7	49	37	1369	71	5041	107	11449	151	22801	193	37249
11	12	4	1681	73	5329	109	11881	157	24649	197	38809
13	169	43	1849	79	6241	113	12769	163	26569	199	39601
17	289	47	2209	83	6889	127	16129	167	27889	211	44521

TO DETERMINE WHETHER ANY GIVEN NUMBER, TERMINATING IN 1, 3, 7, or 9,* IS A PRIME NUMBER.

Find in the table the root of the square next larger than the given number, and divide in succession by all the primes less than that root. If you find no factor by thus dividing, the given number is a prime.

EXAMPLE. Is 20983 a prime number?

The tabular square next larger than 20983 is 22201, the root of which is 149. After dividing by all the primes less than 149, I find that neither of them is a factor of 20983. It is therefore a prime number.

61. EXAMPLES FOR THE PUPIL.

61—80. Find all the prime numbers between 225 and 340.

81—90. Which of the following numbers are prime? 347; 349; 353; 377; 391; 401; 409; 437; 439; 953.

91—100. Find the prime factors of 359; 364; 367; 373; 378; 379; 383; 2674; 4338; 8866.

62. THE LEAST COMMON MULTIPLE.

Every composite number is called a *multiple* of each of its factors. Thus, 9 is a factor of 18, and 18 is a *multiple* of 9.

A *common multiple* of two or more numbers, is one of which those numbers are all factors. 18 is a common multiple of 2, 3, 6, and 9. The *least common multiple* of any series of numbers, is the least number which can be exactly divided by all the given numbers. Thus, 18, 36, 54, and 72, are all common multiples of 2, 3, 6, 9, and 18, but the *least* common multiple of those numbers is 18.

Any number which contains all the prime factors of another number is a multiple of that number. Hence

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* All even numbers can be divided by 2, and all numbers ending in 5, can be divided by 5, (§ 57.) Therefore all prime numbers greater than 5 must terminate in 1, 3, 7, or 9.

the least common multiple of two or more numbers, is the least number that contains all the prime factors of those numbers.

63. Find the least common multiple of 14, 18, 27, 28, 21, and 126.

2)	14	18	27	28	21	126
2)	7	9	27	14	21	63
3)	7	9	27	7	21	63
3)	7	3	9	7	7	21
3)	7	1	3	7	7	7
7)	7	1	1	7	7	7
	1	1	1	1	1	1

Arranging the numbers in a line, we divide as in § **58**, at each division writing the quotient underneath the number from which it is obtained, and bringing down all the numbers that cannot be divided. In this way we obtain all the prime factors that are contained in the given numbers, and the product of those factors, 2 × 2 × 3 × 3 × 3 × 7 = 756, (§**62**,) is the least common multiple sought.

We may if we choose, analyze the numbers separately, we then find that 14 = 2 × 7; 18 = 2 × 3 × 3; 27 = 3 × 3 × 3; 28 = 2 × 2 × 7; 21 = 3 × 7; 126 = 2 × 3 × 3 × 7. 2 is employed twice as a factor in 28, 3 is employed three times in 27, and 7 is employed only once in either number. Then the factors of the least common multiple (§ **62**,) must contain 2^2, (§ **58**,) 3^3, and 7, or 2 × 2 × 3 × 3 × 3 × 7 = 756, as found before. From the first of these methods, we may derive the following

RULE FOR FINDING THE LEAST COMMON MULTIPLE.

Arrange the numbers in a horizontal line, and divide by the prime numbers, 2, 3, 5, 7, 11, *&c., writing the quotients and undivided numbers beneath. Continue this division, (employing each divisor as often as it will divide one or more of the numb* *without a remainder,) until the last quotients are all* 1, *and you will have obtained all the prime factors of the given numbers. The product of these factors is the least common multiple.*

64. Examples for the Pupil.

101—110. Find the least common multiple of 2, 3, and 6; of 4, 3, and 6; of 6, 10, and 15; of 2, 3, and 5; of 3, 6, and 7; of 6, 14, and 21; of 8 and 12; of 9, and 12; of 15, 21, and 35; of 20 and 24.

111—120. Find the least common multiple of 12 and 15; of 18 and 24; of 12, 14, and 21; of 9, 18, and 27; of 14 and 49; of 35 and 49; of 21, 33, and 77; of 27, 33, and 99; of 15, 25, and 75; of 77, 35, and 55.

121—125. Find the least common multiple of 8, 9, 12, 18, and 24; 2, 3, 4, 9, and 12; 8, 7, 11, 44, 56, and 77; 3, 4, 5, 6, 7, and 8; 11, 13, and 15.

126—130. Find the least common multiple of 6, 8, 10, and 12; 9, 10, 11, and 12; 7, 9, 12, and 13; 7, 19, and 23; 36, 20, 45, 30, and 90.

131—135. Find the least common multiple of 96, 32, 12, 24, and 48; 32, 27, 16, 18, and 54; 35, 20, 28, 21, and 12; 19, 27, 18, 12, and 36; 2, 3, 4, 5, 11, 12, and 15.

136—140. Find the least common multiple of 2, 3, 4, 5, 6, 7, 8, 9, and 10; 9, 11, 14, 63, and 99; 8, 9, 12, 18, 24, and 36; 21, 25, 12, 18, and 49; 10, 6, 42, 15, 30, 105, and 210.

65. The Greatest Common Divisor.

Every number that is a factor of two or more other numbers, is called a *common divisor* of those numbers. Thus 9 is a *common* divisor of 36, 54, 27, and 18.

The greatest factor that is common to any series of numbers, is called their *greatest common divisor.* Thus 2, 3, 4, 6, and 12, are all common divisors of 24, *36, 48, and 60,* but their *greatest common divisor* is 12.

Every composite number can be divided by the product of any two or more of its prime factors. Thus 60 = 2 × 2 × 3 × 5, can be divided by 2 × 2, 2 × 3, 2 × 5, 3 × 5, 2 × 2 × 3, 2 × 2 × 5, and 2 × 3 × 5.

Hence the greatest common divisor of any series of numbers is *the product of* ALL *the factors which are common to all the numbers.*

66. Find the greatest common divisor of 540, 378, 594, and 486.

2)	540	378	594	486
3)	270	189	297	243
3)	90	63	99	81
3)	30	21	33	27
	10	7	11	9

Arranging the numbers as in § **63**, we divide as in § **58**, **63**, *employing only those factors which will divide all the numbers.* We thus find that 2 and 3^3 are the only prime factors of all the given numbers, and the *greatest* factor (§ **65**) must be the product 2 × 3 × 3 × 3 = 54.* Hence we derive the following

RULE FOR FINDING THE GREATEST COMMON DIVISOR.

Arrange the numbers in a horizontal line, and divide by any factor† *which will divide them all without a remainder, writing the quotients beneath the line. Divide the quotients in the same manner as the original numbers, and thus continue dividing until you obtain a series of quotients that have no common factor. The product of all the divisors employed, is the greatest common divisor.*

Or, if you cannot readily discover any common factor, proceed in the following manner. (See Note at the end of the Chapter.)

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* For other methods of obtaining the greatest common divisor, see Note at the end of this Chapter.

† We say *any* factor, because we can sometimes perceive at once a large number that will divide all the given numbers. If we discover no such number by inspection, it is best always to proceed with the prime numbers in order, as in the explained *example.*

Arrange the numbers in the order of their magnitudes, by the least number divide all the others, and bring down the remainders. Arrange these remainders and the first divisor in the order of their magnitudes, divide, and bring down as before. Arrange the second divisor and the second remainders in the same manner, and continue, (at each time employing the LEAST *of the arranged numbers as a new divisor,) until you find a divisor that leaves no remainder. The divisor last used, will be the greatest common divisor.*

67. EXAMPLES FOR THE PUPIL.

141—150. Find the greatest common divisor of 12 and 14; of 27 and 33; of 13 and 39; of 8 and 28; of 15 and 35; of 27 and 45; of 81 and 108; of 4, 10, and 12; of 35 and 49; of 18 and 42.

151—160. Find the greatest common divisor of 4, 6, and 10; 9, 12, and 21; 10, 15, and 25; 14, 49, and 35; 30, 18, and 54; 30, 70, and 20; 14, 56, and 42; 15, 75, and 30; 96, 36, and 48; 84, 105, and 63.

161—170. Find the greatest common divisor of 8, 9, 12, and 7; 8, 20, 16, and 28; 45, 81, 27, and 90; 78, 18, 54, and 42; 78, 18, 54, and 39; 98, 28, 70, and 42; 96, 112, 80, and 32; 24, 96, 48, and 120; 84, 252, 168, and 210; 33, 88, 77, and 55.

171—180. Find the greatest common divisor of 48, 72, and 60; 1001, 385, and 539; 405, 567, 729, and 891; 2863 and 1151; 992, 960, 928, and 96; 1177, 1391, and 1819; 2943, 2616, and 4578; 2148, 6444, and 3580; 384 and 672; 539 and 240.

181—190. Find the greatest common divisor of 214, 196, 78, and 180; 322, 230, 92, and 184; 125, 675, and 225; 875, 375, and 625; 932 and 699; 518, 370, and 666; 624, 480, and 1008; 522, 638 and 290; 742, 954, and 318; 842 and 421.

191—195. Find the greatest common divisor of 749 and 535; 3235, 1294, and 4529; 4631 and 2947; 929, 2179, and 534; 5317 and 2863.

196—200. Find the greatest common divisor of 12703 and 12879; 23309 and 10753; 4939, 1347, and 3143; 4199, 1615, and 6137; 4746, 1695, 1017, and 3051.

68. Cancelling. Whenever any division is required, if the divisor and dividend have any common factor, the work may often be abridged by rejecting that factor. The process by which this is done, is called *cancelling.*

Thus, 3 dollars is the same as 3 × 4 quarter-dollars, and 15 dollars is the same as 15 × 4 quarter-dollars. Then 15 dollars will contain 3 dollars just as many times as 15 × 4 quarter-dollars contains 3 × 4 quarter-dollars. In other words, $\frac{15 \times 4}{3 \times 4} = \frac{15}{3} = 5.$ The same illustration may be applied to any other numbers, by substituting other parts for quarters.

$$\frac{\overset{5}{\cancel{15}} \times \overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{4}}}$$

The operation of cancelling is usually performed by drawing a line through the factors that are cancelled, and the numbers that are reduced by cancelling. Thus in the example already given, we say 4 in 4 once, 4 in 4 once, 3 in 15 five times, 3 in 3 once. We have then left only $\frac{5 \times 1}{1 \times 1} = \frac{5}{1} = 5$, the same as before.

Whenever we obtain 1 for a quotient in cancelling, we need not set it down, as it can have no effect upon the answer. But we must observe, that *when all the numbers are cancelled in either the dividend or the divisor,* 1 *must be put in their place.*

Examples.

$$\frac{12 \times 4 \times 5}{3 \times 2 \times 5} = \frac{\overset{4}{\cancel{12}} \times \overset{2}{\cancel{4}} \times \cancel{5}}{\cancel{3} \times \cancel{2} \times \cancel{5}} = \frac{4 \times 2}{1} = 8.$$

$$\frac{13 \times 8 \times 7}{7 \times 13 \times 2} = \frac{\cancel{13} \times \overset{4}{\cancel{8}} \times \cancel{7}}{\cancel{7} \times \underset{8}{\cancel{13}} \times \cancel{2}} = \frac{4}{1} = 4.$$

$$\frac{6 \times 5 \times 4}{9 \times 10 \times 12} = \frac{\overset{\cancel{3}}{\cancel{6}} \times \cancel{5} \times \cancel{4}}{9 \times \underset{\cancel{2}}{\cancel{10}} \times \underset{\cancel{3}}{\cancel{12}}} = \frac{1}{9}.$$

69. EXAMPLES FOR THE PUPIL.

201—210. Find the simplest forms for the following examples by cancelling. $\frac{16}{8}$; $\frac{14}{21}$; $\frac{49}{7}$; $\frac{42}{6}$; $\frac{8}{6}$; $\frac{9}{15}$; $\frac{21}{15}$; 4:10; 15:3; 28:49.

211—220. Cancel the common factors in the following expressions. $\frac{14}{21}$; $\frac{12 \times 9}{18 \times 3}$; $\frac{3 \times 7 \times 11}{6 \times 7 \times 5}$; $\frac{18}{36}$; $\frac{14}{42}$; $\frac{18}{63}$; 42:30; $\frac{15 \times 18}{30 \times 9}$; $\frac{27 \times 25}{75 \times 9}$; $\frac{6 \times 35}{10 \times 63}$.

221—230. Cancel $3 \times 6 \times 10 \div 15 \times 8$; $4 \times 7 \times 9 : 6 \times 14 \times 5$; $\frac{20}{15}$; $\frac{45}{105}$; 81:54; $64 \div 160$; $\frac{28}{196}$; $\frac{44}{12}$; $\frac{16 \times 9 \times 14}{42}$; $\frac{63}{15 \times 12 \times 14}$.

231—240. Cancel $\frac{143}{156}$; $\frac{171}{209}$; $3 \times 7 \times 19 : 285$; $\frac{48}{180}$; $\frac{370}{740}$; $\frac{128}{96}$; $\frac{45}{165}$; $\frac{18 \times 34}{102 \times 12}$; 49:133; $\frac{3 \times 7 \times 11.}{21 \times 5 \times 8.}$

241—250. Cancel $\frac{4 \times 5 \times 9 \times 16}{30 \times 18 \times 14}$; $\frac{27}{15 \times 21 \times 9}$; $\frac{12 \times 14 \times 26}{16}$; $\frac{320}{192}$; $\frac{420}{196}$; $\frac{736}{552}$; $\frac{837}{1395}$; $\frac{938}{1407}$; $\frac{1001}{231}$; $\frac{388}{291}$.

251—265. Determine the prime factors of the following numbers by inspection. 8, 16, 32, 81, 96, 108, 180, 135, 540, 137, 375, 252, 111, 444, 210.

266—270. Find the least common multiple of 1316, and 1974; 2150, 860, and 1290; 1428, 612, and 1020; 1872, 624, and 936; 343, 637, and 539.

271—275. Find the greatest common divisor of 6448 and 8866; 7480 and 4488; 220, 396, and 176; 130, 546, and 832; 1464, 6588, 732 and 4026.

Solve the following examples by cancelling the factors of the divisor from the dividend; as,

$$\frac{\overset{3}{\cancel{24}} \times \overset{9}{\cancel{36}}}{\underset{\cancel{4}}{\cancel{32}}} = 27.$$

276—295. 24 × 36 is how many times 12? 18? 6? 4? 9? 8? 32? 16? 18? 27? 3? 2? 48? 54? 72? 96? 108? 144? 216? 288?

296—315. 72 × 18 is how many times 9? 36? 24? 6? 8? 3? 12? 16? 27? 54? 48? 108? 144? 216? 81? 162? 432? 324? 4? 2?

316—340. 40 × 42 is how many times 84? 21? 20? 80? 10? 7? 6? 8? 240? 280? 560? 56? 28? 14? 112? 120? 60? 30? 15? 210? 105? 48? 70? 35? 140?

341—360. 45 × 54 is how many times 9? 6? 5? 15? 27? 18? 3? 54? 45? 81? 30? 162? 243? 486? 270? 90? 10? 135? 405? 810?

361—390. 75 × 24 is how many times 25? 12? 6? 4? 3? 15? 5? 8? 150? 300? 225? 450? 900? 600? 200? 100? 50? 10? 20? 40? 120? 60? 30? 90? 360? 36? 180? 18? 9? 72?

From § 68 and the examples in § 69, it will be readily seen that *when the divisor is a composite number, the quotient can be obtained by dividing in succession by the several factors of the divisor.*

NOTE TO § 66.

The greatest common divisor, like the least common multiple, can be found by resolving each of the given numbers into its prime factors, and multiplying together the factors that are common to all the given numbers. Thus, in the example explained, 540 = 2 × 2 × 3 × 3 × 3 × 5; 378 = 2 × 3 × 3 × 3 × 7; 594 = 2 × 3 × 3 × 3 × 11; 486 = 2 × 3 × 3 × 3 × 3 × 3. The factors common to all the numbers, are 2 and 3 × 3 × 3, and their product, 2 × 3 × 3 × 3 = 54, is the divisor sought. The following example will illustrate still another method of obtaining the greatest common divisor, which is often more convenient than either of the preceding.

Required the greatest common divisor of 4693, 988, 2223, 2964, and 1482.

If the least of the given numbers will divide all the *others, it is itself the* greatest common divisor. There-

[illegible] numbers in a horizontal line in the order of their magnitude, we divide all the larger numbers by the least of them (988), and bring down the remainders.

```
988)  1482   2223   2964   4693
       988   1976   2964   3952
      ----   ----   ----   ----
       494    247           741

247)   494    741    988
       494    741    988
```

The greatest common divisor of the original numbers, must be the same as that of 988, 494, 247, and 741; because, supposing it to be contained 4 times in 988, and 6 times in 1482, it must be contained 2 times in their difference 494; if it is contained 4 times in 988, and 19 times in 4693, it will be contained 16 times in 4 times 988, 3952, and 3 times in 741, which is the difference between 3952 and 4693. And if it is contained 9 times in 2223, it will be contained 8 times in twice 988, 1976, and once in 247, which is the difference between 1976 and 2223. We need, therefore, only find the greatest common divisor of the first divisor and the first remainders. Arranging them according to their magnitudes as before, we find that 247 divides all the numbers, and is itself the greatest common divisor. If there were any remainders after the second division, we would arrange the second divisor and the second remainders, and again divide, proceeding in like manner with each succeeding divisor and its remainders, until we obtained exact quotients. The divisor last used would be the greatest common divisor.

Whichever method we may adopt for determining the greatest common divisor, if at any step of the process we perceive that a prime factor is contained in one of the numbers, but not in all of them, we may reject it and thus shorten our work.

VII. FRACTIONS.

70. Fractions are *parts* of whole numbers.

If we divide 16 apples among 3 boys, we can give 5 to each, and have one left. If we wish to divide the remaining apple among them, we must cut it into 3 equal parts, and give one of the parts to each of them. Each part would be called *one-third*, and written like 1 divided by 3, $\frac{1}{3}$, (§ **47**.)

Again, if we wished to divide 31 apples among 9 boys, we could give 3 to each, and have 4 left. To divide these 4, we might cut each one into 9 equal parts, or *ninths*, and give each boy 4 of the parts, or 4 *ninths*. 4 *ninths* is, therefore, the same as 4 *divided by* 9, $\frac{4}{9}$.

Fractions may be read in three different ways. Thus, $\frac{3}{4}$ may be called 3-*fourths*, as above; 3 *divided by* 4, as in § **47**; or, $\frac{1}{4}$ of 3, because $\frac{1}{4}$ of any number is by our definition, one of 4 equal parts into which that number is divided. So $\frac{31}{56}$ is 31 *fifty-sixths*, $\frac{1}{56}$ of 31, or 31 *divided by* 56.

The dividend, or upper number in a fraction, is called the *numerator*, and may be considered as showing *the number of parts that are taken.* The divisor, or lower number, is called the *denominator*, and shows how many of the *parts* would make a *whole.* The numerator and denominator are also called the *terms* of the fraction.

A *mixed number* is one which contains a whole number and a fraction, as $3\frac{2}{3}$, (*read* three and two-thirds,) $17\frac{4}{5}$.

EXERCISES FOR BEGINNERS.

1—8. If you divide an apple into 8 equal parts, what would one of these parts be called? 2 parts? 3 parts? 4 parts? 5 parts? 6 parts? 7 parts? 8 parts?

9—20. What part of 10 is 1? 3? 7? 9? 2? 4? 6? 8? 5? 10? 15? 33?

21—40. What part of 15 is 1? 2? 5? 3? 7? 4? 8?

[illegible]

[illegible] there will be as [illegible] ninths [illegible] in the [illegible] by 9. [illegible] by 11, and 3 by 4

2. If we regard $\frac{18}{9}$ as denoting $\frac{1}{9}$ of 18, by our definitions we find $\frac{1}{9}$ of any number by dividing that number by 9. We must, therefore, divide 18 by 9, and in like manner, 27 by 11, and 3 by 4.

3. We then see that in whatever light the several fractional expressions are regarded, the answer is to be found in the same manner as when we consider them as denoting simple division. Therefore $\frac{18}{9} = 18 \div 9 = 2$; $\frac{27}{11} = 27 \div 11 = 2\frac{5}{11}$; $\frac{3}{4} = 3 \div 4 = .75$. (§ 56, Exs. 627—675.) Hence, *to reduce a fraction to a whole or mixed number, or to a decimal;*

RULE.

PERFORM *the division which the fraction* EXPRESSES.

EXAMPLES FOR THE PUPIL.

1—20. Reduce to a whole number, $\frac{8}{4}$; $\frac{6}{3}$; $\frac{15}{3}$; $\frac{12}{6}$; $\frac{12}{2}$; $\frac{12}{3}$; $\frac{12}{4}$; $\frac{6}{3}$; $\frac{8}{2}$; $\frac{15}{5}$; $\frac{18}{2}$; $\frac{18}{3}$; $\frac{18}{6}$; $\frac{18}{9}$; $\frac{21}{3}$; $\frac{21}{7}$; $\frac{35}{5}$; $\frac{35}{7}$; $\frac{33}{3}$; $\frac{33}{11}$.

21—40. Reduce to a mixed number, $\frac{9}{2}$; $\frac{11}{4}$; $\frac{11}{3}$; $\frac{11}{5}$; $\frac{16}{7}$; $\frac{22}{5}$; $\frac{31}{9}$; $\frac{47}{10}$; $\frac{98}{11}$; $\frac{100}{12}$; $\frac{85}{13}$; $\frac{111}{14}$; $\frac{129}{15}$; $\frac{487}{16}$; $\frac{329}{17}$; $\frac{519}{18}$; $\frac{842}{19}$; $\frac{353}{20}$; $\frac{592}{31}$; $\frac{647}{35}$.

41—60. Reduce to a decimal, $\frac{1}{2}$; $\frac{1}{4}$; $\frac{2}{4}$; $\frac{3}{4}$; $\frac{1}{5}$; $\frac{2}{3}$; $\frac{3}{5}$; $\frac{1}{6}$; $\frac{2}{6}$; $\frac{3}{6}$; $\frac{4}{6}$; $\frac{5}{6}$; $\frac{1}{8}$; $\frac{2}{8}$; $\frac{3}{8}$; $\frac{4}{8}$; $\frac{7}{8}$; $\frac{1}{9}$; $\frac{2}{9}$; $\frac{7}{9}$.

61—80. Reduce all of the following fractions which are less than 1,* to decimals, and the others to whole or mixed numbers: $\frac{3}{9}$; $\frac{4}{9}$; $\frac{5}{9}$; $\frac{6}{9}$; $\frac{8}{9}$; $\frac{27}{3}$; $\frac{169}{13}$; $\frac{798}{38}$; $\frac{487}{11}$; $\frac{999}{97}$; $\frac{876}{35}$; $\frac{597}{17}$; $\frac{460}{23}$; $\frac{460}{20}$; $\frac{460}{46}$; $\frac{1901}{44}$; $\frac{6482}{65}$; $\frac{987}{13}$; $\frac{432}{9}$; $\frac{114}{3}$.

81—100. Reduce the following fractions, as above. $\frac{8}{13}$; $\frac{11}{3}$; $\frac{3}{11}$; $\frac{8}{6}$; $\frac{6}{8}$; $\frac{5}{8}$; $\frac{99}{5}$; $\frac{284}{19}$; $\frac{7}{82}$; $\frac{9763}{101}$; $\frac{944}{58}$; $\frac{9}{15}$; $\frac{3}{14}$; $\frac{59}{3}$; $\frac{878}{29}$; $\frac{13}{125}$; $\frac{4}{375}$; $\frac{375}{4}$; $\frac{125}{15}$; $\frac{64021}{83}$.

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* Fractions in which the numerator is less than the denominator, are sometimes called *proper fractions;* those in which the numerator is equal to, or greater than the denominator, *improper fractions*. All improper fractions are equal to 1, or greater than 1.

CASE II. TO REDUCE WHOLE OR MIXED NUMBERS, OR DECIMALS, TO FRACTIONS.

72. Reduce 18 to *twelfths*; $25\frac{7}{9}$ to *ninths*; .015 to *hundredths*. 18 = 18 × 1, and 1 = 12 *twelfths*. Then 18 = 18 × 12 twelfths = $\frac{216}{12}$. $25\frac{7}{9} = 25\frac{7}{9} \times 1$, and 1 = 9 *ninths*. Then $25\frac{7}{9} = 25\frac{7}{9} \times 9$ ninths = $\frac{232}{9}$. For 9 × 25 = 225 and $9 \times \frac{7}{9}$ (§ **70**,) = 7, which added to 225 = 232. Therefore, $9 \times 25\frac{7}{9} = 232$.*

.015 = .015 × 1, and 1 = 100 *hundredths*. Then, .015 = .015 × 100 hundredths, = $\frac{1.5}{100}$. Hence, *to reduce a whole or mixed number, or a decimal, to a fraction having any given denominator;*

RULE.

Multiply the given number by the given denominator. The product will be the numerator.

EXAMPLES FOR THE PUPIL.

101—120. Reduce 18 to halves, thirds, fourths, fifths, sixths, sevenths, eighths, ninths, tenths, elevenths, twelfths, twentieths, twenty-fifths, thirtieths, fortieths, fiftieths, sixtieths, seventy-fifths, hundredths, thousandths.

121—140. Reduce the following mixed numbers to fractions. $1\frac{3}{4}$; $2\frac{5}{8}$; $3\frac{3}{8}$; $5\frac{4}{5}$; $7\frac{4}{11}$; $9\frac{1}{6}$; $8\frac{5}{7}$; $8\frac{5}{12}$; $4\frac{8}{13}$ $10\frac{4}{7}$; $19\frac{1}{4}$; $18\frac{1}{2}$; $20\frac{1}{2}$; $32\frac{1}{3}$; $15\frac{2}{3}$; $29\frac{3}{4}$; $84\frac{3}{4}$; $9\frac{1}{13}$; $9\frac{5}{25}$; $1\frac{19}{73}$.

141—160. Reduce .13 to tenths, hundredths, thousandths, ten-thousandths; .197, .8, 21.5, .09, *each* to tenths, hundredths, thousandths, and millionths.

161—180. Write *in the form of a fraction*, .9; .7; 16; .04; .247; .013; .005; .84; 1.84; 2.06; 59.3; 5.7; .0019; 80.2; 8.02; .3; .62; 1.19; .04; .083.

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* This mode of illustration is given, in order to make all the [illegible] in a uniform manner. Perhaps the following will be more readily understood.

25 = 25 × 1 = 25 × 9 *ninths* = $\frac{225}{9}$. Then $25\frac{7}{9} = \frac{225}{9} + \frac{7}{9} = \frac{232}{9}$. By either method, we may say 9 times 5 are 45 and 7 are 52, 9 times 2 are 18 and 5 are 23.

181—200. Reduce to a fractional form, $2\frac{7}{12}$; .87; $4\frac{3}{17}$; 91 ÷ 11; 4:37; .084; 11.6; .49; $15\frac{3}{5}$; $33\frac{1}{3}$; $16\frac{4}{5}$; $14\frac{3}{7}$; $12\frac{4}{5}$; $11\frac{1}{5}$; $9\frac{1}{11}$; $8\frac{4}{12}$; 1.9; 2.7; 3.05; 19 to *forty-ninths.*

73. CASE III. TO REDUCE COMPOUND FRACTIONS TO SIMPLE FRACTIONS.

A compound fraction is a fraction of a fraction; as $\frac{1}{2}$ of $\frac{3}{4}$; $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{1}{3}$.

Reduce $\frac{3}{4}$ of $\frac{2}{7}$ to a simple fraction.

The denominator of any fraction shows the number of parts into which the unit is divided. If we make each of the parts only $\frac{1}{4}$ as large, there will be 4 times as many parts. Therefore, to obtain $\frac{1}{4}$ of any fraction, *multiply the denominator by* 4.

As the numerator of any fraction shows how many parts are taken, in $\frac{3}{4}$ of any fraction there will be 3 times as many parts as in $\frac{1}{4}$ of the same fraction; then

$$\text{as } \frac{1}{4} \text{ of } \frac{2}{7} = \frac{2}{4 \times 7},\ \frac{3}{4} \text{ of } \frac{2}{7} = \frac{3 \times \cancel{2}}{\underset{2}{\cancel{4}} \times 7} = (\S\ 68)\ \frac{3}{14}.$$

Hence, *to reduce a compound fraction to a simple fraction,*

RULE.

Multiply all the numerators together for a new numerator, and all the denominators for a new denominator.

Before performing the multiplication, *cancel all factors that are common to the numerators and denominators.*

$$\text{EXAMPLE. } \frac{3}{7} \text{ of } \frac{14}{18} \text{ of } \frac{6}{11} = \frac{\cancel{3} \times \overset{2}{\cancel{14}} \times \cancel{6}}{\cancel{7} \times \underset{\cancel{6}}{\cancel{18}} \times 11} = \frac{2}{11}.$$

EXAMPLES FOR THE PUPIL.

201—220. Reduce to simple fractions $\frac{1}{2}$ of $\frac{1}{3}$; $\frac{1}{2}$ of $\frac{1}{4}$; $\frac{1}{3}$ of $\frac{1}{5}$; $\frac{1}{4}$ of $\frac{1}{6}$; $\frac{1}{2}$ of $\frac{1}{7}$; $\frac{1}{2}$ of $\frac{3}{8}$; $\frac{1}{2}$ of $\frac{7}{9}$; $\frac{1}{2}$ of $\frac{5}{11}$; [illegible] of $\frac{1}{3}$; $\frac{2}{3}$ of $\frac{1}{5}$; $\frac{3}{5}$ of $\frac{5}{8}$; $\frac{2}{3}$ of $\frac{6}{8}$; $\frac{3}{5}$ of $\frac{9}{12}$; $\frac{2}{5}$ of $\frac{3}{4}$; $\frac{2}{3}$ of $\frac{4}{11}$; $\frac{2}{3}$ of $\frac{6}{15}$; $\frac{3}{4}$ of $\frac{1}{6}$; $\frac{3}{4}$ of $\frac{3}{8}$; $\frac{1}{4}$ of $\frac{12}{17}$; $\frac{3}{4}$ of $\frac{5}{6}$.

221—230. Reduce to simple fractions $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$; $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{1}{2}$ of $\frac{2}{5}$ of $\frac{5}{7}$; $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{7}{8}$; $\frac{2}{5}$ of $\frac{3}{4}$ of $\frac{4}{8}$; $\frac{2}{3}$ of $\frac{1}{5}$ of $\frac{3}{7}$; $\frac{1}{6}$ of $\frac{2}{3}$ of $\frac{1}{5}$; $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{2}{3}$; $\frac{3}{7}$ of $\frac{1}{5}$ of $\frac{6}{11}$; $\frac{3}{8}$ of $\frac{1}{5}$ of $\frac{7}{11}$.

231—240. Simplify $\frac{2}{3}$ of $\frac{9}{13}$; $\frac{3}{8}$ of $\frac{7}{17}$; $\frac{9}{12}$ of $\frac{14}{3}$; $\frac{1}{3}$ of $2\frac{1}{2}$ or $\frac{1}{4}$ of $\frac{5}{2}$; $\frac{1}{3}$ of $2\frac{1}{4}$; $\frac{2}{3}$ of $2\frac{1}{2}$; $\frac{3}{7}$ of $1\frac{2}{3}$; $\frac{4}{11}$ of $2\frac{3}{4}$; $\frac{3}{5}$ of $\frac{4}{7}$ of $\frac{5}{6}$; $\frac{1}{6}$ of $\frac{3}{7}$ of $\frac{3}{4}$.

241—250. Simplify $\frac{5}{11}$ of 3 ($\frac{5}{11}$ of $\frac{3}{1}$); $\frac{4}{5}$ of 7; $\frac{3}{7}$ of 15; $\frac{2}{5}$ of $\frac{5}{11}$ of $\frac{10}{13}$; $\frac{1}{4}$ of .5; $\frac{2}{5}$ of $6\frac{1}{4}$; $\frac{1}{7}$ of $5\frac{1}{4}$; $\frac{3}{10}$ of $4\frac{5}{8}$; $\frac{2}{5}$ of $\frac{1}{2}$ of $\frac{11}{13}$; $\frac{1}{6}$ of $\frac{3}{4}$ of $7\frac{2}{3}$.

74. CASE IV. REDUCTION OF FRACTIONS TO THEIR LOWEST TERMS.

$$\frac{3}{5} = \frac{3 \times 4}{5 \times 4} = \frac{3 \times 7}{5 \times 7} = \frac{3 \times 49}{5 \times 49} \text{ \&c. } (\S\ 68, 72)$$

We may therefore either multiply or divide both terms of a fraction by the same number, without altering its value.

When we divide both numerator and denominator by any common factor, the fraction is said to be *reduced to lower terms.* If our divisor is the greatest common divisor, the fraction will be *reduced to its* LOWEST *terms.*

$\frac{2}{2})\frac{18}{36}(\frac{9}{18}$

$\frac{3}{3})\frac{18}{36}(\frac{6}{12}$

$\frac{18}{18})\frac{18}{36}(\frac{1}{2}$

Thus, $\frac{18}{36} \div \frac{2}{2} = \frac{9}{18}$; $\frac{18}{36} \div \frac{3}{3} = \frac{6}{12}$; $\frac{18}{36} \div \frac{18}{18} = \frac{1}{2}$. $\frac{9}{18}$ and $\frac{6}{12}$ are equivalent to $\frac{18}{36}$ in *lower terms*; $\frac{1}{2}$ is equivalent to the same fraction in its *lowest terms.* Hence, *to reduce a fraction to its lowest terms,*

RULE,

Divide both terms of the fraction by their greatest common divisor.

EXAMPLES FOR THE PUPIL.

251—270. Reduce to their lowest terms $\frac{2}{4}$; $\frac{2}{6}$; $\frac{2}{8}$;

$\frac{4}{18}$; $\frac{4}{8}$; $\frac{4}{10}$; $\frac{6}{8}$; $\frac{6}{10}$; $\frac{6}{12}$; $\frac{6}{18}$; $\frac{8}{10}$; $\frac{8}{12}$; $\frac{8}{14}$; $\frac{8}{16}$; $\frac{8}{48}$; $\frac{9}{12}$; $\frac{9}{15}$; $\frac{9}{18}$; $\frac{9}{27}$; $\frac{9}{81}$.

271—290. Find the lowest terms of $\frac{12}{14}$; $\frac{12}{15}$; $\frac{12}{16}$; $\frac{12}{17}$; $\frac{12}{18}$; $\frac{12}{20}$; $\frac{12}{24}$; $\frac{12}{30}$; $\frac{12}{84}$; $\frac{12}{90}$; $\frac{18}{20}$; $\frac{18}{21}$; $\frac{18}{22}$; $\frac{18}{24}$; $\frac{18}{27}$; $\frac{18}{30}$; $\frac{18}{33}$; $\frac{18}{36}$; $\frac{18}{81}$; $\frac{18}{29}$.

291—310. What are the lowest terms of $\frac{27}{30}$; $\frac{27}{36}$; $\frac{27}{63}$; $\frac{27}{54}$; $\frac{27}{108}$; $\frac{27}{99}$; $\frac{36}{48}$; $\frac{36}{51}$; $\frac{36}{52}$; $\frac{36}{54}$; $\frac{36}{66}$; $\frac{36}{63}$; $\frac{36}{84}$; $\frac{36}{72}$; $\frac{36}{90}$; $\frac{36}{108}$; $\frac{32}{38}$; $\frac{32}{44}$; $\frac{32}{72}$; $\frac{32}{112}$.

311—325. Reduce to the lowest terms $\frac{40}{58}$; $\frac{40}{52}$; $\frac{40}{88}$; $\frac{40}{90}$; $\frac{40}{60}$; $\frac{72}{94}$; $\frac{72}{93}$; $\frac{72}{92}$; $\frac{72}{102}$; $\frac{72}{88}$; $\frac{72}{99}$; $\frac{72}{132}$; $\frac{72}{90}$; $\frac{72}{120}$; $\frac{72}{252}$.

75. CASE V. TO REDUCE FRACTIONS TO A COMMON DENOMINATOR.

We can reduce any fraction to another, having ANY *denominator which is a multiple of its own denominator.* (§ **74.**)

Accordingly, we can reduce two or more fractions to others having ANY *common denominator which is a common multiple of* ALL *the original denominators.* The *least common denominator* will be the *least common multiple* of the original denominators.

MENTAL EXERCISES.

1—20. How many 24ths in $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{6}$? $\frac{1}{8}$? $\frac{1}{12}$? $\frac{2}{3}$? $\frac{2}{4}$? $\frac{2}{6}$? $\frac{2}{8}$? $\frac{2}{12}$? $\frac{3}{4}$? $\frac{3}{6}$? $\frac{3}{8}$? $\frac{3}{12}$? $\frac{4}{6}$? $\frac{4}{8}$? $\frac{4}{12}$? $\frac{5}{6}$? $\frac{5}{8}$?

21—50. How many 60ths in $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{10}$? $\frac{1}{12}$? $\frac{1}{15}$? $\frac{1}{20}$? $\frac{1}{30}$? $\frac{2}{3}$? $\frac{2}{4}$? $\frac{2}{5}$? $\frac{2}{6}$? $\frac{2}{10}$? $\frac{2}{12}$? $\frac{2}{15}$? $\frac{2}{20}$? $\frac{2}{30}$? $\frac{11}{30}$? $\frac{7}{30}$? $\frac{8}{30}$? $\frac{10}{30}$? $\frac{13}{30}$? $\frac{15}{30}$? $\frac{17}{30}$? $\frac{19}{30}$? $\frac{21}{30}$? $\frac{23}{30}$? $\frac{29}{30}$?

51—100. How many 120ths in each of the previous examples?

EXAMPLE. Reduce to their least common denominator, $\frac{5}{9}$, $\frac{2}{3}$, $\frac{7}{12}$, $\frac{13}{18}$, and $\frac{5}{6}$. The least common multiple *of all the given* denominators is 36.

In 1 there are $\frac{36}{36}$, and in $\frac{1}{9}$ there are $\frac{1}{9}$ as many 36ths as there are in 1. Therefore, (§ 73,)

$$\frac{5}{9} = \frac{5}{9} \text{ of } \frac{36}{36} = \frac{5 \times \overset{4}{\cancel{36}}}{\cancel{9} \times 36} = \frac{20}{36} \qquad \frac{1}{9} = \frac{4}{36} \;\&\; \frac{5}{9} = \frac{20}{36}$$

$$\frac{2}{3} = \frac{2}{3} \text{ of } \frac{36}{36} = \frac{2 \times \overset{12}{\cancel{36}}}{\cancel{3} \times 36} = \frac{24}{36} \qquad \frac{1}{3} = \frac{12}{36} \;\&\; \frac{2}{3} = \frac{24}{36}$$

$$\frac{7}{12} = \frac{7}{12} \text{ of } \frac{36}{36} = \frac{7 \times \overset{3}{\cancel{36}}}{\cancel{12} \times 36} = \frac{21}{36} \qquad \frac{1}{12} = \frac{3}{36} \;\&\; \frac{7}{12} = \frac{21}{36}$$

$$\frac{13}{18} = \frac{13}{18} \text{ of } \frac{36}{36} = \frac{13 \times \overset{2}{\cancel{36}}}{\cancel{18} \times 36} = \frac{26}{36} \qquad \frac{1}{18} = \frac{2}{36} \;\&\; \frac{13}{18} = \frac{26}{36}$$

$$\frac{5}{6} = \frac{5}{6} \text{ of } \frac{36}{36} = \frac{5 \times \overset{6}{\cancel{36}}}{\cancel{6} \times 36} = \frac{30}{36} \qquad \frac{1}{6} = \frac{6}{36} \;\&\; \frac{5}{6} = \frac{30}{36}$$

or, as there are $\frac{36}{36}$ in 1

Hence, *to reduce two or more fractions to a common denominator,*

RULE.

Reduce all the fractions to their lowest terms:

Find the least common multiple of all the denominators, for the common denominator.

Multiply both terms of each fraction, by the common denominator, first cancelling the original denominator of the fraction.

326—345. Reduce to a common denominator, $\frac{1}{2}$ and $\frac{1}{4}$; $\frac{1}{2}$ and $\frac{1}{3}$; $\frac{1}{2}$ and $\frac{4}{5}$; $\frac{1}{2}$ and $\frac{5}{8}$; $\frac{2}{3}$ and $\frac{3}{4}$; $\frac{2}{3}$ and $\frac{5}{6}$; $\frac{1}{3}$ and $\frac{5}{9}$; $\frac{3}{4}$ and $\frac{1}{6}$; $\frac{3}{4}$ and $\frac{7}{10}$; $\frac{4}{5}$ and $\frac{7}{15}$; $\frac{4}{9}$ and $\frac{7}{12}$; $\frac{6}{7}$ and $\frac{4}{28}$; $\frac{8}{24}$ and $\frac{2}{8}$; $\frac{9}{27}$ and $\frac{5}{9}$; $\frac{2}{3}$ and $\frac{7}{8}$; $\frac{5}{12}$ and $\frac{13}{18}$; $\frac{3}{10}$ and $\frac{18}{25}$; $\frac{1}{7}$ and $\frac{5}{21}$; $\frac{4}{10}$ and $\frac{7}{25}$; $\frac{4}{9}$ and $\frac{5}{12}$.

346—360. Reduce to common denominators, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$; $\frac{1}{3}$, $\frac{2}{4}$, and $\frac{2}{5}$; $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{1}{6}$; $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{7}{8}$; $\frac{2}{3}$, $\frac{1}{4}$ and $\frac{10}{24}$; $\frac{2}{7}$, $\frac{2}{3}$, and $\frac{10}{35}$; $\frac{5}{9}$, $\frac{3}{4}$, and $\frac{11}{12}$; $\frac{1}{6}$, $\frac{1}{4}$, and $\frac{4}{9}$; $\frac{7}{18}$, $\frac{7}{12}$, and

$\frac{3}{8}$; $\frac{5}{18}$, $\frac{7}{9}$, and $\frac{3}{4}$; $\frac{2}{15}$, $\frac{8}{9}$, and $\frac{2}{3}$; $\frac{5}{7}$, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{9}{14}$; $\frac{5}{14}$, $\frac{3}{7}$, and $\frac{3}{5}$; $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$; $\frac{7}{8}$, $\frac{1}{4}$, and $\frac{2}{3}$.

361—375. Reduce to common denominators, $\frac{3}{4}$ and $\frac{3}{11}$; $\frac{5}{9}$ and $\frac{5}{12}$; $\frac{7}{8}$ and $\frac{7}{12}$; $\frac{5}{6}$, $\frac{3}{10}$, and $\frac{8}{15}$; $\frac{1}{6}$, $\frac{9}{14}$, and $\frac{8}{21}$; $\frac{7}{10}$, $\frac{8}{14}$, and $\frac{9}{35}$; $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{9}{16}$; $\frac{1}{2}$, $\frac{2}{7}$ $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{5}{6}$; $\frac{7}{15}$ and $\frac{9}{35}$; $\frac{8}{21}$ and $\frac{4}{33}$; $\frac{2}{15}$, $\frac{11}{18}$, and $\frac{17}{30}$; $\frac{5}{6}$, $\frac{4}{9}$, and $\frac{5}{12}$; $\frac{2}{7}$ and $\frac{9}{13}$; $\frac{1}{3}$, $\frac{3}{8}$, and $\frac{5}{7}$; $\frac{7}{8}$, $\frac{4}{5}$, and $\frac{15}{20}$.

76. CASE VI. TO REDUCE COMPLEX FRACTIONS TO SIMPLE FRACTIONS.

A *complex fraction* is one which contains a fraction in one or both of its terms; as,

$$\frac{2\frac{1}{2}}{3};\ \frac{1}{8\frac{1}{3}};\ \frac{3\frac{1}{8}}{4\frac{2}{3}};\ \frac{\frac{3}{4}}{7};\ \frac{\frac{5}{9}}{\frac{3}{8}}.$$

$$\frac{2\frac{1}{2}}{3}=\frac{\frac{5}{2}}{\frac{3}{1}}=(\S\ 74)\,\frac{\frac{5\times\cancel{2}\times1}{\cancel{2}}}{\frac{3\times2\times1}{1}}=\frac{5\times1}{2\times3}=\frac{5}{6}$$

$$\frac{1}{8\frac{1}{3}}=\frac{\frac{1}{1}}{\frac{25}{3}}=\frac{\frac{1\times1\times3}{1}}{\frac{25\times1\times\cancel{3}}{\cancel{3}}}=\frac{1\times3}{1\times25}=\frac{3}{25}$$

$$\frac{3\frac{1}{8}}{4\frac{2}{3}}=\frac{\frac{25}{8}}{\frac{14}{3}}=\frac{\frac{25\times\cancel{8}\times3}{\cancel{8}}}{\frac{14\times8\times\cancel{3}}{\cancel{3}}}=\frac{25\times3}{8\times14}=\frac{75}{112}$$

$$\frac{\frac{3}{4}}{7}=\frac{\frac{3}{4}}{\frac{7}{1}}=\frac{\frac{3\times\cancel{4}\times1}{\cancel{4}}}{\frac{7\times1\times4}{1}}=\frac{3\times1}{4\times7}=\frac{3}{28}$$

$$\frac{\frac{5}{9}}{\frac{3}{8}}=\frac{\frac{5\times\cancel{9}\times8}{\cancel{9}}}{\frac{3\times9\times\cancel{8}}{\cancel{8}}}=\frac{5\times8}{9\times3}=\frac{40}{27}$$

In reducing complex to simple fractions, we first reduce both numerator and denominator to a fractional form, (writing 1 for a denominator, if either term is a whole number.) Then multiplying the numerators of each term by both the denominators, (which (§ 74,) does not alter the value of the fraction,) we cancel the denominators, and the product of the remaining terms is a simple fraction.

It will be seen in every instance, that the result is the same as if we had first inverted *the denominator, and multiplied the numerators of the resulting fractions together for a new numerator, and the denominators together for a new denominator.* For the denominator of the upper term and the denominator of the lower term are always cancelled, while the products of the numerator of the upper term by the denominator of the lower—and of the numerator of the lower term by the denominator of the upper, are left in the result. Hence, *to reduce a complex fraction to a simple fraction,*

RULE.

First change both terms of the complex fraction to a fractional form, and invert the denominator.

Take the resulting fractions and multiply the numerators together for a new numerator, and the denominators for a new denominator.

Cancel before multiplying, if possible.

EXAMPLES FOR THE PUPIL.

376—395. Reduce to simple fractions, $\frac{1}{2\frac{1}{2}}$; $\frac{1}{2\frac{2}{3}}$; $\frac{2}{1\frac{7}{8}}$; $\frac{3}{\frac{5}{8}}$; $\frac{1\frac{1}{2}}{2}$; $\frac{2\frac{1}{3}}{7}$; $\frac{\frac{3}{4}}{9}$; $\frac{\frac{1}{2}}{\frac{1}{3}}$; $\frac{3\frac{2}{3}}{4\frac{1}{2}}$; $\frac{3\frac{2}{3}}{5}$; $\frac{7}{6\frac{3}{4}}$; $\frac{1\frac{3}{7}}{3}$; $\frac{\frac{2}{5}}{1\frac{2}{7}}$; $\frac{5\frac{3}{5}}{\frac{2}{7}}$; $\frac{\frac{3}{11}}{9}$; $\frac{\frac{4}{7}}{\frac{1}{3}}$; $\frac{2}{\frac{8}{11}}$; $\frac{3\frac{1}{2}}{4}$; $\frac{7\frac{2}{3}}{5}$; $\frac{\frac{3}{7}}{\frac{2}{5}}$.

396—410. Simplify, $\frac{2\frac{1}{2}}{9}$; $\frac{2\frac{1}{2}}{4}$; $\frac{2\frac{1}{2}}{5\frac{1}{2}}$; $\frac{3}{5\frac{1}{2}}$; $\frac{4\frac{2}{3}}{9\frac{1}{3}}$; $\frac{4\frac{2}{3}}{5\frac{1}{6}}$; $\frac{4\frac{2}{3}}{3\frac{1}{2}}$; $\frac{16}{\frac{1}{2}}$; $\frac{14}{\frac{2}{7}}$; $\frac{3\frac{6}{7}}{1\frac{2}{7}}$; $\frac{\frac{3}{4}}{9}$; $\frac{\frac{1}{2}}{7}$; $\frac{2}{5\frac{3}{8}}$; $\frac{2\frac{4}{5}}{\frac{7}{10}}$; $\frac{\frac{3}{5}}{\frac{1}{2}}$.

411—425. Simplify, $\frac{1\frac{1}{2}}{1\frac{1}{3}}$; $\frac{\frac{1}{2}\text{ of }\frac{3}{4}}{\frac{2}{7}\text{ of }5}$; $\frac{16}{17\frac{1}{2}}$; $\frac{3}{3\frac{5}{8}}$; $\frac{4\frac{5}{6}}{7}$; $\frac{8}{9\frac{2}{3}}$; $\frac{\frac{2}{7}}{\frac{3}{8}}$; $\frac{3\frac{4}{5}}{3\frac{7}{8}}$; $\frac{7\frac{1}{2}}{8\frac{1}{4}}$; $\frac{4\frac{4}{5}}{9\frac{3}{5}}$; $\frac{1\frac{1}{3}}{9\frac{3}{5}}$; $\frac{9\frac{3}{5}}{1\frac{1}{5}}$; $\frac{7\frac{1}{2}}{11}$; $\frac{\frac{2}{3}}{9}$; $\frac{5\frac{1}{3}}{\frac{3}{7}}$.

77. CASE .VII. TO REDUCE FRACTIONS TO OTHERS HAVING ANY GIVEN NUMERATOR OR DENOMINATOR.

Reduce $\frac{3}{11}$ to a fraction with 7 for its numerator, and also to a fraction with 5 for its denominator.

$$\frac{3}{11}=\frac{3\times\frac{7}{3}}{11\times\frac{7}{3}}=\frac{7}{25\frac{2}{3}}$$

$$\frac{3}{11}=\frac{3\times\frac{5}{11}}{11\times\frac{5}{11}}=\frac{1\frac{4}{11}}{5}$$

Seven, (§ **70**,) is $\frac{7}{3}$ of 3. If we multiply both terms of a fraction by the same number, (§ **74**,) we do not alter its value. Therefore, if we make the numerator $\frac{7}{3}$ as large as it was at first, we must also make the denominator $\frac{7}{3}$ as large, and $\frac{7}{3}$ of 11 = $25\frac{2}{3}$.

The proposed denominator, 5, is $\frac{5}{11}$ of the original denominator. Taking also $\frac{5}{11}$ of the numerator, we obtain $1\frac{4}{11}$ for the new numerator. Hence, *to reduce any fraction to an equivalent fraction, whose numerator or denominator is given,*

RULE,

Multiply both terms of the given fraction by that fraction which represents the part that the PROPOSED *numerator or denominator is of the* GIVEN *numerator or denominator.*

426—435. Reduce $\frac{4}{9}$ to thirds, halves, fourths, sevenths, elevenths, tenths, sixths, ninths, twentieths, twelfths.

436—445. Reduce $\frac{7}{8}$ to fractions whose numerators are 5, 9, 2, 1, 4, 8, 11, 14, 15, 19.

446—465. Reduce $\frac{3}{4}$ to fractions whose numerators are 9, 5, 1, 8, 2, 16, 4, 6, 20, 17, 7, 27, 10, 11, 12, 31, 42, 13, 15, 14.

466—485. Reduce $\frac{5}{8}$ to halves, 3ds, 7ths, 12ths, 4ths,

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20ths, 11ths, 5ths, 8ths, 10ths, 13ths, 9ths, 25ths, 49ths, 14ths, 18ths, 30ths, 15ths, 19ths, 96ths.

486—490. Reduce $\frac{3}{11}$ to thirds; $\frac{2}{17}$ to ninths; $\frac{4}{15}$ is 5 of what denomination? $\frac{11}{18}$ is 3 of what denomination? $\frac{2}{23}$ is 8 of what denomination?

491—500. Reduce $29\frac{61}{184}$ to a fraction; $\frac{8847}{131}$ to a mixed number; $\frac{287}{1463}$ to a decimal; $\frac{792}{1037}$ to its lowest terms; $\frac{12}{36}$, $\frac{7}{34}$, and $\frac{4}{9}$ to a common denominator; .004625 to a fractional form; 295 to nineteenths; $\frac{4}{41}$ to eighths; $\frac{3}{8}$ of $\frac{5}{6}$ of $\frac{12}{23}$ of $\frac{4}{5}$ to a simple fraction; $\frac{9\frac{2}{3}}{15}$ to a simple fraction.

78. Addition of Fractions.

Since the numerator of any fraction shows how many parts are taken, (§ **70**,) we may add any number of fractions with the same denominators, by simply adding the numerators. Thus, as 5 houses + 3 houses + 2 houses = 10 houses, so $\frac{5}{7} + \frac{3}{7} + \frac{2}{7} = \frac{10}{7}$, and $\frac{5}{13} + \frac{3}{13} + \frac{2}{13} = \frac{10}{13}$.

But if the fractions to be added have different denominators, we must first reduce them to a common denominator. Thus, $\frac{3}{4}$ and $\frac{5}{6}$ would neither make $\frac{8}{4}$ nor $\frac{8}{6}$. But as $\frac{3}{4} = \frac{9}{12}$, and $\frac{5}{6} = \frac{10}{12}$, $\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{12} = 1\frac{7}{12}$.

RULE FOR THE ADDITION OF FRACTIONS.

Reduce all the given fractions to a common denominator. Add the numerators, and place their sum over the common denominator.

If the result is an improper fraction, ***reduce it to a whole or mixed number.***

Examples for the Pupil.

1—10. Add $\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{3}$, and $\frac{5}{3}$; $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, and $\frac{4}{6}$; $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$; $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$; $\frac{3}{7}$, $\frac{4}{7}$, and $\frac{5}{7}$; $\frac{6}{7}$, $\frac{8}{7}$, and $\frac{9}{7}$; $\frac{1}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$; $\frac{9}{11}$, $\frac{6}{11}$, *and* $\frac{10}{11}$; $\frac{21}{23}$ and $\frac{22}{23}$; $\frac{5}{8}$, $\frac{4}{8}$, and $\frac{19}{8}$.

11—20. Add $\frac{11}{12}$, $\frac{5}{12}$, and $\frac{9}{12}$; $\frac{3}{17}$, $\frac{4}{17}$, $\frac{5}{17}$, and $\frac{6}{17}$; $\frac{9}{13}$, $\frac{10}{13}$, and $\frac{11}{13}$; $\frac{1}{9}$, $\frac{8}{9}$, and $\frac{7}{9}$; $\frac{5}{4}$, $\frac{3}{4}$, and $\frac{13}{4}$; $\frac{11}{19}$, $\frac{18}{19}$, and $\frac{4}{19}$; $\frac{1}{13}$, $\frac{3}{13}$, and $\frac{7}{13}$; $\frac{3}{8}$, $\frac{13}{8}$, and $\frac{25}{8}$; $\frac{49}{3}$ and $\frac{58}{3}$; $\frac{5}{2}$, $\frac{11}{2}$ and $\frac{21}{2}$.

21—30. Add $1\frac{2}{3}$, $3\frac{1}{3}$, and 5; $2\frac{1}{2}$, $4\frac{1}{2}$, and $7\frac{1}{2}$; $8\frac{3}{4}$, $2\frac{2}{4}$, and $\frac{1}{4}$; $1\frac{4}{5}$, $\frac{3}{5}$, and $\frac{2}{5}$; $\frac{5}{6}$, $9\frac{5}{6}$, and $\frac{1}{6}$; $\frac{2}{7}$, $\frac{4}{7}$, and 3; $\frac{6}{9}$, 7, and $\frac{7}{9}$; $1\frac{7}{8}$, $1\frac{5}{8}$, and $2\frac{3}{8}$; $5\frac{7}{9}$, $\frac{8}{9}$, and $2\frac{4}{9}$; $\frac{4}{5}$, $\frac{3}{5}$, and $7\frac{2}{5}$.

31—40. Add $\frac{1}{2}$ and $\frac{2}{3}$; $\frac{4}{2}$ and $\frac{3}{4}$; $\frac{2}{3}$, and $\frac{1}{4}$; $\frac{1}{3}$ and $\frac{4}{5}$; $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$; $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$; $\frac{1}{2}$ and $\frac{5}{7}$; $\frac{5}{6}$ and $\frac{5}{9}$; $\frac{2}{3}$, $\frac{1}{6}$, and $\frac{4}{9}$; $\frac{1}{7}$, $\frac{3}{14}$, and $\frac{3}{4}$.

41—50. Add $5\frac{1}{2}$ and $8\frac{3}{4}$; $4\frac{1}{2}$ and $\frac{1}{4}$; $1\frac{2}{3}$ and $5\frac{5}{6}$; $\frac{1}{3}$, $2\frac{1}{3}$, and $2\frac{1}{6}$; $2\frac{1}{3}$, $\frac{1}{2}$, $15\frac{1}{4}$, and $28\frac{5}{6}$; $\frac{2}{3}$ of $\frac{1}{4}$, $\frac{1}{2}$ of $\frac{5}{6}$, and $19\frac{3}{4}$; $\frac{5}{9}$, $\frac{2}{7}$, $\frac{11}{21}$, and $\frac{19}{63}$; $4\frac{3}{4}$, $\frac{7}{9}$, and $1\frac{11}{18}$; $11\frac{5}{6}$, $2\frac{7}{12}$, and $1\frac{1}{9}$; $4\frac{1}{2}$, $\frac{2}{3}$, and $7\frac{4}{9}$.

51—60. Add $\frac{1}{2}$ of $1\frac{1}{3}$, $\frac{1}{2}$ of $2\frac{1}{3}$, and $8\frac{5}{6}$; $8\frac{7}{8}$ and $3\frac{2}{3}$, $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{3}{4}$, and $7\frac{1}{6}$; $3\frac{5}{7}$ and $2\frac{8}{11}$; $\frac{5}{7}$, $\frac{8}{9}$, and $\frac{15}{21}$; $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$; $1\frac{1}{2}$ and $5\frac{9}{11}$; $2\frac{3}{8}$, $5\frac{1}{2}$, and $7\frac{3}{4}$; $16\frac{1}{2}$ and $\frac{7}{9}$; $\frac{4}{5}$, and $2\frac{5}{7}$.

79. SUBTRACTION OF FRACTIONS.

From $9\frac{2}{7}$ take $6\frac{5}{7}$.

$9\frac{2}{7}$
$6\frac{5}{7}$
—
$2\frac{4}{7}$

The difference between any two numbers, (§ **29**,) is the number which must be added to the less to make the greater. If we add $\frac{2}{7}$ to $6\frac{5}{7}$, we shall obtain 7, and adding $2\frac{2}{7}$ *more*, (making $2\frac{4}{7}$ in all,) we obtain $9\frac{2}{7}$. $2\frac{4}{7}$ is therefore the remainder.

Or, as $\frac{5}{7}$ cannot be taken from $\frac{2}{7}$, we add 1 of the 9 units to the $\frac{2}{7}$, making $\frac{9}{7}$, and say $\frac{5}{7}$ from $\frac{9}{7}$ leaves $\frac{4}{7}$. As we have already taken 1 of the 9 units, there are but 8 units left, and 6 from 8 leaves 2. The difference would have been the same, if instead of considering the minuend to be 1 smaller, we had considered the subtrahend to be 1 larger than it really is, and said, *7 from 9 leaves 2.*

RULE FOR THE SUBTRACTION OF FRACTIONS.

Reduce the fractions to a common denominator. Subtract the numerator of the subtrahend from the numerator of the minuend, and write the remainder over the common denominator.

When the fractional part of the subtrahend exceeds that of the minuend, *add the minuend fraction to the difference between the subtrahend fraction and* 1, *and add* 1 *to the units' figure of the subtrahend.*

EXAMPLES FOR THE PUPIL.

61—70. Subtract $\frac{3}{7}$ from $\frac{6}{7}$; $\frac{9}{13}$ from $\frac{10}{13}$; $\frac{4}{13}$ from $\frac{8}{13}$; $\frac{7}{10}$ from $1\frac{3}{10}$; $\frac{1}{2}$ from 5; $\frac{3}{8}$ from $8\frac{1}{3}$; $\frac{1}{8}$ from 9; $\frac{1}{4}$ from 7; $\frac{3}{4}$ from $8\frac{1}{4}$; $\frac{3}{4}$ from $19\frac{3}{4}$.

71—80. How much is $5\frac{1}{2} - \frac{1}{4}$; $9\frac{1}{8} - 2\frac{3}{4}$; $4 - 1\frac{3}{8}$; $6\frac{3}{8} - 2$; $6\frac{3}{8} - 2\frac{4}{5}$; $10\frac{1}{5} - 5\frac{3}{5}$; $8\frac{1}{4} - 7\frac{1}{3}$; $3\frac{3}{4} - 2\frac{1}{2}$; $5\frac{1}{3} - 4\frac{7}{8}$; $9\frac{1}{2} - 1\frac{5}{8}$.

81—90. From $11\frac{5}{12}$ take $10\frac{3}{12}$, $4\frac{1}{12}$, $3\frac{1}{12}$, $7\frac{1}{3}$, $5\frac{4}{12}$, $3\frac{7}{12}$, $2\frac{2}{3}$, $1\frac{1}{2}$, $5\frac{3}{4}$, $6\frac{5}{6}$.

91—110. From $10\frac{1}{3}$ take $9\frac{1}{18}$, $6\frac{1}{9}$, $4\frac{1}{6}$, $7\frac{2}{9}$, $7\frac{5}{18}$, $9\frac{7}{18}$, $4\frac{4}{9}$, $3\frac{1}{2}$, $1\frac{5}{9}$, $5\frac{11}{18}$, $2\frac{2}{3}$, $4\frac{13}{18}$, $6\frac{7}{9}$, $5\frac{5}{6}$, $1\frac{8}{9}$, $9\frac{17}{18}$, $4\frac{11}{12}$, $2\frac{7}{18}$, $5\frac{5}{18}$, $7\frac{7}{18}$.

111—120. Take $5\frac{3}{7}$ from $9\frac{2}{7}$; $8\frac{5}{12}$ from $9\frac{1}{7}$; $\frac{6}{7}$ from $5\frac{3}{14}$; $\frac{3}{4}$ from $9\frac{1}{8}$; $1\frac{5}{6}$ from $2\frac{4}{13}$; $2\frac{5}{6}$ from $8\frac{2}{9}$; $3\frac{3}{4}$ from $4\frac{1}{6}$; $\frac{5}{6}$ from $8\frac{1}{4}$; $\frac{9}{10}$ from $2\frac{3}{4}$; $8\frac{1}{2}$ from $10\frac{3}{10}$.

80. MULTIPLICATION OF FRACTIONS.

If we multiply any number by 4, the product will be 4 times as large as if we multiplied by 1. If we multiply by $\frac{3}{4}$, the product will be $\frac{3}{4}$ as large as if we multiplied by 1. Then $\frac{3}{4} \times 11 = \frac{3}{4}$ of $\frac{11}{1}$; $\frac{2}{7} \times \frac{5}{8} = \frac{2}{7}$ of $\frac{5}{8}$; $4\frac{3}{5} \times 2\frac{1}{2} = \frac{23}{5}$ of $\frac{5}{2}$. Multiplication of fractions, is therefore performed in the same way as the reduc*tion of compound* to simple fractions. (§ 78.)

RULE FOR THE MULTIPLICATION OF FRACTIONS.

Reduce the numbers which are to be multiplied together, to a fractional form, (§ **76**,) *and multiply the numerators together for a new numerator, and the denominators for a new denominator. Cancel before multiplying, if possible.*

Examples for the Pupil.

121—130. Multiply $\frac{1}{2}$ by $\frac{1}{4}$; $\frac{3}{8}$ by $\frac{2}{3}$; $\frac{4}{7}$ by $\frac{4}{1}$; $\frac{4}{7}$ by 3; $\frac{2}{7}$ by 9; $\frac{7}{8}$ by $\frac{8}{7}$; $\frac{3}{4}$ by $\frac{5}{6}$; $\frac{7}{8}$ by $\frac{9}{10}$; $\frac{6}{7}$ by $\frac{2}{3}$; $\frac{7}{8}$ by $\frac{6}{8}$.

131—140. Find the product of $\frac{1}{2}$ of $\frac{3}{4} \times \frac{2}{7}$; $\frac{5}{9} \times 1\frac{4}{5}$; $\frac{7}{8} \times 2\frac{2}{3}$; $3\frac{1}{2} \times 2\frac{5}{9}$; $4 \times 6\frac{2}{3}$; $\frac{3}{4} \times 7\frac{1}{2}$; $\frac{3}{5}$ of $2\frac{1}{2} \times \frac{7}{9}$ of 9; $3\frac{1}{3} \times \frac{7}{15}$; $5\frac{1}{2} \times 5\frac{2}{3}$; $2\frac{3}{4} \times 1\frac{1}{3}$.

141—150. Multiply $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{5}$ by $\frac{5}{7}$ of $\frac{7}{9}$ of 9; $\frac{2}{3}$ by $1\frac{2}{3}$; $\frac{5}{9}$ by $1\frac{4}{5}$; $2\frac{7}{11}$ by $\frac{22}{29}$; $3\frac{1}{8}$ by $\frac{16}{25}$; $5\frac{1}{2}$ by $1\frac{1}{11}$; $\frac{2}{7}$ by $4\frac{5}{6}$; $2\frac{4}{5}$ by $5\frac{4}{7}$; $1\frac{3}{4}$ by $\frac{7}{18}$; $\frac{2}{5}$ of $7\frac{1}{2}$ by $\frac{1}{3}$ of $4\frac{5}{6}$.

151—160. Multiply $3\frac{1}{2}$ by $3\frac{2}{3}$; 6 by $1\frac{5}{9}$; $8\frac{3}{4}$ by 4; $\frac{7}{11}$ by 11; 5 by $8\frac{3}{5}$; 14 by $1\frac{9}{14}$; $2\frac{7}{8}$ by 8; $5\frac{1}{2}$ by 2; $4\frac{3}{4}$ by 12; $\frac{1}{2}$ by $\frac{7}{11}$; $3\frac{1}{2}$ by $\frac{5}{7}$.

161—170. Multiply $4\frac{3}{13}$ by 13; $2\frac{7}{19}$ by 19; $8\frac{3}{5}$ by 15; $5\frac{2}{7}$ by $\frac{7}{37}$; $3\frac{8}{9}$ by 9; $4\frac{5}{6}$ by 6; $3\frac{7}{10}$ by 10; $1\frac{6}{17}$ by 17; $5\frac{2}{11}$ by 11; $5\frac{2}{11}$ by $\frac{22}{57}$.

171—180. Multiply $\frac{3}{4}$ by 4; $\frac{3}{4}$ by $1\frac{1}{3}$; $\frac{7}{9}$ by 9; $\frac{7}{9}$ by $1\frac{2}{7}$; $\frac{6}{11}$ by 11; $\frac{5}{11}$ by $2\frac{1}{5}$; $3\frac{1}{2}$ by $4\frac{2}{3}$; $5\frac{3}{4}$ by $6\frac{4}{7}$; $1\frac{7}{8}$ by $2\frac{1}{5}$; $\frac{3}{7\frac{1}{8}}$ by $\frac{2\frac{1}{2}}{3}$.

81. Division of Fractions.

Divide $\frac{2}{3}$ by $\frac{3}{4}$. $\frac{\frac{2}{3}}{\frac{3}{4}} = \frac{2 \times 4}{3 \times 3} = \frac{8}{9}$. (§ **76**.)

We make the dividend the numerator, and the divisor the denominator of a complex fraction, which is reduced in the usual manner.

The same result may be obtained analytically, as follows: 1 is contained in $\frac{2}{3}$, $\frac{2}{3}$ of a time. $\frac{1}{4}$ will be contained 4 times as often as 1, therefore $\frac{2}{3} \div \frac{1}{4} = \frac{4}{1} \times \frac{2}{3}$. $\frac{3}{4}$ is contained only $\frac{1}{3}$ as often as $\frac{1}{4}$, therefore $\frac{2}{3} \div \frac{3}{4} = \frac{1}{3}$ of $\frac{4}{1} \times \frac{2}{3} = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$.

RULE FOR THE DIVISION OF FRACTIONS.

Reduce both divisor and dividend to a fractional form, invert the divisor, and proceed as in multiplication.

181—190. Divide each of the following numbers by $\frac{1}{4}$. $1\frac{1}{4}$, $1\frac{3}{4}$, 2, $2\frac{1}{4}$, $2\frac{3}{4}$, 9, $3\frac{1}{2}$, $\frac{1}{2}$, $5\frac{1}{2}$, $7\frac{3}{4}$.

191—200. Divide $2\frac{7}{8}$ by $\frac{1}{8}$, $\frac{3}{8}$, $\frac{7}{8}$, $1\frac{3}{8}$, $2\frac{1}{4}$, $2\frac{3}{4}$, $1\frac{1}{2}$, $3\frac{4}{5}$, $3\frac{2}{7}$, 5.

201—210. Divide $2\frac{8}{9}$ by $4\frac{2}{9}$; $1\frac{5}{6}$ by $2\frac{2}{3}$; $\frac{8}{9}$ by $7\frac{1}{3}$; $4\frac{1}{5}$ by $6\frac{4}{5}$; $9\frac{5}{7}$ by 10; $3\frac{9}{10}$ by $4\frac{3}{10}$; $2\frac{1}{2}$ by $4\frac{3}{4}$; $1\frac{1}{8}$ by 2; $6\frac{5}{7}$ by $7\frac{6}{7}$; $2\frac{7}{10}$ by $3\frac{4}{5}$.

211—220. Divide $\frac{5}{8}$ by $2\frac{1}{2}$; $1\frac{4}{9}$ by $2\frac{1}{3}$; $2\frac{2}{3}$ by 8; 7 by $1\frac{3}{4}$; $3\frac{2}{11}$ by $1\frac{1}{2}$; $2\frac{5}{7}$ by $3\frac{4}{5}$; $2\frac{5}{6}$ by $5\frac{1}{3}$; $\frac{2}{7}$ by 3; 5 by $\frac{5}{9}$; 4 by $7\frac{5}{8}$.

221—230. How much is $3\frac{3}{4} \div \frac{3}{4}$; $5\frac{6}{7} \div 8\frac{9}{10}$; $4 : 9\frac{1}{6}$; $2\frac{3}{8} : 7$; $5\frac{1}{2} \div 2\frac{3}{4}$; $2\frac{1}{2} : 7\frac{1}{2}$; $2\frac{2}{3} : 8$; $1\frac{7}{8} \div 4\frac{3}{7}$; $9 \div 16\frac{2}{3}$; $4\frac{4}{11} : 5\frac{1}{8}$.

231—240. Divide $\frac{1}{4}$ of $\frac{3}{7}$ by $\frac{2}{5}$ of 9; $\frac{2}{3}$ of $1\frac{5}{8}$ by $\frac{8}{13}$ of 3; 5 by $\frac{7}{8}$ of $4\frac{1}{2}$; 16 by $\frac{2}{3}$ of $\frac{1}{4}$ of $\frac{6}{13}$; $\frac{12}{35}$ by $\frac{6}{7}$; $2\frac{11}{13}$ by $5\frac{1}{2}$; $2\frac{11}{13}$ by $8\frac{2}{7}$; $14\frac{1}{2}$ by 10; $\frac{1}{2}$ of $\frac{5}{8}$ of $\frac{3}{7}$ by $6\frac{9}{11}$; $11\frac{3}{16}$ by $81\frac{4}{7}$.

82. INFINITE DECIMALS.

Reduce to decimals, $\frac{1}{9}$, $\frac{1}{99}$, $\frac{1}{999}$, $\frac{1}{9999}$.

$\frac{1}{9} = .1111 +$; $\frac{1}{99} = .010101 +$; $\frac{1}{999} = .001001001 +$; $\frac{1}{9999} = .00010001 +$.

In each of these examples, it will be observed that one or more figures are repeated in the same order, and if we examine our work carefully, we shall see that they would continue to repeat, however far we may carry the division. Decimals that contain such repeating figures, are called *infinite*, or *repeating decimals.* The figures that are repeated, are called the *repetend.* The repetend is usually distinguished by placing a point over the first and last figure. Thus, $\dot{3} = .3333+ = \frac{3}{9}$; $.\dot{0}1\dot{7} = .017017+ = \frac{17}{999}$; $14.3\dot{0}\dot{2} = 14.30202+ = 14.3\frac{2}{99} = 14\frac{3\frac{2}{99}}{10} = 14\frac{299}{990}$.

Every repetend is equivalent to a fraction, having the repetend for its numerator, and as many 9's as there are figures in the repetend for its denominator. When there are decimal figures preceding the repetend, they are called the *finite part* of the decimal.

RULE FOR REDUCING INFINITE DECIMALS TO FRACTIONS.

Make the repeating figures the numerator, and as many 9's the denominator of a fraction. If the decimal has a finite part, *write the whole decimal, including the reduced repetend, in the form of a complex fraction, and reduce it to a simple fraction.*

241—250. Reduce the following fractions to infinite decimals, and *prove* the result, by reducing the decimals back to simple fractions: $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$, $\frac{1}{12}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{11}{12}$.

251—270. Reduce the following fractions in like manner: $\frac{1}{6}$; $\frac{5}{6}$; $\frac{1}{11}$; $\frac{3}{11}$; $\frac{7}{11}$; $\frac{9}{11}$; $\frac{10}{11}$; $\frac{1}{18}$; $\frac{5}{18}$; $\frac{7}{18}$; $\frac{11}{18}$; $\frac{13}{18}$; $\frac{17}{18}$; $\frac{1}{24}$; $\frac{5}{24}$; $\frac{7}{24}$; $\frac{11}{24}$; $\frac{13}{24}$; $\frac{17}{24}$; $\frac{19}{24}$.

271—280. Reduce the decimals in the following examples to fractions, and *prove* the result by reducing the fractions back to decimals: $.1\dot{5}$; $.2\dot{3}\dot{0}$; $5.1\dot{6}$; $.00\dot{3}$; $1.0\dot{2}\dot{7}$; $.18\dot{6}\dot{3}$; $.\dot{2}70\dot{9}$; $4.7\dot{5}$; $4.\dot{7}\dot{5}$; $.8\dot{3}$.

281—290. Reduce the following decimals in like manner: $7.\dot{6}$; $41.\dot{3}$; $.8\dot{2}\dot{9}$; $15.\dot{9}$; $4.\dot{0}\dot{9}$; $5.0\dot{3}\dot{6}$; $.0\dot{1}\dot{6}$
$7.\dot{8}\dot{3}$; $.07\dot{8}\dot{3}$; $.09\dot{1}\dot{6}$.

291—300. Perform the following examples, after r
ducing each number to a fractional form: 18 × .i
$28.\dot{5} \div 13.\dot{1}\dot{6}$; $4.\dot{0}7 : 1.\dot{0}\dot{6}$; $4.\dot{6}\dot{3} : 1.\dot{5}\dot{4}$; $2.0\dot{1}$ × 3.
$.0\dot{8}\dot{2}$ of $6.\dot{7}$; $.01\dot{8} : 9$; $5 \times 7.0\dot{1}\dot{6}$; $4.\dot{4} \times .\dot{3}$; $.\dot{0}\dot{8}1$:

Addition and subtraction of infinite decimals may usually be performed with sufficient accuracy, by extending the decimals to four or five places.

83. MISCELLANEOUS EXERCISES IN FRACTIONS.

301—310. Reduce to whole numbers, $\frac{129}{3}$; $\frac{48}{16}$; $\frac{165}{11}$; $\frac{165}{15}$; $\frac{169}{13}$; $\frac{361}{19}$; $\frac{567}{27}$; $\frac{899}{31}$; $\frac{308}{14}$; $\frac{800}{16}$.

311—320. Reduce 28 to 4ths, 7ths, 11ths, 15ths, 22ds, 39ths, 63ds, 80ths, 100ths, 250ths.

321—330. Reduce to simple fractions $\frac{3}{4}$ of $\frac{4}{5}$; $\frac{1}{3}$ of $\frac{7}{18}$ of 6; $\frac{2}{3}$ of $\frac{10}{13}$ of 39; $\frac{3}{4}$ of $\frac{7}{2}$ of $2\frac{2}{11}$; $\frac{7}{18}$ of $\frac{3}{8}$; $\frac{5}{9}$ of $1\frac{4}{5}$; $\frac{2}{7}$ of $\frac{11}{3}$ of $2\frac{5}{8}$; $\frac{16}{5}$ of $9\frac{2}{7}$; $\frac{2}{11}$ of $\frac{4}{7}$ of $8\frac{5}{9}$; $\frac{3}{8}$ of 90.

331—340. Simplify $\frac{1\frac{1}{2}}{3}$; $\frac{2}{8\frac{3}{5}}$; $\frac{1}{9\frac{1}{3}}$; $\frac{2\frac{4}{5}}{7\frac{3}{5}}$; $\frac{\frac{3}{7}}{9}$; $\frac{4}{\frac{3}{5}}$; $\frac{11}{6\frac{2}{3}}$; $\frac{5\frac{2}{3}}{8}$; $\frac{3\frac{3}{7}}{9\frac{3}{5}}$; $\frac{4\frac{1}{2}}{\frac{1}{3}}$.

341—350. Add $2\frac{1}{2}$ and $3\frac{5}{8}$; $\frac{1}{3}$, $\frac{1}{4}$, and $3\frac{3}{4}$; $2\frac{1}{3}$, $\frac{5}{6}$, and $7\frac{1}{2}$; $\frac{3}{8}$, $\frac{7}{8}$, and $9\frac{5}{8}$; $7\frac{2}{3}$, $5\frac{3}{8}$, and $2\frac{1}{4}$; $3\frac{1}{3}$, $4\frac{1}{2}$, and $\frac{3}{4}$; $\frac{5}{6}$, $\frac{4}{9}$, and $6\frac{1}{2}$; $5\frac{5}{9}$ and $1\frac{3}{4}$; $\frac{11}{12}$ and $\frac{17}{18}$; $4\frac{5}{6}$, $3\frac{7}{10}$, and $9\frac{8}{15}$.

351—360. How much is $\frac{3}{11} : \frac{5}{9}$; $\frac{4}{7} : \frac{2}{3}$; $3\frac{1}{2} \div 4\frac{2}{5}$; $\frac{1}{3}$ of $\frac{3}{7}$ of $1\frac{13}{15} : \frac{2}{5}$ of $\frac{9}{2}$ of $4\frac{4}{9}$; $\frac{3}{7} \div 2\frac{6}{11}$; $5\frac{1}{9} \div 3\frac{1}{2}$; $13 : 4\frac{4}{5}$; $3\frac{4}{5} \div 17$; $6\frac{2}{3} \div \frac{5}{7}$; $\frac{4}{9} : \frac{3}{44}$.

361—370. Reduce to mixed numbers, $\frac{62}{11}$; $\frac{95}{13}$; $\frac{140}{19}$; $\frac{87}{8}$; $\frac{224}{15}$; $\frac{154}{17}$; $\frac{980}{23}$; $\frac{999}{29}$; $\frac{999}{18}$; $\frac{1001}{31}$.

371—380. Reduce to a fractional form, $7\frac{4}{5}$; $13\frac{3}{7}$; $12\frac{5}{9}$; $8\frac{4}{11}$; $5\frac{6}{13}$; $18\frac{5}{6}$; $14\frac{9}{10}$; $23\frac{12}{17}$; $49\frac{3}{19}$; $108\frac{3}{56}$.

381—390. Reduce to their lowest terms, $\frac{18}{27}$; $\frac{72}{208}$; $\frac{219}{292}$; $\frac{266}{418}$; $\frac{119}{323}$; $\frac{87}{401}$; $\frac{339}{584}$; $\frac{240}{624}$; $\frac{436}{724}$; $\frac{164}{988}$.

391—400. Find the denominators that are indicated by a —, in the following fractions: (§ **77**,) $\frac{1}{2} = \frac{5}{\quad}$; $\frac{3}{7} = \frac{2}{\quad}$; $\frac{6}{11} = \frac{18}{\quad}$; $\frac{15}{2} = \frac{1}{\quad}$; $5\frac{1}{3} = \frac{4}{\quad}$; $\frac{3}{19} = \frac{19}{\quad}$; $\frac{4}{23} = \frac{7}{\quad}$; $7\frac{2}{3} = \frac{2}{\quad}$; $4\frac{7}{11} = \frac{1}{\quad}$; $\frac{3}{7} = \frac{9}{\quad}$.

401—410. Subtract $3\frac{1}{2}$ from $4\frac{5}{8}$; $2\frac{1}{3}$ from $8\frac{7}{8}$; 5

from $15\frac{1}{4}$; $7\frac{5}{8}$ from $8\frac{1}{15}$; $4\frac{6}{7}$ from $9\frac{3}{4}$; $\frac{9}{10}$ from $6\frac{3}{5}$; $\frac{13}{15}$ from $2\frac{2}{3}$; $4\frac{6}{7}$ from $7\frac{1}{2}$; $13\frac{11}{12}$ from $17\frac{7}{30}$; $49\frac{17}{21}$ from $93\frac{6}{35}$.

411—420. Reduce to fractions or mixed numbers, $.\dot{1}3$; $2.0\dot{7}$; $.\dot{8}\dot{4}$; $7.\dot{6}\dot{2}$; $13.9\dot{5}\dot{4}$; $.8\dot{0}\dot{2}$; $.36\dot{7}$; $1.4\dot{9}$; $.30\dot{9}$; $.87\dot{6}\dot{3}$.

421—430. Reduce to a decimal, $\frac{5}{6}$; $\frac{4}{19}$; $\frac{37}{128}$; $\frac{9}{13}$; $\frac{7}{15}$; $\frac{29}{30}$; $\frac{77}{96}$; $\frac{7}{80}$; $\frac{4}{40}$; $\frac{7}{111}$.

431—440. Reduce .984 to tenths, hundredths, thousandths, and ten-thousandths; .0684 to hundredths; 1.923 to hundredths and thousandths; 48.2 to hundredths; .0075 to thousandths and millionths.

441—450. Reduce to a common denominator $\frac{14}{35}$ and $\frac{10}{21}$; $\frac{4}{77}$ and $\frac{13}{88}$; $\frac{10}{33}$ and $\frac{5}{21}$; $\frac{7}{2}$, $\frac{5}{6}$, $\frac{11}{14}$, and $\frac{8}{21}$; $\frac{5}{11}$ and $\frac{8}{9}$; $\frac{4}{5}$ and $\frac{8}{13}$; $\frac{3}{10}$, $\frac{1}{14}$, and $\frac{19}{35}$; $\frac{9}{11}$ and $\frac{9}{17}$; $\frac{5}{6}$ and $\frac{5}{13}$; $\frac{4}{7}$, $\frac{2}{3}$, and $\frac{2}{5}$.

451—460. Find the numerators indicated by a —, in the following fractions; $\frac{1}{2} = \frac{}{5}$; $\frac{2}{7} = \frac{}{9}$; $\frac{3}{17} = \frac{}{2}$; $\frac{8}{9} = \frac{}{3}$; $\frac{4}{5} = \frac{}{11}$; $\frac{8}{8} = \frac{}{9}$; $\frac{1}{17} = \frac{}{4}$; $\frac{3}{8} = \frac{}{11}$; $\frac{3}{7} = \frac{}{9}$; $8\frac{7}{11} = \frac{}{15}$.

461—470. Find the following products: $8\frac{1}{2} \times 2\frac{2}{3}$; $15 \times \frac{7}{9}$; $\frac{4}{11} \times 23$; $6\frac{2}{5} \times 5$; $7 \times 8\frac{3}{7}$; $14\frac{1}{9} \times 81$; $5\frac{7}{8} \times 9\frac{3}{5}$; $4\frac{5}{6} \times 7\frac{8}{9}$; $\frac{2}{7}$ of $15\frac{1}{3} \times 5\frac{1}{4}$; $\frac{1}{2}$ of $\frac{5}{7}$ of $11\frac{5}{11} \times \frac{11}{15}$ of $\frac{11}{15}$.

471. How many potatoes at $\frac{3}{8}$ of a dollar a bushel, can be bought for $1\frac{1}{4}$ dollars? (See § **55**.)

472—475. How much corn at $\frac{3}{4}$ of a dollar per bushel, would cost $5\frac{7}{8}$ dollars? $2\frac{1}{5}$ dollars? $3\frac{3}{4}$ dollars? 6 dollars?

476—480. If $\frac{7}{19}$ barrels of flour cost $1\frac{3}{4}$ dollars, what is the cost of 1 barrel? of $2\frac{2}{3}$ barrels? $3\frac{1}{2}$ barrels? 11 barrels? $5\frac{7}{8}$ barrels?

481—485. What must I give per yard for cloth, if $2\frac{3}{4}$ yards cost, $15\frac{1}{8}$ dollars? $13\frac{1}{5}$ dollars? $14\frac{2}{3}$ dolls.? $7\frac{9}{16}$ *dolls.? 11 dolls.?*

486—490. How long will it take a stage to run [illegible] miles, if in 1 hour it runs $9\frac{1}{4}$ miles? 5 miles? $7\frac{2}{3}$ miles? $6\frac{1}{4}$ miles? $6\frac{7}{11}$ miles?

491—495. If a family consume $\frac{3}{8}$ of a barrel of flour in a month, how long will it take them to use 5 barrels? $4\frac{3}{4}$ barrels? $7\frac{1}{3}$ barrels? $3\frac{7}{8}$ barrels? $12\frac{3}{4}$ barrels?

496—500. If a man walk a mile in $\frac{2}{5}$ of an hour, how far will he walk in $1\frac{3}{10}$ hours? in 4 hours? 5[illegible] hours? $8\frac{2}{3}$ hours? $7\frac{4}{5}$ hours?

501—505. At $28\frac{3}{4}$ dollars for $6\frac{3}{8}$ yards of cloth, what is the price of 1 yard? 8yds.? 16yds.? $7\frac{1}{2}$yds.? $9\frac{1}{3}$yds.?

506—510. What is the price of butter per pound, when $2\frac{2}{3}$ pounds cost $\frac{1}{2}$ dollar? $\frac{3}{4}$doll.? 1doll.? $\frac{5}{6}$doll.? $\frac{8}{15}$doll.?

511—515. When oats are $\frac{3}{8}$ of a dollar per bushel, how many can be bought for $1\frac{1}{2}$ dollars? $\frac{4}{5}$dolls.? [illegible] dolls.? $\frac{1}{5}$dolls.? $6\frac{2}{3}$dolls.?

516—520. What is the price of sugar per pound, if $5\frac{2}{3}$ pounds cost 34 cents? $25\frac{1}{2}$cts.? $29\frac{3}{4}$cts.? $38\frac{1}{4}$cts.? $59\frac{1}{2}$cts.?

521—530. If a man can build $15\frac{2}{3}$ rods of wall in $5\frac{7}{8}$ days, how much can he build in 1 day? in $7\frac{1}{2}$ days? $3\frac{3}{8}$ days? $6\frac{5}{9}$ days? 8 days? $\frac{3}{8}$ of a day? $\frac{2}{3}$ of a day? $5\frac{1}{4}$ days? $13\frac{1}{3}$ days? $2\frac{7}{9}$ days?

531—540. If $\frac{7}{8}$ of an acre of land cost $31\frac{1}{2}$ dollars, what is the cost of 1 acre? of $6\frac{1}{3}$, $4\frac{3}{4}$, $8\frac{5}{8}$, $15\frac{1}{3}$, $9\frac{7}{1}$[illegible], $4\frac{4}{5}$, $11\frac{3}{8}$, $5\frac{2}{3}$, $6\frac{13}{24}$ acres?

VIII. COMPOUND NUMBERS.

84. When different denominations of money, weights, or measures, (as pounds, shillings, and pence; pounds, ounces, and drams; gallons, quarts, and pints,) are embraced in one expression, they are called COMPOUND NUMBERS. The operations upon them may be *performed* by regarding each denomination as a *fraction of the next* higher. The following Tables, which are the ones most frequently employed, should be ***thoroughly*** committed to memory.

85. Federal Money.

1 eagle is 10 dollars.
1 dollar is 10 dimes.
1 dime is 10 cents.
1 cent is 10 mills.

1 mill is $\frac{1}{10}$ of a cent.
1 cent is $\frac{1}{10}$ of a dime.
1 dime is $\frac{1}{10}$ of a dollar.
1 dollar is $\frac{1}{10}$ of an eagle.

ea.		$		di.		ct.		m.
1	=	10	=	100	=	1000	=	10000
		1	=	10	=	100	=	1000
				1	=	10	=	100
						1	=	10

86. English Money.

1 pound (£) is 20 shillings s.
1 shilling is 12 pence (d.)
1 penny is 4 farthings (qr.)

1 farthing is $\frac{1}{4}$ of a penny.
1 penny is $\frac{1}{12}$ of a shilling.
1 shilling is $\frac{1}{20}$ of a pound.

£		s.		d.		qr.
1	=	20	=	240	=	960
		1	=	12	=	48
				1	=	4

A guinea is 21 shillings. In exchanging the values of English and Federal Money, 1£ is estimated at $\frac{40}{9}$\$, and an allowance is afterwards made for premium, (§ **136**.)

87. Troy Weight.

1 pound (lb.) is 12 ounces (oz.)
1 ounce is 20 pennyweights (dwt.)
1 pennyweight is 24 grains (gr.)

1 gr. is $\frac{1}{24}$ of a dwt.
1 dwt. is $\frac{1}{20}$ of an oz.
1 oz. is $\frac{1}{12}$ of a lb.

lb.		oz.		dwt.		gr.
1	=	12	=	240	=	5760
		1	=	20	=	480
				1	=	24

Troy Weight is employed in weighing liquids and the precious metals.

88. APOTHECARIES' WEIGHT.

1 pound (℔) is 12 ounces (℥.)
1 ounce is 8 drams (ʒ.)
1 dram is 3 scruples (℈.)
1 scruple is 20 grains (gr.)

1 gr. is $\frac{1}{20}$ of a ℈.
1 ℈ is $\frac{1}{3}$ of a ʒ.
1 ʒ is $\frac{1}{8}$ of an ℥.
1 ℥ is $\frac{1}{12}$ of a ℔.

℔	℥	ʒ	℈	gr.
1 =	12 =	96 =	288 =	5760
	1 =	8 =	24 =	480
		1 =	3 =	60
			1 =	20

Apothecaries' Weight is used in compounding medicines.

89. AVOIRDUPOIS WEIGHT.

1 ton (T.) is 20 hundred-weight cwt.
1 hundred-weight is 4 quarters (qr.)
1 quarter is 28 pounds (lb.)
1 pound is 16 ounces (oz.)
1 ounce is 16 drams (dr.)
1 dram is 27$\frac{11}{32}$ Troy grains.

1 gr. Ty. is $\frac{32}{875}$ of a dr.
1 dr. is $\frac{1}{16}$ of an oz.
1 oz. is $\frac{1}{16}$ of a lb.
1 lb. is $\frac{1}{28}$ of a qr.
1 qr. is $\frac{1}{4}$ of a cwt.
1 cwt. is $\frac{1}{20}$ of a T.

T.	cwt.	qr.	lb.	oz.	dr.	gr. Troy.
1 =	20 =	80 =	2240 =	35840 =	573440 =	15680000
	1 =	4 =	112 =	1792 =	28672 =	784000
		1 =	28 =	448 =	7168 =	196000
			1 =	16 =	256 =	7000
				1 =	16 =	437$\frac{1}{2}$
					1 =	27$\frac{11}{32}$

In some sections of the country the hundred-weight is estimated at 100 pounds, and the ton at 2000 pounds. But the cwt. of 112 lb., and the ton of 2240 lb., are still in general use, particularly in weighing heavy or cheap materials, such as iron, coal, plaster, &c.

The pound, ounce, and grain, are the same in Troy and Apothecaries' Weight. The Avoirdupois ounce is less than the Troy ounce, and the Avoirdupois pound is $\frac{175}{144}$ of the Troy pound.

90. LONG MEASURE.

1 mile (m.) is 8 furlongs (f.)
1 furlong is 40 rods (r.)
1 rod is 5½ yards (yd.)
1 yard is 3 feet (ft.)
1 foot is 12 inches (in.)

1 in. is $\frac{1}{12}$ of a ft.
1 ft. is ⅓ of a yd.
1 yd is $\frac{2}{11}$ of a r.
1 r. is $\frac{1}{40}$ of a f.
1 f. is ⅛ of a m.

m.	f.	r.	yd.	ft.	in.
1 =	8 =	320 =	1760 =	5280 =	63360
	1 =	40 =	220 =	660 =	7920
		1 =	5½ =	16½ =	198
			1 =	3 =	36
				1 =	12

Long Measure is used in measuring lengths and distances.

91. CLOTH MEASURE.

Cloth Measure is used in measuring Dry Goods. The yard is of the same length as the yard in Long Measure.

1 yard (yd.) is 4 quarters (qr.)
1 quarter is 4 nails (na.)
1 nail is 2¼ inches.

1 in. is $\frac{4}{9}$ of a na.
1 na. is ¼ of a qr.
1 qr. is ¼ of a yd.

yd.	qr.	na.	in.
1 =	4 =	16 =	36
	1 =	4 =	9
		1 =	2¼

92. SQUARE MEASURE.

1 square mile is 640 acres (A.)
1 acre is 4 roods (R.)
1 rood is 40 rods (r.)
1 rod is 30¼ sq. yards.
1 yard is 9 sq. feet.
1 foot is 144 sq. inches.

1 sq. in. is $\frac{1}{144}$ of a sq. ft.
1 sq. ft. is ⅑ of a sq. yd.
1 sq. yd. is $\frac{4}{121}$ of a sq. r.
1 sq. rod is $\frac{1}{40}$ of a R.
1 rood is ¼ of an acre.
1 A. is $\frac{1}{640}$ of a sq. mile.

m.	A.	R.	sq. r.	sq. yd.	sq ft.	sq. in.
1 =	640 =	2560 =	102400 =	3097600 =	27878400 =	4014489600
	1 =	4 =	160 =	4840 =	43560 =	6272640
		1 =	40 =	1210 =	10890 =	1568160
			1 =	30¼ =	272¼ =	39204
				1 =	9 =	1296
					1 =	144

A square inch, foot, or mile, is a square, each side of which is 1 inch, 1 foot, or 1 mile in length. To find the number of square inches, feet, or rods in any rectangular surface,* (§ **45**, Ex. 580—582,) *multiply the number of inches, feet, or rods in the length, by the number in the breadth.*

93. CUBIC MEASURE.

1 cubic yard is 27 c. feet.	1 c. in. is $\frac{1}{1728}$ of a c. ft.
1 cubic foot is 1728 c. inches.	1 c. ft. is $\frac{1}{27}$ of c. yd.

A foot of wood is 16 cubic feet. 8 feet of wood, or 128 cubic feet, make a cord. A ton of round timber is 50 cubic feet. A ton of square timber is 40 cubic feet. A ton of storage or shipping is 40 cubic feet.

A cubic inch, foot, or yard, is a cube, the length, breadth, and thickness of which are each 1 inch, 1 foot, or 1 yard. To find the number of cubic inches, feet, or yards, in any solid bounded by rectangular surfaces, *multiply together the number of inches, feet, or yards, in the length, breadth, and thickness.* For example, if one surface of a solid contain 5 square feet, and its thickness be 1 foot, the solid will contain 5 cubic feet; if the thickness be 4 feet, it will contain 4 × 5 cubic feet. But the surface is found (§ **92**) by multiplying the length by the breadth; therefore the solid contents must be found by multiplying together the length, breadth, and thickness.

94. DRY MEASURE.

1 bushel is 4 pecks (pk.)	1 pt. is ½ of a qt.
1 peck is 8 quarts (qt.)	1 qt. is ⅛ of a pk.
1 quart is 2 pints (pt.)	1 pk. is ¼ of a bu.

bu.		pk.		qt.		pt.
1	=	4	=	32	=	64
		1	=	8	=	16
				1	=	2

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* *A rectangular surface* is a surface bounded by four sides, in *which the opposite* sides are equal, and the corners are square.

Dry Measure is used in measuring most articles of
d that are not sold by weight. The half-peck is
8$\frac{4}{5}$ cubic inches.

95. Liquid Measure.

ogshead (hhd.) is 63 gallons.
allon is 4 quarts.
uart is 2 pints.
int is 4 gills.

1 gi. is $\frac{1}{4}$ of a pt.
1 pt. is $\frac{1}{2}$ of a qt.
1 qt. is $\frac{1}{4}$ of a gall.
1 gall. is $\frac{1}{63}$ of a hhd.

hhd.	gal.	qt.	pt.	gi.
1 =	63 =	252 =	504 =	2016
	1 =	4 =	8 =	32
		1 =	2 =	8
			1 =	4

The hogshead is used only in estimating the con-
ts of cisterns, wells, or large bodies of water. The
mmon gallon is 231 cubic inches. A gallon of milk
malt liquor is 282 cubic inches.

96. Circular and Astronomical Measure.

ircumference is 360 degrees (°.)
egree is 60 minutes (′.)
inute is 60 seconds (″.)

1″ is $\frac{1}{60}$ of a ′.
1′ is $\frac{1}{60}$ of a °.
1° is $\frac{1}{360}$ of a circ.

circ.	°	′	″
1 =	360 =	21600 =	1296000
	1 =	60 =	3600
		1 =	60

A *quadrant* is 90 degrees. A *sign* of the zodiac is
degrees.

97. Time Measure.

year is 365 days (dy.)
ay is 24 hours (h.)
hour is 60 minutes (min.)
minute is 60 seconds (sec.)

1 sec. is $\frac{1}{60}$ of a min.
1 min. is $\frac{1}{60}$ of an hour.
1 hour is $\frac{1}{24}$ of a day.
1 dy. is $\frac{1}{365}$ of a y.

y.	dy.	h.	min.	sec.
1 =	365 =	8760 =	525600 =	31536000
	1 =	24 =	1440 =	86400
		1 =	60 =	3600
			1 =	60

A *Leap Year* is 366 days. The Leap Years are all those that are divisible by 4; (as 1852, 1856;) except the centurial years, which are Leap Years only when the number of hundreds is divisible by 4, as 2000, 2400, 2800. In business transactions, 30 days are considered a month, and 12 months a year.

The number of days in each month of the common year, may be readily learned by the following lines:

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except the second month alone,
To which we twenty-eight assign,
Till Leap Year gives it twenty-nine.

98. Exercises upon the Tables.

1—10. How many farthings in 9 pence? in 11d.? 16d.? 27d.? 19d.? 44d.? 35d.? 50d.? 75d.? 39d.?

11—20. How many pence in 2 shillings? in 3s.? 4s.? 5s.? 7s.? 9s.? 14s.? 15s.? 19s.? 49s.?

21—30. How many shillings in 4 pounds? in 2£? 7£? 9£? 13£? 15£? 20£? 25£? 37£? 49£?

31—40. How many pence in 8 farthings? in 12qr.? 24qr.? 36qr.? 48qr.? 60qr.? 96qr.? 120qr.? 256qr.? 592qr.?

41—50. How many shillings in 24 pence? in 36d.? 72d.? 108d.? 180d.? 240d.? 276d.? 384d.? 492d.? 672d.?

51—60. How many pounds in 40 shillings? in 80s.? 120s.? 180s.? 220s.? 360s.? 580s.? 640s.? 720s.? 980s.?

61—70. Reduce £5 3s. to shillings; 9s. 5d. to d.; 11d. 3qr. to qr.; 4d. 1qr. to qr.; 8s. 10d. to d. and to qr.; £4 to s., to d., and to qr.; £1 11s. to qr.

71—80. Reduce 49qr. to d. and to s.; 492d. to s. and to £; 480d. to s. and to £; 144qr. to d. and to s.; 1000qr. to d. and to s.

81—90. How many pennyweights in 48 grains? in 72gr.? 120gr.? 192gr.? 264gr.? 300gr.? 375gr.? 590gr.? 742gr.? 960gr.?

91—100. How many ounces in 60 pennyweights? in 100dwt.? 140dwt.? 210dwt.? 440dwt.? 960dwt.? 950dwt.? 740dwt.? 900dwt.? 630dwt.?

101—110. How many pounds in 24 ounces Troy? in 48oz.? 72oz.? 96oz.? 120oz.? 192oz.? 250oz.? 310oz.? 519oz.? 647oz.?

111—120. How many ounces in 5 pounds? in 5lb. 11oz.? 10lb.? 10lb. 9oz.? 15lb. 3oz.? 7lb. 7oz.? 8lb. 10oz.? 11lb. 8oz.? 6lb. 4oz.? 13lb. 6oz.?

121—130. How many pennyweights in 4 ounces? in 7oz.? 7oz. 13dwt.? 9oz. 19dwt.? 10oz. 11dwt.? 5oz. 18dwt.? 29oz. 4dwt.? 47oz. 11dwt.? 74oz. 17dwt.? 4lb. 3oz. 16dwt.?

131—140. How many grains in 3 pennyweights? in 3dwt. 17gr.? 4dwt. 23gr.? 5dwt. 19gr.? 10dwt. 22gr.? 15dwt. 7gr.? 13dwt. 20gr.? 5dwt. 21gr.? 11dwt. 18gr.? 4oz. 2dwt. 9gr.?

141—150. How many drams in 5 ounces? in 5℥ 7ʒ? 7℥ 3ʒ? 9℥ 6ʒ? 13℥ 4ʒ? 19℥ 1ʒ? 27℥ 5ʒ? 36℥ 2ʒ? 2℔ 9℥ 4ʒ? 5℔ 4℥ 6ʒ?

151—160. How many grains in 2 scruples? in 5℈? 4℈ 19gr.? 7℈ 13gr.? 11℈ 17gr.? 14℈ 14gr.? 3ʒ 2℈ 5gr.? 7ʒ 1℈ 18gr.? 1℥ 3ʒ 2℈ 16gr.? 5℥ 7ʒ 1℈ 12gr.?

161—170. Reduce 6000 grains to scruples, to drams, to ounces, and to pounds; 700℈ to ʒ, to ℥, and to ℔; 500ʒ to ℥, and to ℔; 470℥ to ℔.

171—180. How many hundred weight in 4 tons? in 4T. 17cwt.? 5T. 3cwt.? 11T. 15cwt.? 4T. 9cwt.? 8T. 13cwt.? 15T. 19cwt.? 12T. 16cwt.? 10T. 10cwt.? 7T. 18cwt.?

181—190. How many pounds in 3 quarters? in 7qr.? 7qr. 19lb.? 11qr. 27lb.? 2cwt. 3qr. 15lb.? 5cwt. 2qr. 20lb.? 7cwt. 1qr. 8lb.? 4cwt. 1qr. 25lb.? 13cwt. 13lb.? 3cwt. 3qr. 23lb.?

191—200. How many drams in 8 ounces? in 1lb.? 1lb. 3oz.? 1lb. 7oz. 9dr.? 2lb. 4oz. 10dr.? 5lb.? 6lb. 2oz.? 3lb. 15oz. 12dr.? 7lb. 7oz. 7dr.? 9lb.?

201—210. Reduce to pounds, ounces, and drams, 90dr.; 200dr.; 480dr.; 576dr.; 830dr.; 729dr. *553dr.; 872dr.*; 928dr.; 898dr.

211—220. Reduce to T., cwt., qr., and lb., 97lb.; 72lb.; 84lb.; 196lb.; 270lb.; 308lb.; 560lb.; 756lb.; 4905lb.; 7632lb.

221—230. How many feet in 72 inches? in 48in.? 144in.? 192in.? 200in.? 540in.? 935in.? 999in.? 4320in.? 8763in.?

231—240. How many yards in 18 feet? in 39ft.? 48ft.? 87ft.? 99ft.? 200ft.? 250ft.? 440ft.? 555ft.? 879ft.?

241—250. How many furlongs in 80 rods? in 120r.? 160r.? 280r.? 720r.? 960r.? 832r.? 875r.? 963r.? 979r.?

251—260. How many miles in 24 furlongs? in 72fur.? 96fur.? 176fur.? 150fur.? 235fur.? 478fur.? 923fur.? 550fur.? 387fur.?

261—270. Reduce 5m. to fur. and to r.; 7m. 6fur. to fur. and to r.; 18m. 7fur. to fur. and to r.; 4m. 3fur. 29r. to r.; 4yd. 2ft. to ft. and to in.; 395yd. to in.

271—280. How many acres in 5 square miles? in 7sq. m.? 3sq. m.? 2sq. m. 139A.? 7sq. m. 625A.? 4sq. m.? 6sq. m. 500A.? 13sq. m.? 40sq. m. 40A.? 79sq. m.?

281—290. How many square yards in 4 square rods? 8r.? 20r.? 79r.? 2R. 5r.? 8R.? 1R. 24r.? 2R. 16r.? 5R.? 2A.?

291—300. How many square feet in 432 sq. inches? in 864sq. in.? 720sq. in.? 936sq. in.? 1500sq. in.? 1728sq. in.? 2730sq. in.? 3456sq. in.? 9876sq. in.? 8740sq. in.?

301—310. How many sq. rods, how many roods, and how many acres, in a piece of land 80 rods long, and 4 rods wide? 60r. long and 7r. wide? 20r. and 18r.? 47r. and 9r.? 55r. and 10r.? 80r. and 20r.? 90r. and 29r.? 37r. and 36r.? 44r. and 30r.? 70r. and 7r.?

311—320. How many cubic feet in 3c. yards? 5yd.? 7yd.? 13yd.? 13yd. 19ft.? 6yd. 26ft.? 20yd. 20ft.? 8yd. 17ft.? 4yd. 25ft.? 12yd. 21ft.?

321—330. How many cubic inches in 2c. feet? 5ft.?

8ft.? 3ft. 140in.? 19ft.? 4ft. 1720in.? 1yd. 3ft.? 2yd. 26ft.? 3yd. 5ft. 1670in.? 4yd.?

331—340. Reduce to cubic feet and to yards, 2720c. inches; 5990in.; 5400in.; 54000in.; 77260in.; 95350in.; 89765in.; 93274in.; 125000in.; 89775in.

341—350. How many pecks in 3 bushels? 7bu.? 11bu.? 13bu.? 15bu. 3pk.? 17bu. 1pk.? 29bu. 2pk.? 87bu. 2pk.? 93bu. 1pk.? 48bu. 3pk.?

351—360. How many quarts in 5 pecks? 12pk.? 3pk. 7qt.? 2pk. 6qt.? 19pk.? 2bu. 2pk.? 1bu. 3pk. 5qt.? 5bu. 1pk. 4qt.? 8bu. 3pk. 2qt.? 10bu. 2pk. 7qt.?

361—370. How many pints in 7 quarts? 15qt.? 37qt.? 69qt. 1pt.? 5pk. 3qt.? 7pk. 7qt.? 4bu.? 1bu. 3pk.? 2bu. 2pk.? 3bu. 1pk. 5qt. 1pt.?

371—380. Reduce to quarts, pecks, and bushels, 19 pints; 64pt.; 192pt.; 384pt.; 370pt.; 181pt.; 527pt.; 831pt.; 1263pt.; 2972pt.

381—390. How many quarts and how many pints in 5 gallons? 12 gallons? 17gal.? 27gal. 3qt.? 9gal. 2qt.? 31gal. 1qt.? 44gal.? 2hhd.? 3hhd.? 48gal. 2qt.?

391—400. How many gallons in 2 hogsheads? 5hhd.? 7hhd.? 15hhd.? 20hhd.? 8hhd. 29gal.? 4hhd. 60gal.? 15hhd. 47gal.? 60hhd.? 97hhd.?

401—410. How many gills in 7 pints? 27pt.? 63pt.? 12qt.? 19gal.? 47gal.? 2hhd.? 5gal. 3qt.? 7gal. 2qt. 1pt.? 5gal. 3qt. 1pt. 2gi.?

411—420. Reduce to pints, to quarts, and to gallons, 64 gills; 160gi.; 278gi.; 352gi.; 512gi.; 416gi.; 864gi.; 900gi.; 537gi.; 1319gi.

421—430. Reduce to quarts, to gallons, and to hogsheads, 128pt.; 740pt.; 834pt.; 1116pt.; 1380pt.; 1597pt.; 4820pt.; 1500pt.; 2520pt.; 4536pt.

431—440. How many degrees in 180 minutes? 720′? 900′? 1036′? 1144′? 1872′? 3600′? 4380′? 9000′? 7259′?

441—450. How many degrees in 2 signs of the zodiac? in 7 signs? 11s.? 4s. 28°? 9s. 17°? 8s. 9°? 10s. 13°? 5s. 29°? 7s. 12°? 11s. 22°?

451—460. How many degrees in 3 circumferences?

in 9circ.? 12circ.? 18circ.? 27circ.? 14circ.? 11circ.? 17circ.? 3circ. 180°? 7 circ. 269°?

461—470. Reduce to minutes and to hours 480 seconds; 1200sec.; 2520sec.; 2940sec.; 4000sec.; 9500sec.; 9832sec.; 4597sec.; 10800sec.; 39780sec.

471—480. Reduce to days 72 hours; 120h.; 768h.; 1560h.; 2880min.; 7200min.; 86400min.; 8760h.; 5920h.; 13278h.

481—490. How many days in 5 years? in 12y.? 7y.? 19y.? 25y.? 13y.? 4y. 360dy.? 8y. 280dy.? 3y. 105dy.? 13y. 55dy.?

491—500. How many seconds in 47 minutes? in 38min.? 3h.? 17h.? 2dy.? 12dy.? 5dy. 8h.? 7h. 59min.? 1dy. 9h. 17min.? 4y.?

99. REDUCTION OF COMPOUND NUMBERS.

1. Reduce 2m. 7fur. 39r. 3yd. 1ft. 7in. to inches.

2 m. 7fur. 39r. 3yd. 1ft. 7in.
8
23 fur.
40
959 r.
5½
5277½ yd.
3
15833½ ft.
12
190009 in.

As there are 8 furlongs in 1 mile, in 2 miles there are 16 fur., and in 2m. 7 fur. there are 23 fur. As there are 40 rods in 1 furlong, in 23 fur. there are 920r. and adding the 39r. there are 959 r. As there are 5½ yards in 1 rod, in 959r. (§ 80,) there are 5274½yd., and adding the 3yd., there are 5277½yd. As there are 3ft. in 1yd., in 5277½yd., there 15832½ft., and adding the 1ft., there are 15833½ft. As there are 12 inches in 1 foot, in $15833\frac{1}{2}$ft. there are 190002in., and adding the 7in., there are 190009in.

2. Reduce $\frac{5}{13}$ tons to its value in lower denominations.

There are 20 times as many cwt. as tons, and $20 \times \frac{5}{13} = \frac{100}{13} = 7\frac{9}{13}$cwt. To reduce the $\frac{9}{13}$cwt. to qr., since there are 4 times as many qr. as cwt., $4 \times \frac{9}{13} = \frac{36}{13} = 2\frac{10}{13}$qr. To reduce the $\frac{10}{13}$qr. to lb., since there are 28 times as many lb. as qr. $28 \times \frac{10}{13} = \frac{280}{13} = 21\frac{7}{13}$lb. To reduce the $\frac{7}{13}$lb. to oz., since there are 16 times as many oz. as lb., $16 \times \frac{7}{13} = \frac{112}{13} = 8\frac{8}{13}$oz. To reduce the $\frac{8}{13}$oz. to drams, since there are 16 times as many dr. as oz. $16 \times \frac{8}{13} = \frac{128}{13} = 9\frac{11}{13}$dr. Therefore, $\frac{5}{13}$T. = 7cwt. 2qr. 21lb. 8oz. $9\frac{11}{13}$dr.

3. Reduce 6248 inches to its value in higher denominations.

12)6248

3) 520ft. 8in.

$5\frac{1}{2}$)173yd. 1ft.

31r. $2\frac{1}{2}$yd.

Ans. 31r. 3yd. 2in.

As 1 in. = $\frac{1}{12}$ ft. 6248 in. = $\frac{6248}{12}$ft. = 520ft. 8in. As 1ft. = $\frac{1}{3}$yd. 520ft. = $\frac{520}{3}$yd. = 173yd. 1ft. As 1yd. = $\frac{1}{5\frac{1}{2}}$r. 173yd. = 31r. $2\frac{1}{2}$yd.

Or, as 1ft. = 12in. to find the number of feet, (§ **55**,) we must divide the number of inches by 12. In 6248in. there are 520ft. and 8 inches over. To find the number of yards, we must divide the number of feet by 3. In 520ft. there are 173yd. and 1 foot over. To find the number of rods, we must divide the number of yards by $5\frac{1}{2}$. In 173yd. there are 31r. and $2\frac{1}{2}$yd over. Therefore, 6248 inches = 31r. $2\frac{1}{2}$yd. 1ft. 8in. But $\frac{1}{2}$yd. 1ft. 8in. = 38in. = 1yd. 2in., which added to 31r. 2yd., gives 31r. 3yd. 2in.

4. Reduce 3mo. 17dy. to the fraction of a mercantile year.

As there are 30 days in 1 month, to reduce any number of days to months, we must divide by 30. (§ **55**.) Then 17 days = $\frac{17}{30}$mo., and 3mo. 17dy. = $3\frac{17}{30}$*mo. As there* are 12 months in 1 year, to reduce any number of months to years, we must divide by 12. $3\frac{17}{30} \div 12 = \frac{107}{360}$, therefore, $3\frac{17}{30}$mo. = $\frac{107}{360}$ years.

From the four preceding examples, we may deduce the following

RULE FOR REDUCTION OF COMPOUND NUMBERS.

To reduce to lower denominations. ***Multiply the number of the highest denomination by the number which is required of the next lower denomination to make one of the higher, adding to the product the number of that lower denomination, (if there be any.) Proceed in the same manner with each result, until you reach the lowest denomination sought.***

To reduce to higher denominations. ***Divide the number of the lowest denomination by the number which is required of that denomination to make one of the next higher, adding to the quotient the number of the higher denomination, (if there be any.) Proceed in the same manner with each result, until you reach the highest denomination sought.***

In each question, observe whether the WHOLE *of the given number is to be reduced to a* SINGLE *denomination,* (as in Nos. 1 and 4 of the preceding examples,) *or whether* PARTS *are to be retained in the intermediate denominations,* (as in Nos. 2 and 3.)

100. EXAMPLES FOR THE PUPIL.

501—510. Reduce to pence, 7s. 9d.; 12s. 3d.; 8s. 7d. 2qr.; 1£ 3s. 9d.; 2£ 8d.; 5£ 14s.; .19s.; 1.8£; 7£ 2s. 11d.; 15£ 15s.

511—520. Reduce to higher denominations, (Troy weight,) 769gr.; 107oz.; 449oz.; 783dwt.; 1540dwt.; 9827gr.; 47592gr.; 8409dwt.; 9999dwt.; 96083gr.

521—530. Reduce to lower denominations, $\frac{15}{17}$℔ (Apoth.); $\frac{9}{10}$ʒ; $\frac{16}{17}$T.; .93cwt.; .87qr.; $\frac{5}{19}$ miles; $\frac{11}{19}$fur.; $\frac{6}{7}$yd. (Cloth); $\frac{5}{9}$sq. miles; .086A.

531—540. Reduce to the fraction of a business year, 7mo. 7dy.; 5mo. 24dy.; 8mo. 13dy.; 11mo. 5dy.; 9mo. 15dy.; 27dy.; 15h.; 9dy. 8h.; 10mo. 10dy.; 4mo. 18dy.

541—550. Reduce to the decimal of a bushel, 1qt. 1pt.; 7qt.; 3pk. 2qt. 1pt.; 1pk. 6qt.; 2pk. 4qt.; 5qt. 1pt.; 1pk. 1pt.; 3pk. 3qt.; 2pk. 1pt.; 2pk. 6qt. 1pt.

551—560. Reduce to gills, 2gal. 3qt.; 5gal. 1pt.; 3qt. 3gi.; 3hhd.; 59gal. 2gi.; 8gal. 1pt. 1gi.; 5hhd. 62gal.; 4hhd. 3qt.; $\frac{7}{8}$hhd.; 2hhd. 1pt. 2gi.

561—570. Reduce to higher denominations, 97600 cub. in.; 543c. ft.; 729c. ft. of wood; 2775c. ft. of round timber; 9430c. ft. of hewn timber; 8720″; 9400′; 4796°; 98930sec.; 44920h.

571—580. Reduce to lower denominations, $\frac{5}{11}$dy.; $\frac{4}{7}$h.; .93°; .7bu.; .09hhd.; .875 tons of shipping; $\frac{2}{7}$A.; $\frac{10}{13}$sq. rods; $\frac{3}{17}$ miles; $\frac{2}{3}$£.

581—590. Reduce to the decimal of a business year, 3mo. 18dy.; 27dy.; 1mo. 24dy.; 6mo. 9dy.; 7mo. 6dy.; 11mo. 12dy.; 3mo. 15dy.; 8mo.; 9mo. 18dy.; 4mo. 26dy.

591—600. Reduce to the fraction of a mile, 3fur. 7r.; $\frac{5}{8}$r.; .7r.; 28r. 3yd.; 2yd. 1ft. 11in.; 4yd. 7in.; 2fur. 4yd.; 39r. 2ft. 6in.; 5yd.; 7fur. 33r.

101. ADDITION OF COMPOUND NUMBERS.

Find the sum of £20 16s. 9d. 3qr., £11 7d. 1qr., and £9 19s. 2qr.

£	s.	d.	qr.
20	16	9	3
11	0	7	1
9	19	0	2
41	16	5	2

Writing the numbers as in Simple Addition, placing all that are of the same denomination in the same perpendicular column, we commence with the column of farthings, and find its sum, 6 qr., = 1d. 2qr. We write the 2 qr. beneath the column of qr., and carry the 1d. to the column of d. The sum of the column of d., (including the 1d. from qr.,) is 17d. = 1s. 5d. Write the 5d. beneath the column of d., and carry the 1s. to the column of s. The sum of the column of s., (including the 1s. carried from d.,) is 36s. = 1£ 16s. Write 16 beneath the column of s., and carry 1 to £. Finally, the sum of the £ is 41, and the entire sum desired, £41 16s. 5d. 2qr. *Observe the analogy between this process and Simple Addition.*

RULE FOR COMPOUND ADDITION.

Write the numbers as in Simple Addition, so that all of the same denomination may stand in the same vertical column. Commence at the right hand to add, and reduce the sum of each column to the next higher denomination. Write the remainder underneath, and add the quotient to the next column.

102. EXAMPLES FOR THE PUPIL.

601. Can Simple Addition be performed by the rule for Compound Addition?

602—605. Add £7 15s. 9d., and £1 19s. 3d.; £2 4s. 11d., and £5 7s. 5d.; £3 18s. 7d., and £3 17s. 4d.; £9 19s. 11d., and 18s. 10d.

606—610. Add 5lb. 11oz. 17dwt., and 3lb. 8oz. 5 dwt.; 4lb. 17dwt. 18gr., 10oz. 23gr., and 7lb. 18dwt.; 5oz. 15dwt. 15gr., 8oz. 19dwt. 22gr., and 18dwt. 23 gr.; 1℔ 11℥ 6ʒ, 10℥ 5ʒ 2℈, and 4℔ 7ʒ 2℈; 9℥ 1℈ 10 gr., 7ʒ 2℈ 17gr., and 8℥ 5ʒ.

611—615. Add 2T. 19cwt. 27lb., 3qr. 15oz., and 5 cwt. 2qr. 7oz.; 19cwt. 3qr. 26lb., 1qr. 25lb., and 18lb. 11oz.; 15oz. 15dr., 14oz. 14dr., and 27lb. 13oz.; 18 cwt. 2qr., 3qr. 19lb., and 3qr. 10lb.; 7fur. 39r., 6fur. 20r., and 5fur. 37r.

616—620. Add 39r. 5yd. 2ft., 38r. 4yd. 1ft. and 7 fur. 5yd.; 2ft. 11in., 5yd. 2ft. 7in., and 1ft. 10in.; 3 yd. 3qr. 3na., 2qr. 1na., and 17yd. 3qr. 2na.; 3qr. 1 na., 1qr. 3na., and 2qr. 2na.; 5yd. 3qr., 2qr. 3na., and 2yd. 2na.

621—625. Add 639A. 3R. 39r. and 5m. 72A. 14r.; 30sq. yd. 8sq. ft. 140sq. in., 28sq. yd. 7sq. ft. 127sq. in. and 5sq. ft. 94sq. in.; 5m. 628A., 638A. 3R. and 99A. 2R.; 7c. yd..26c. ft., 11c. yd. 18c. ft., and 24c. ft.; 4 cords 7ft., 11 cords 3ft. and 15 cords 6ft.

626—630. Add 7bu. 3pk., 2pk. 7qt., and 9bu. 3qt.; *3pk. 6qt. 1pt.* and 2pk. 5qt. 1pt.; 3pk. 7qt., 6qt. 1pt.

and 7qt. 1pt.; 359° 59′ 59″ and 41° 18′ 27″; 2circ. 324° 58″ and 187° 59′ 37″.

631—635. Add 5hhd. 62gall. 3qt. and 58gal. 2qt. 1pt.; 62gal. 1pt. 3gi. and 3qt. 1pt. 2gi.; 9gal. 2qt. 1gi., 3qt. 3gi., and 2qt. 1pt. 2gi.; 60gal. 3qt., 29gal. 2qt. 1pt., and 18gal. 1qt. 1pt.; 62gal., 59gal. 3qt., and 2qt.

636—640. Add 3y. 319dy. 23h., and 5y. 287dy. 11h.; 22h. 59min., 40sec., and 13h. 37sec.; 4y. 360dy., 7y. 344dy., and 283dy.; 12h. 47min., 23h. 59min., and 5h. 18min.; 1y. 67dy. 14h., and 5y. 323dy. 19h.

103. SUBTRACTION OF COMPOUND NUMBERS.

A farmer had 13T. 7cwt. 26lb. of hay. How much was left, after selling 9T. 8cwt. 3qr. 18lb.

T.	cwt.	qr.	lb.
13	7	0	26
9	8	3	18
3	18	1	8

18lb. from 26lb. leave 8lb.; 3qr. cannot be taken from 0qr., but if we add 1qr. to 8cwt. 3qr., we obtain 9cwt. 0qr., therefore, (§ **29**,) 1qr. must be written in the remainder. 9cwt. cannot be taken from 7cwt., but if we add 11cwt. to 9T. 9cwt., we obtain 10T., and 7cwt. *more*, (making 18cwt. to be added in all,) make 10T. 7cwt. 10T. from 13T. leave 3T.

RULE FOR COMPOUND SUBTRACTION.

Write the numbers as in Compound Addition, and commence subtracting at the right hand. If the number representing any denomination in the minuend, is less than that in the subtrahend, increase the upper number by as many as make one of the next higher denomination, and add 1 *to the next higher denomination of the subtrahend.*

104. EXAMPLES FOR THE PUPIL.

641. Can simple subtraction be performed by the rule for compound subtraction?

642—645. Subtract £5 17. 6d. from £8 3s. 9d.; 4s

11d. 3qr. from 5s. 3d. 1qr.; 19s. 7d. 2qr. from £1 7s.; 4d. from £2.

646—650. Subtract 3qr. from £3 1qr.; 11oz. 17dwt, from 2lb. 5dwt.; 18gr. from 4lb.; 7lb. 10oz. from 9lb. 3oz.; 8oz. 13dwt. 21gr. from 1lb. 2dwt. 3gr.

651—655. Subtract 4℔ 6℥ from 5℔ 3℥; 9℥ 7ʒ from 2℔ 8℥; 5ʒ 2℈ 12gr. from 2℥ 1℈ 17gr.; 19cwt. 3qr. from 1T.; 3qr. 27lb. from 2cwt. 1qr.

656—660. Take 9lb. 15oz. 11dr. from 1cwt. 7oz. 12dr.; 4lb. 13oz. 5dr. from 1T.; 3cwt. 3qr. from 2T. 1qr.; 7fur. 39r. from 2m. 7r.; 2ft. from 1m.

661—665. Take 5yd. 2ft. 11in. from 2r.; 5fur. 18r. from 3m. 7r.; 5yd. 3qr. 2na. from 7yd. 3qr. 1na.; 3na. from 3yd.; 1yd. 2qr. from 7yd. 2na.

666—670. Find the difference between 5sq. m. and 5sq. r.; 3A. 3R. and 1m. 2R.; 7sq. yd. 139sq. in. and 1sq. r. 5sq. ft.; 1c. yd. and 1640c. in.; 3pk. 7qt. 1pt. and 2bu. 5qt.

671—675. Find the difference between 5bu., and 4bu. 7qt. 1pt.; 2bu. 3pk. 7qt. 1pt. and 3bu.; 1hhd., and 62gal. 3qt.; 2qt. 1pt. and 9gal. 1qt.; 59° 28′ 27″ and 1circ.

676—680. Take 1776y. 7th mo. 4th dy. from 1859y. 3d mo. 7th dy.; 1848y. 3d mo. 28th dy. from 1853y. 2d mo. 11th dy.; 1820y. 8th mo. 18th dy. from 1827y. 6th mo. 16th dy.; 1620y. 12th mo. 11th dy. from 1732y. 2d mo. 22d dy.; 1492y. 10th mo. 11th dy. from 1775y. 4th mo. 19th dy.

105. MULTIPLICATION OF COMPOUND NUMBERS.

How much gold in 23 ingots, each ingot weighing 7lb. 9oz. 18gr.?

lb.	oz.	dwt.	gr.
7	9	0	18
			23
178	3	17	6

23 × 18gr. = 414gr. = 17dwt. 6gr. Write the 6gr. in the column of gr., and carry the 17dwt. to the product of the dwt. 23 × 0dwt. = 0dwt., and 17dwt. *to carry*, make 17dwt., which being less than an ounce, is written in the dwt.

column. 23 × 9oz. = 207oz. = 17lb. 3oz. Write the 3oz. in the column of oz., and carry the 17lb. to the product of the lb. 23 × 7lb. = 161lb., and 17lb. *to carry* make 178lb., which we write in the lb. column.

RULE FOR COMPOUND MULTIPLICATION.

Multiply the several denominations in order, commencing with the smallest. Reduce each product to the next higher denomination, writing the remainder underneath, and adding the quotient to the succeeding product.

106. Examples for the Pupil.

(The answers to § **106**, may be found in § **108**.)

681. Can simple multiplication be performed by the rule for compound multiplication?

682—685. Multiply £5 7s. 3d. by 5; £4 13s. 9d. by 7; 19s. 4d. 3qr. by 12; £8 7s. 6d. by 8.

686—690. Find the product of £4 11d. 2qr. × 10; 6 × 3lb. 10oz. 17dwt.; 2 × 11oz. 19dwt. 12gr.; 7lb. 5oz. 19gr. × 3; 19lb. 4oz. 18dwt. × 9.

691—695. Find the product of 3℔ 8℥ 7ʒ by 5; 6℥ 6ʒ 2℈ by 8; 2℈ 10gr. by 25; 5℔ 4ʒ 9gr. by 30; 4℔ 11℥ 2ʒ by 9.

696—700. Multiply 5T. 18cwt. 3qr. by 6; 2qr. 25lb. 12oz. by 7; 2lb. 10oz. 10dr. by 27; 7fur. 30r. by 15; 2m. 3fur. 18r. by 40.

701—705. Multiply 28r. 4yd. 2ft. 11in. by 4; 9yd. 2qr. 1na. by 36; 7yd. 3qr. 2na. by 10; 14yd. 3na. by 17; 5A. 2R. 38r. by 11.

706—710. Multiply 13bu. 1pk. 7qt. by 3; 3pk. 6qt. 1pt. by 9; 2pk. 4qt. 1pt. by 39; 7qt. 1pt. by 127; 5bu. 6qt. by 18.

711—715. Multiply 4hhd. 60gal. 3qt. by 5; 2qt. 1pt. 3gi. by 31; 7gal. 1pt. 2gi. by 75; 19hhd. 62gal. 3qt. by 12; 94° 27′ 51″ by 9.

716—720. Multiply 5y. 319dy. 23h. by 7; 14dy.

12h. 38min. by 8; 5h. 19min. 49sec. by 9; 7mo. 25d. by 30; 2y. 3mo. 19dy. by 63.

107. DIVISION OF COMPOUND NUMBERS.

If 23 ingots of gold weigh 178lb. 3oz. 17dwt. 6gr., what is the weight of each ingot.

	lb.	oz.	dwt.	gr.		lb.	oz.	dwt.	gr.
23)	178	3	17	6	(	7	9	0	18

161
——
17 lb.
12
——
207 oz.
207 oz.
——
0 oz.
20
——
17 dwt.
24
——
414 gr.
414
——

178 lb. ÷ 23, gives a quotient of 7lb. and a remainder of 17lb. Reducing the 17lb. to oz., and adding the 3oz., we have 207 oz. 207 oz. ÷ 23, gives a quotient of 9oz. and no remainder. 17 dwt. ÷ 23, gives 0dwt. Reducing the 17dwt. to gr., and adding the 6 gr., we have 414 gr. 414gr. ÷ 23, gives 18gr. The entire quotient is therefore, 7lb. 9oz. 18gr.

RULE FOR COMPOUND DIVISION.

Divide the several denominations in order, commencing with the highest. Whenever there is any remainder, reduce it to the next lower denomination, add it to the number, (if there is any,) already in that denomination, and divide the sum.

If the divisor and dividend are both compound numbers, *reduce them to the same denomination, and divide as in simple division.*

108. Examples for the Pupil.

(The answers to § **108**, may be found in § **106**.)

721. Can simple division be performed by the rule for compound division?

722—725. Divide £26 16s. 3d. by 5; £32 16s. 3d. by 7; £11 12s. 9d. by 12; £67 by 8.

726—730. Divide £40 9s. 7d. by 10; 23lb. 5oz. 2dwt. by 6; 1lb. 11oz. 19dwt. by 2; 22lb. 3oz. 2dwt. 9gr. by 3; 174lb. 8oz. 2dwt. by 9.

731—735. Divide 18℔ 8℥ 3ʒ by 5; 4℔ 6℥ 5ʒ 1℈ by 8; 2℥ 4ʒ 2℈ 10gr. by 25; 151℔ 3℥ 4ʒ 1℈ 10gr. by 30; 44℔ 5℥ 2ʒ by 9.

736—740. Divide 35T. 12cwt. 2qr. by 6; 5cwt. 12lb. 4oz. by 7; 2qr. 15lb. 14oz. 14dr. by 27; 14m. 4fur. 10r. by 15; 97m. 2fur. by 40.

741—745. Divide 2fur. 35r. 3yd. 1ft. 2in. by 4; 344yd. 1qr. by 36; 78yd. 3qr. by 10; 241yd. 3na. by 17; 63A. 18r. by 11.

746—750. Divide 40bu. 1pk. 5qt. by 3; 8bu. 2pk. 2qt. 1pt. by 9; 24bu. 3pk. 7qt. 1pt. by 39; 29bu. 3pk. 1pt. by 127; 93bu. 1pk. 4qt. by 18.

751—755. Divide 24hhd. 51gal. 3qt. by 5; 22gal. 1qt. 1gi. by 31; 8hhd. 35gal. 2gi. by 75; 239hhd. 60gal. by 12; 2circ. 130° 10′ 39″ by 9.

756—760. Divide 41y. 49dy. 17h. by 7; 116dy. 5h. 4min. by 8; 1dy. 23h. 58min. 21sec. by 9; 19y. 7mo. by 25; 145y. 27dy. by 63.

761—770. Divide the compound numbers in Ex. 751—760, by the corresponding compound numbers in Ex. 711—720.

109. Miscellaneous Examples.

771. Five pieces of cloth measured as follows: 29yd. 2qr. 2na.; 18yd. 1qr. 3na.; 23yd. 3na.; 30yd. 3qr.; and 25yd. 1qr. 1na. How much in all?

772. Paid for a carriage £120 7s. 3d. 3qr.; for horses £93 18s. 11d.; for harness £18 4s. 9d. 2qr., and for a livery £17 13s. 6d. What was the whole cost?

773. How much sugar is left after selling 6cwt. 1qr. 5lb. 11oz. from 19cwt. 27lb. 6oz.?

774. A farmer harvested 97bu. 2pk. 2qt. of wheat, from sowing 5bu. 1pk. 3qt. 1pt. What was his gain?

775. A road contractor agreed to build 16m. 3fur. 5yd. 1ft. of road. How much remained to be done, after building 3m. 7fur. 2r. 2yd. 2ft.?

776. How much iron in 9 loads, each weighing 1T. 7cwt. 3qr. 20lb. 5oz.?

777. How much corn will 29 barrels hold, if 2bu. 3 pk. 7qt. 1pt. fill a barrel?

778. How far will a locomotive run in 11 hours, at the rate of 16m. 25r. 2yd. an hour?

779. What is the weight of 31 bars of silver, each weighing 4lb. 11oz. 6gr.?

780. How much will make 1 suit of clothes, if 17yd. 3qr. 2na. make 5 suits?

781. A jeweller bought 6 agates of equal size, weighing in all 1lb. 5oz. 13gr. What did each weigh?

782. A railway train went 103m. 7fur. in 4½ hours. What was the rate per hour?

783. The horses in a livery stable eat 42bu. 3pk. 6qt. of oats in 39 days. How many were used daily.

784. How many bins, each holding 48bu. 7qt., will contain 197bu. 3pk. of oats?

785. A jeweller's stock of gold was 5lb. 5oz. 18gr. How much was left after selling 1lb. 3oz. 16dwt. 19 gr.?

786. A grocer had 5 hogsheads of molasses, each holding 113¼ gallons. How much was left after selling 49gal. 2qt. 1pt.?

787. After selling 9yd. 3qr. 2na. from a piece of broadcloth containing 21yd. 3qr. 3na., how much was left?

788. If a man's income is £229 11s. a year, and his expenses £179 14s. 3qr., what does he save?

789. There are 4T. 18cwt. 3qr. of sugar in 47 boxes. How much in each box?

790. How much molasses in 43 casks, each holding 97gal. 1pt. 2gi.?

791. How many yards of sheeting in 37 pieces, each measuring 31yd. 3qr. 2¼na.?

792. What will 59yd. of cloth cost, at £1 19s. 3qr. per yd.?

793. What is the weight of 39 prescriptions, each weighing 1℥ 6ʒ 11gr.

794. From a medicinal mixture, containing 3lb. 6ʒ 2℈, an apothecary sold 1lb. 10℥ 7ʒ 2℈ 7gr. How much was left?

795. From a farm of 149A. 3R. the owner sold a field 31r. long, and 30r. wide. How much land did he sell, and how much had he left?

796. From a pile of wood 48ft. long, 8ft. wide, and 5ft. high, a load was sold 8ft. long, 5ft. wide, and 6ft. high. How many cords, &c., were there in the original pile?—how many in the load that was sold?—and how many were left?

797. If 5gal. 3qt. 1pt. leak from a hogshead holding 113gal. 2qt., how much will be left?

798. If 31lb. of indigo cost £19 14s., what is the price per lb.?

799. How many cords, &c. of wood in a lumber yard, there being 17 piles, each of which is 11ft. long, 4ft. wide, and 6ft. high?

800. A farm of 41A. 3R. was divided into 33 lots of equal size. How much did each lot contain?

801. James left home 2yr. 31dy. ago, expecting to return in 3 years. How soon may his friends look for him?

802. How many tons of hewn timber in 13 logs, each 1½ft. thick, 2ft. wide, and 48½ft. long?

803. A man started on a journey of 69 miles, and rode 24m. 7fur. 39r. the first day. How far had he still to go?

804. Subtract 1829y. 11mo. 27dy. from 1844y. 7mo. 28dy.

805. On October 1st, 1848, how old was a man who was born August 18th, 1820?

806. A man borrowed a sum of money Sept. 16th,

1837, which he paid with interest, March 1st, 1841. How long did he keep the money?

807. How many pints of chestnuts in 9 baskets, each holding 1.37 bushels?

808. If 19 logs of hewn timber contain 21T. 39c. ft., what does each log measure?

809. If 25 barrels of flour hold 2T. 3cwt. 3qr., what does 1 barrel hold? How many pounds?

810—815. Reduce 4yd. 10in. to the fraction of a rod; 13h. 11min. 11sec. to the fraction of a day; 7s. 6d. to the fraction of a £; 3qt. 1pt. 2gi. to the fraction of a gal.; 5mo. 18dy. to the fraction of a year.

816—820. Reduce 9s. 3d. 2qr. to the decimal of a £; 4mo. 15dy. to the decimal of a year; 2ft. 3in. to the decimal of a rod; 87$\frac{1}{2}$ cents to the fraction of a dollar; 3qt. 3gi. to the decimal of a gallon.

821—830. Reduce to lower denominations, $\frac{7}{16}$cwt.; .481£; $\frac{4}{53}$y.; $\frac{3}{11}$hhd.; $\frac{3}{8}$dy.; \$$\frac{3}{8}$; $\frac{7}{11}$ miles; $\frac{5}{7}$lb. Troy; .783cwt.; .4583yr.

831. In a certain farm that is 142.5 rods long, and 119$\frac{3}{4}$ rods wide, there is a pond containing 13$\frac{3}{7}$ acres. How many acres are not covered by water?

832. The pyramid of Cheops measures 763.4ft. on each side of its base. How many acres does it cover?

833. A farmer purchased 49A. 3R. 31r. of woodland, 56$\frac{5}{8}$A. of pasture, 119A. 27r. of meadow, and 38$\frac{2}{7}$A. of tillage. What did he give for the whole, at \$28.50 per acre?

834. What would be the weight of a gold wire that would reach from Boston to Liverpool, estimating the distance at 2997m. 3fur. 28r., and the weight of the wire at $\frac{1}{54000}$dr. avoirdupois per foot?

835. What is the cost of 9T. 11cwt. 3qr. of sugar, at 5$\frac{3}{4}$d. per pound?

836. How many potatoes at 2s. 6d. per bu., will pay for 49$\frac{7}{8}$yd. of cloth, at £1$\frac{1}{7}$ per yd.

837. How many times will a seconds' pendulum tick in 3y. 19dy. 4h.?

838. What is the value of 3$\frac{7}{8}$yd. of cloth, if 2$\frac{5}{8}$yd. cost *43$\frac{3}{4}$ cents?*

839. When wood is \$5.37$\frac{1}{2}$ per cord, what must I pay for 19 loads, each measuring 8.3ft. long, 3.5ft. wide, and 4$\frac{1}{4}$ft. high?

840. A grocer bought 19$\frac{4}{7}$cwt. of sugar, at 6$\frac{1}{2}$ct. a pound, and sold the whole for \$156.75. How much did he gain by the sale?

841. Bought 95yd. 3qr. 2na. of broadcloth, at \$3.50 per yard. For how much must the whole be sold, to gain \$35.75?

842. Find the least common multiple of 130, 546, 156, and 182.

843. Find the greatest common divisor of 222, 333, 444, 666, 999, and 814.

844. If 4$\frac{7}{8}$ yards of calico cost 5$\frac{1}{3}$ shillings, what is the price per yard?

845. How much land in a field that is 73$\frac{1}{4}$ rods long, and 49$\frac{1}{3}$r. wide?

846. If from a piece of broadcloth containing $\frac{4}{5}$ of $\frac{7}{8}$ of 32$\frac{1}{3}$yd., $\frac{1}{11}$ of $\frac{4}{7}$ of 49yd. be cut, how much will be left?

847. If $\frac{2}{3}$ of 15$\frac{3}{4}$ bushels of wheat cost \$11$\frac{1}{3}$, what is the price of 1 bushel?

848. A laborer received \$1614 for 3y. 115$\frac{1}{2}$dy. wages. How much was that per day?

849. Find the sum of 19$\frac{4}{5}$, 2$\frac{7}{8}$, $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{5}{6}$ of 8, $\frac{119}{120}$, and 63$\frac{11}{15}$.

850. What must I give for 49T. 5cwt. 1qr. of flour, at \$4.875 per barrel of 196 pounds?

IX. PERCENTAGE.

110. In business and other transactions, we often have occasion to find a certain number of hundredths of a given amount. The number of hundredths required is called the *rate per cent.*, (from the Latin *per centum*, which signifies *by the hundred.*) Thus, $\frac{1}{2}$ per cent. is .00$\frac{1}{2}$, or .005; 6 per cent. is .06; 5$\frac{1}{4}$ per cent. is

$.05\frac{1}{4}$, or .0525; $4\frac{7}{8}$ per cent. is $.04\frac{7}{8}$, or .04875; $7\frac{1}{3}$ per cent. is $.07\frac{1}{3}$, or $.07\dot{3}$.

Since .05 *of* any number, is the same as .05 *times* that number, (§ 80,) in order to compute any percentage, *multiply by the rate expressed decimally.* Thus, $7\frac{1}{2}$ per cent. of \$28.40, is equivalent to \$28.40 × .075 = \$2.13.

111. EXAMPLES FOR THE PUPIL.

1—10. Find 6 per cent. of 87, 64, 316, 932, 102.7, 394.06, 9900, 182.57, .059, 5.24.

11—20. Find $4\frac{1}{2}$ per cent. of 200, 750, 3.18, .09, 140, 64.7, 9830, 12000, 8.7, 153.

21—30. Find $2\frac{3}{4}$ per cent. of 9, 27, 80, 300, 700, .095, 950, 2000, $33\frac{1}{3}$, $12\frac{1}{2}$.

31—40. Find $\frac{1}{4}$ per cent. of 96, 144, 2.08, $72\frac{1}{2}$, $3\frac{7}{8}$, 500, 875, 53.75, $44\frac{4}{9}$, $36.\dot{8}$.

41—50. Find $6\frac{1}{4}$ per cent., $8\frac{1}{3}$ per ct., $12\frac{1}{2}$ per ct., $16\frac{2}{3}$ per ct., 25 per ct., $33\frac{1}{3}$ per ct., $37\frac{1}{2}$ per ct., $66\frac{2}{3}$ per ct., $87\frac{1}{2}$ per ct., and 100 per ct., of \$2500.

51—60. How many per cent. (§ 77,*) in $\frac{1}{12}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{3}{8}$, $\frac{2}{3}$, $\frac{7}{8}$, and in the whole of any number?

61—70. Find $2\frac{3}{4}$ per cent. of 1687; of \$500; \$340; \$1.15; 813; .095; 4720; \$16.80; 9000; 72.

71—90. What rate per cent. is equivalent to $\frac{1}{2}$? to $\frac{5}{8}$, $\frac{5}{6}$, $\frac{5}{12}$, $\frac{1}{16}$, $\frac{3}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{9}{16}$, $\frac{11}{16}$, $\frac{13}{16}$, $\frac{15}{16}$, $\frac{7}{25}$, $\frac{1}{125}$, $\frac{1}{5}$, $\frac{1}{625}$, $\frac{23}{25}$, $\frac{7}{9}$, $\frac{3}{7}$, $\frac{15}{17}$?

91—100. Find 17 per cent. of 64; 100 per cent. of 47; 25 per cent. of 50; $33\frac{1}{3}$ per cent. of 255; 89 per cent. of 13; $\frac{7}{8}$ per cent. of 250; 2.5 per cent of 600; .25 per cent. of 43; .9 per cent. of 136; 44 per cent. of 92.

112. COMMISSION is an allowance made to an agent, for the purchase, sale, or care of property. The agent is sometimes called a factor, correspondent, or broker.

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* To obtain the number of hundredths, or the rate per cent., to *which* any fraction is equivalent, we need only reduce the fraction *to a decimal.*

101. A broker sold goods for $1963.50, at a commission of 4½ per cent. What was the amount of his commission, and how much did he pay over to his employer?*

102. How much shall I receive for my real estate, by selling it for $9550, after allowing a commission of 3⅓ per cent.?

103. My correspondent has expended $4762 in the purchase of cloths, by my order. What will he receive, if I allow him 2¾ per cent. on the transaction?

104. What is the commission on £963 17s. 6d., at 4⅔ per cent.?

```
     £      s.    d.
   9.63    .17   .06
                  14
  ─────────────────    £     s.    d.
3)134.82   2.38  .84(44    19    7⅖
  132
  ────
    2.82£
     20
   ────
   58.78s.
   57
   ─────
    1.78s.
     12
   ─────
   22.20d.
   21
   ─────
    1.2
```

In computing interest on English Money, it is most convenient to first obtain .01 of each denomination, and multiply by the number of per cent. required. 1 per cent. of £963 17s. 6d. is £9.63 .17s. .06d., and 4⅔ (or $\frac{14}{3}$) times this amount, will be 4⅔ per cent. But multiplying by $\frac{14}{3}$ is equivalent to multiplying by 14, and dividing by 3. In the division, the several remainders are reduced and added to the lower denominations, as in ordinary Compound Division. The final remainder, $\frac{1.2}{3}=\frac{12}{10}\div 3=\frac{12}{30}$d.

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* In all business transactions, any fraction less than ½ a cent is usually disregarded. If the fraction is equivalent to ½ a cent, or more, it is called a cent. Thus, $.124, would be called 12 cents; $.137 *would be* called 14 cents.

105. At 2.5 per cent., what commission must I allow my factor for purchasing goods to the amount of £153 7s. 11d.?

113. Insurance is a security against loss by fire or other accidents, and is generally obtained from an incorporated company, by the payment of a certain percentage on the amount insured.

The amount paid for insurance, is called the *Premium.* The *Policy* is the certificate of insurance. The Insurers are sometimes called *Underwriters.*

106. At a premium of 2 per cent., how much must be paid to insure a cargo of merchandise for £144. 8s. 6d.?

107. How much must be paid to insure $9795 on a store, at a premium of $2\frac{1}{8}$ per cent.?

108. A farmer insured his house for $950 at $1\frac{1}{4}$ per cent., his barn at $375 at $1\frac{1}{2}$ per cent., and his granary for $275 at 1 per cent. What premium did he pay for the whole?

109. An English merchant effected an insurance of £5163 10s. on the ship Mary Ann, at a premium of $1\frac{3}{4}$ per cent. What did he pay, including a charge of 3s. 6d. for the policy?

110. What premium must I pay, to insure the following amounts for one year: $10875 on my store, and $4500 on the merchandise in the same, at $\frac{3}{4}$ per cent.; $4650 on my house, and $1750 on the furniture, books and clothing contained in it, at $\frac{1}{2}$ per cent.; and $975 on my barn, at $\frac{5}{8}$ per cent.?

114. Taxes are certain amounts levied upon all property holders, for the support of government.

111. When taxes are rated at $1\frac{1}{4}$ per cent., what amount will be paid by a man whose property is valued at $5680?

112. At $\frac{2}{3}$ per cent., what amount of tax must be paid by property valued at $18875.25?

113. What is the tax on a house valued at $4750, at .85 per cent.?

114. The State tax being .5 per cent., the county tax .1 per cent., and the city tax $\frac{1}{5}$ per cent., how much must be paid by property worth $7500?

115. At $\frac{7}{8}$ per cent., what amount of tax must be paid by property valued at $2775?

115. Stocks are sums of money invested in government funds, or incorporated companies, and are bought and sold by *shares*. The *par value* of a share is the original cost. When a share will sell for more than it originally cost, it is said to be *above par*, or *at an advance;* when for less, *below par*, or *at a discount*. Thus the *par value* of a stock is $\frac{100}{100}$, or 100 per cent. of the first cost. When a stock sells at 25 per cent. *above par*, or at 25 per cent. *advance*, it brings $\frac{125}{100}$, or 125 per cent. of the original cost. When it sells at $12\frac{1}{2}$ per cent. *below par*, or at $12\frac{1}{2}$ per cent. *discount*, it brings $100 - 12\frac{1}{2} = 87\frac{1}{2}$ per cent. of the par value.

The profits made by any company, are usually divided once a year or oftener. The amount paid to the stockholders at every such division, is called a *dividend.*

116. What must I pay for 27 shares of the Boston and Worcester Rail Road, when the stock is $16\frac{1}{2}$ per cent. above par, the par value being $100 per share?

117. The par value of United States Bank stock, was $100 per share. How much was received for 31 shares, at $9\frac{3}{4}$ per cent. below par?

118. A man purchased $5000 Pennsylvania stock at par, and sold it at a discount of $19\frac{1}{2}$ per cent. How much did he receive for the whole?

119. What must I pay for 43 shares of bank stock, at an advance of $7\frac{5}{8}$ per cent. on the par value, which is $250 per share?

120. What is the value of $9500 of stock, at 25 per cent. advance? at $19\frac{1}{3}$ per cent. discount? at 50 per cent. *below par?*

121. When a dividend of 4½ per cent. was declared on Eastern Rail Road stock, how much was received by a man who owned 27 shares, the par value being $100 per share?

116. DUTIES are imposts levied by government on goods imported into a country. A *specific* duty, is a fixed amount charged without reference to the cost of the article. An *ad valorem* duty, is a certain percentage charged on the cost of the merchandise before importation. Before computing duties, certain deductions are made.

For *leakage* and *breakage*, 2 per cent. is allowed on all merchandise paying duty by the gallon, contained in casks; 10 per cent. on all beer, ale, and porter, in bottles, and 5 per cent. on all other liquors in bottles.

Draft is an allowance made for waste. The following is the present legal rate. On all packages weighing

1cwt. or less,	1lb.	10cwt. or less,	4lb.
2cwt. "	2lb.	18cwt. "	7lb.
3cwt. "	3lb.	more than 18cwt.	9lb.

Tare is an allowance made after deducting the draft, for the weight of the box, cask, or other article, in which the merchandise is contained. The weight before making the deductions, is called the *gross weight*,—after the deductions are made, the *net weight*. Duties are computed on the net weight.

122. What is the duty on 5cwt. 3qr. 11lb. of currants, the tare being 12 per cent., and duty 3 cents a pound?

123. What duty must be paid on 150 blankets, purchased in England at 94 cents each, supposing them subject to an ad valorem duty of 25 per cent.?

124. What is the duty on 156lb. of chocolate, tare 10 per cent., duty 4 cents a pound?

125. At 40 per cent. ad valorem, what will be the duty on 125 pounds of sewing silk, purchased in Lyons at $2.75 per pound?

126. What is the duty on 459gal. 2qt. of wine, at 35cts. a gallon, leakage 2 per cent.?

127. What would be the duty on 7 casks of olive oil, averaging 32 gallons per cask, if the oil is worth 78 cents a gallon in Florence, and is subject to a duty of 30 per cent. ad valorem?

117. Loss and Gain are frequently estimated by percentage.

In order to find what per cent. is gained or lost by any transaction, we first *find what part of the original value is equivalent to the gain or loss,* (§ **70**, Ex. 1—50,) *and reduce the fraction representing that part, to hundredths.* Thus, if I sell merchandise that cost $12, for $19, my gain will be $7, which is $\frac{7}{12} = .58\frac{1}{3}$ of the prime cost.

128. How much per cent. do I gain, by selling at 90 cts. a pound, tea that cost me 75 cts.?

129. Bought 15 yards of broadcloth at $3.75 per yard, and sold the whole for $67.50. How much, and what per cent. did I gain.

130. A mercantile firm with a capital of $20000, lost $3000 in one year by bad debts. What percentage of their capital did they lose?

131. When sugar is $6.50 per cwt., what must be the price per lb. to gain 40 per cent.?

132. Anthracite coal is usually sold by the ton of 2240lbs. What per cent. is lost by those who pay the regular market price, for a ton of 2000lbs.?

133. One miller asks me the same price for 1cwt. of buckwheat flour, as another asks for 100lb. What per cent. should I lose by purchasing of the latter?

134. Bought 13cwt. 3qr. 17lb. of coffee, at 9½cts. a pound, but it having been damaged, the whole was sold for $125. How much, and what per cent. was the loss?

135. A bankrupt has property worth only $2700, to meet his debts, which amount to $3600. What per cent. of their claims will his creditors lose?

136. If a merchant makes $3000 a year, upon a

capital of $12000, what per cent. of his capital are his annual profits?

137. A flour speculator bought wheat at $1.20 per bushel, which he was obliged to sell for 95cts. a bushel. What per cent. did he lose?

138. If the wheat in the last example had been sold at $1.50 per bushel, what would have been the gain per cent.?

139. What per cent. is gained by selling sugar that cost $5 a cwt., at 6 cents a pound?

140. If the census of a town in 1830 was 3500, and in 1840 it was 4000, what per cent. was the increase in ten years?

118. INTEREST is the allowance of a certain per cent. for the use of money. The *Principal* is the sum on which interest is paid. The *Rate* is the percentage paid annually. It is regulated by statute, the usual rate being 6 per cent.* The *Amount* is the sum of principal and interest.

In *Simple Interest,* the interest is not allowed to increase the principal. In *Compound Interest,* the interest is added to the principal as it becomes due, and interest is afterwards computed on the amount.

141—150. At .06 per annum, what is the annual interest on $300; $450; $2.75; $98; $762; $119.50; $87.35; $49; $115; $630?

151—160. At 5 per cent. what is the yearly interest on $400; $700; $2000; $1.18; $.68; $4.30; $18.50; $345; $53.40; $187?

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* In each of the New England States, in New Jersey, Pennsylvania, Delaware, Maryland, Virginia, North Carolina, Tennessee, Kentucky, Ohio, Indiana, Illinois, Missouri, Arkansas, and the District of Columbia, and on U. S. notes, the rate is 6 per cent. In New York, South Carolina, Michigan, Wisconsin, and Iowa, it is 7 per cent. In Georgia, Alabama, Mississippi, and Florida, 8 per cent. In Louisiana, 5 per cent. though the bank interest is .06, and conventional interest may be as high as .10. In Maryland the interest on tobacco contracts is .08. In Mississippi, Missouri, *and* Arkansas, the interest by agreement may be as high as .10, *and in Illinois,* Wisconsin, and Iowa. as high as .12.

161—170. When the rate is 8 per cent. what is the annual interest on $125; $375; $8.75; $625; $4.125; $291; $43.69; $880; $12.34; $75.90?

171—180. At the rate of .07, what is the yearly interest on $130; $45.25; $14.60; $91; $800; $.97: $450; $162.83; $47.50; $33.87?

181—190. At the rate of $4\frac{1}{2}$ per cent., what is the interest on $729.50 for 1 year; 2 years; 7yr.; 4.5yr.; $6\frac{2}{3}$yr.; $3\frac{7}{18}$yr.; $\frac{5}{11}$yr.; .625yr.; 5.875yr.; $\frac{19}{72}$yr.?

191—200. The rate being $6\frac{1}{3}$ per cent., what is the interest on $847.20 for 1yr.; 2yr. 3mo. ($2\frac{1}{4}$yr.); 7yr. 6mo. 15dy. ($7\frac{13}{24}$yr.); 9mo.; 11mo. 10dy.; 5yr. 8mo.; 3yr. 9mo. 18dy.; 4yr. 24dy.; 8yr. 7mo.; 6yr. 5mo. 20dy.?

119. We obtain the interest on any sum for 1 year, by multiplying the principal by the rate, (§ **118**, **119**.) And the interest for any number of years, (§ **45**,) may be obtained by multiplying the interest for 1 year by the number of years. By this method of computing interest, months and days must be reduced either to the decimal, or to the fraction of a year, (§ **99**, No. 4,) as in Ex. 181—200. Hence, the

GENERAL RULE FOR INTEREST.

Multiply the time, (expressed in years and parts of a year,) by the annual rate, and multiply the principal by this product.

In the following examples the rate of 6 per cent. is to be always understood, unless some other is expressed.

201—210. What is the interest on $100 for 3yr.; 5y. 6mo.; 11mo. 27dy; 7yr. 3mo.; 1mo. 15dy.; 2yr. 4mo. 15dy.; 7mo. 10dy.; 13yr.; 7mo.; 5yr. 5dy.?

211—220. What is the interest on $950 for 6mo.; 11mo. 5dy.; 7mo. 18dy.; 3mo. 10dy.; 1yr. 8mo.; 5yr. 5mo.; 8yr. 2mo. 18dy.; 2yr. 7mo. 25dy.; 10mo.; 9yr. 24dy.?

221—230. Find the interest on $2000 for 12 days; 27dy.; 2mo. 21dy.; 5mo. 13dy.; 3yr. 5mo.; 1yr.

9mo.; 8mo.; 7mo. 15dy.; 6yr. 2mo. 9dy.; 4yr. 11mo.

231—240. Find the interest on $375 at 5 per cent., for 2mo. 12dy.; 1mo. 15dy.; 6mo.; 4yr. 3mo.; 17yr.; 20yr.; 8yr. 4mo. 7dy.; 11mo.; 3yr. 3mo. 27dy.; 8yr.

241—250. What is the interest on $450 at 8 per cent., for 7yr.; 5mo.; 12yr. 6mo.; 9mo. 24dy.; 3yr. 7mo. 15dy.; 2yr. 11mo.; 10mo. 10dy.; 4mo. 27dy.; 6yr. 3mo.; 5yr. 4mo. 3dy.?*

120. If the rate is .06 for one year, for one month it will be $\frac{1}{12}$ as much, or .005; for 6 days, or $\frac{1}{5}$ of 1 month, it will be $\frac{1}{5}$ as much as for one month, or .001. For 1 day it will be $\frac{1}{6}$ as much as for 6 days, or $\frac{1}{6}$ of .001 = $\frac{1}{6000}$. Then for 17 days it will be $\frac{17}{6000}$, for 29days $\frac{29}{6000}$, and generally *as many* 6000*ths of the principal as are equivalent to the number of days, if we estimate each year at* 12 *months, and each month at* 30 *days.* Hence the

BANK AND BUSINESS RULE.

Multiply the principal by the number of days, (allowing 30 *days to each month,) and divide by* 6000; *the quotient will be the interest at* 6 *per cent. Cancel before multiplying, if possible.*

For any other rate than 6 per cent., *multiply and divide as above, and take as many* SIXTHS *of the result as are equivalent to the number of per cent. For* 5½ *per cent., take* 5½ *sixths, or* $\frac{11}{12}$; *for* 7 *per cent., take* $\frac{7}{6}$; *for* .04¾, *take* 4¾ *sixths, or* $\frac{19}{24}$.

251—260. Find the interest on $500 for 33 days; 63dy.; 93dy.; 123dy.; 3mo.; 5mo.; 7mo.; 11mo; 6mo. 18dy.; 4mo. 27dy.

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* The following is a very convenient GENERAL RULE. To find the interest of any principal at 6 per cent.: *To one half as many cents as there are months, add one sixth as many mills as there are days, and the sum will be the interest of* $1. *Multiply the interest of* ONE *dollar by the* NUMBER *of dollars in the principal, and the product will be the interest required.*

261—270. Compute the interest on $350 for 228dy.; 9mo. 15dy.; 2yr. 3mo.; 3yr. 7mo.; 1yr. 3mo. 18dy.; 6mo. 24dy.; 11mo. 12dy.; 4yr. 6mo.; 2mo. 3dy.; 9mo. 8dy.

271—280. What is the interest on $100 at 7 per cent., for 94dy.? 64dy.? 124dy.? 6mo.? 3mo.? 1yr. 7mo.? 3yr. 5mo.? 2yr. 2mo. 11dy.? 4mo. 15dy.? 7mo. 28dy.?

281—290. What would be the interest on Exs. 271—280 at 5 per cent.?

291—300. What would be the interest on Exs. 251—260 at 8 per cent.?

121. PARTIAL PAYMENTS. It is customary for any person who borrows money, to give in return a written promise to pay the amount borrowed, with interest. Such a writing is called a *promissory note.*

Whenever any part of the note is paid, an *endorsement* is made on the back of the note, stating the amount and time of payment.

When partial payments have been made upon notes that are settled within a year from their date, it is customary to compute interest upon the whole note, and also on each payment, until the time of settlement, and deduct the amount of the payments and interest from the amount of the principal and interest. This would be an *equitable* rule for all periods of time, but the following is the

LEGAL RULE.*

If any payment exceeds the interest due, deduct it from the AMOUNT *due at the time, and compute interest on the balance. If the payment is less than the interest, reserve the excess of interest (to be added to the succeeding interest) and continue the interest on the former principal.*

EXAMPLE FOR ILLUSTRATION.

$1000.00 Philadelphia, March 4th, 1841.

For value received, I promise to pay John Smith, or order, one thousand dollars on demand, without defalcation.

WILLIAM BROWN.

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* *See Johnson's* Chancery Reports, Vol. 1, p. 17.

December 1st, 1841, received $75.00. July 17th, 1842, received $15.50. August 18th, 1843, received $30.50. December 11th, 1843, received $500.00. January 3d, 1844, received $150.00. What was due on the note, Aug. 18th, 1844?

First principal, on interest from March 4, 1841 - - - - - - -		$1000.00
Interest to Dec. 1, 1841, (8mo. 27dy.) -		44.50
Amount,		$1044.50
First payment, exceeding the interest due,		75.00
Balance for a new principal - - -		969.50
Interest from Dec. 1, 1841, to July 17, 1842, (7mo. 16dy.) -	36.52	
Second payment, less than interest due - - - - - -	15.50	
Excess of interest - -	21.02	
Interest from July 17, 1842, to Aug. 18, 1843, (13mo. 1dy.) - -	63.18	
Interest due Aug. 18, 1843 -	84.20	
Third payment, less than interest due - - - - - -	30.50	
Excess of interest - -	53.70	
Interest from Aug. 18, 1843, to Dec. 11, 1843, (3mo. 23dy.) - -	18.26	71.96
Amount due Dec. 11, 1843 - -		1041.46
Fourth payment, exceeding the interest due		500.00
Balance for a new principal - - -		541.46
Interest from Dec. 11, 1843, to Jan. 3, 1844, (23dy.) - - - - - -		2.07
Amount due Jan. 3, 1844 - - -		543.53
Fifth payment, exceeding the interest due		150.00

Balance for a new principal - - -	393.53
Interest from Jan. 3, 1844, to Aug. 18, 1844, (7mo. 15dy.) - - - -	14.76
Amount due Aug. 18, 1844 - -	$408.29

122. Examples for the Pupil.

301. $500.00 Worcester, July 4, 1844.

For value received, I promise to pay William Mason, or order, five hundred dollars on demand, with interest.

John Winter.

Endorsements. Sept. 5, 1844, received $97.25. June 3, 1845, received $15.75. Aug. 1, 1845, received $217. What was due Jan. 1, 1846?

302. $862.50 Boston, May 5, 1846.

For value received we promise to pay Brown & Oliver, or order, eight hundred and sixty-two dollars and fifty cents, on demand. Mason & Thomas.

Endorsements. Sept. 29, 1846, received $17.00. April 17, 1847 received $27.50. Dec. 13, 1847, received $800. How much was due on the settlement of the note, Sept. 3, 1848?

303. $1000.00 New York, Jan. 1, 1847.

For value received, we promise to pay George Appleton, or order, one thousand dollars on demand, with interest at 7 per cent. James Thompson & Co.

Endorsements. Sept. 25, 1847, received $243.50. Dec. 13, 1847, received $15.89. May 5, 1848, received $34.75. What was due Oct 1, 1848?

123. Compound Interest.

It is sometimes stipulated that the interest shall be paid quarterly, semi-annually, or annually. If not paid at the specified time, the interest is added to the principal, and the amount is regarded as a new principal, on which the next interest is to be computed. Interest calculated in this manner is called *compound interest.*

To find the amount of any sum at compound interest, *whenever the interest becomes due, add it to the* *principal, and consider the amount as a new principal,* as in the following example.

What is the amount of $280 for 1yr. 10mo. 22dy., the interest being payable semi-annually?

```
         280
         .03 rate for 6 mo.
        ----
        8.40
       280
      ------
      288.40 amt. for 6mo.
         .03
      ------
      8.6520
     288.40
    --------
     297.052 amount for 1yr.
         .03
     -------
     8.91156
    297.052
    --------
    305.963 amt. for 1yr. 6mo.
       0236 rate for 4mo. 22dy,
    -------
  7.2207268
  305.963
  ---------
  313.183 amt for 1yr. 10mo. 22dy.
```

As the interest is .06 per year, it will be .03 for 6mo. Increasing the principal by the interest, each time it becomes due, we find that $280 in 1yr. and 6mo. will amount to $305.963. For the remaining 4 mo. 22dy., the rate is .0236, and the interest is .0236 × 305.963 = 7.2207268, which added to $305.963, gives the whole amount desired.

In adding the interest, the result may usually be obtained with sufficient accuracy, by rejecting all decimals below the place of mills.

124. EXAMPLES FOR THE PUPIL.

304—310.. Find the amount of each of the following *sums for* 1yr. 10mo. 22dy., interest payable semi-annu*ally.* *$200;* $150; $375; $500; $87.25; $93.50; $844.

311—315. Find the amount of $180, interest payable semi-annually, at 5 per cent. per annum, for 1yr.; 2yr. 6mo.; 3yr. 3mo. 18dy.; 1yr. 9mo.; 11mo. 27dy.

316—320. Find the *compound interest** of $500 for 4yr., at 5 per cent.; 3yr. 7mo. at 4½ per cent.; 6yr. at 6 per cent.; 2yr. 11mo. 15dy. at 7 per cent.; 3yr. 3mo. 18dy. at 4 per cent.—the interest payable annually.

125. Problems in Interest.

I. TO FIND THE RATE, WHEN THE PRINCIPAL, INTEREST, AND TIME ARE GIVEN.

The *unknown quantity* in this problem, is the *number* of per cent. The value of the unknown number of per cent. (the interest,) is given, and to find the *number* of per cent., (§ **55**,) we must *divide the given interest by the interest at* ONE *per cent. for the given time.*

321. If I pay $37.50 for the use of $500 1yr. 6mo., what is the rate per cent.?

322. If the interest of $400 for 2yr. 3mo. is $40.50, what is the rate per cent.?

323—330. At what rate per cent. will $100 yield $100 interest, in 16yr. 8mo.?—in 20yr.?—in 25yr.?—22$\frac{2}{9}$yr.?—in 14$\frac{2}{7}$yr.?—in 12yr. 6mo.?—in 18yr. 9mo.?—in 16yr.?

II. TO FIND THE TIME, WHEN THE PRINCIPAL, INTEREST, AND RATE ARE GIVEN.

The *unknown quantity* in this problem, is the number of years. The value for the unknown number of years, (the interest,) is given, and to find the *number* of years, (§ **55**,) we must *divide the given interest by the interest for* ONE *year at the given rate.*

331. In what time will $1500 at 7 per cent., yield $420 interest?

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* The *interest* may be found by deducting the principal from the *amount.*

332. In what time will $900 amount to $1044 at 6 per cent., simple interest? (How do you find the interest, if you know the amount?)

333. A note for $500 amounted at its settlement to $510. How long was it on interest, the rate being 5 per cent.?

334. The interest on a note of $750 was $42.50, and the rate was .06. What was the time?

335—340. To double any principal, there must be a gain of $\frac{100}{100}$, or 100 per cent. Then in how many years will any investment be doubled at 6 per cent.? 4 per cent? 5 per cent.? 7 per cent.? 5½ per cent.? 6¼ per cent.?

III. TO FIND THE PRINCIPAL, WHEN THE TIME, RATE, AND INTEREST ARE GIVEN.

The *unknown quantity* in this problem, is the number of dollars that will yield the given interest. To find the *number* of dollars, (§ **55**,) *divide the given interest by the interest of* ONE *dollar for the given time.*

341. What principal at 6 per cent., will yield $150 in 18 months?

342. If a man's property is all invested at 5 per cent., what is he worth, supposing his income to be $1375 per annum?

343. If a man's property yields 5½ per cent. interest, and his annual income is $1093.12½, what is he worth?

344. What fund at 6 per cent. will support a charity school, the annual expenditure being $1575?

345—350. How much money at 5½ per cent. will yield $1200 in 2yr. 6mo.; 3yr.; 1yr. 8mo.; 9mo.; 4yr. 3mo.; 2yr. 11mo.?

IV. TO FIND THE PRINCIPAL, WHEN THE TIME, RATE, AND AMOUNT ARE GIVEN; OR, TO FIND THE PRESENT WORTH OF A DEBT DUE AT SOME FUTURE TIME.

The *unknown quantity* in this problem, is the number of dollars that will amount to the given sum in the *given time.* To find the *number* of dollars, (§ **55**,) *divide the given amount by the amount of* ONE *dollar for the given time.*

351. What sum of money at 6 per cent., will amount to $537.50 in 1yr. 3mo.?

352. What is the present worth of $1399.50 due in 2yr. 4mo. 15dy., if money will yield 7 per cent. interest?

353. What sum of money at 8 per cent., will amount to $3201 in 3yr. 8mo.?

354. When money yields but 5 per cent. interest, what is the present worth of $5316.84 due in 1yr. 7mo. 6dy.?

355—360. What is the present worth of $2000 due in 1yr. 6mo.? in 2yr.? 3yr. 3mo.? 9mo.? 11mo. 24dy.? 1yr. 5mo. 19dy.?

126. DISCOUNT is an allowance made for the payment of money before it is due.

In the settlement of accounts between individuals, the discount is usually found by subtracting the *present worth*, (found by Prob. IV., § **125**,) from the amount of the debt.

361. What discount should be made for the payment of a debt of $900, 1yr. 9mo. before it becomes due?

362. If I am to receive 7 per cent. of the amount of my purchases, and if I have received $1750 to pay for the whole amount of purchases and commission, how much may I retain for my commission?

363. If I gain 25 per cent. on the original cost of merchandise by selling it for $1718.75, what is the amount of gain?

364. When money is worth 7 per cent., what discount should be made on $700 paid 9mo. 27dy. before it becomes due?

365—370 What discount should be made on $1500 due in 2yr.? in 9mo.? 1yr. 6mo.? 8mo. 18dy.? 3yr. 3mo.? 1yr. 4mo.?

127. BANK DISCOUNT is the same as Bank Interest.

In borrowing money at a bank, it is customary to deposite one *or more notes, payable* at some specified time, (as 30, 60, or 90 days,) "after date." If the bank accepts and *discounts* a note, the interest is computed on the *face of the note** until the time when it becomes due, and deducted from the face of the note.

A note discounted at bank, is not usually considered as due until 3 days after the time specified for its payment. Thus a note for $500, dated May 4, and payable in 90 days after date, need not be paid until 93 days after May 4. The 3 additional days are called *days of grace*.

Therefore, to determine the amount that will be received upon a discounted note, ***deduct from the face of the note, the bank interest*** FOR THREE DAYS MORE ***than the time that will elapse before the specified time of payment.***

371. A note for $2000 payable in 30 days, was discounted at a bank. What was received for the note?

372—376. What would be received for the following notes, if discounted at a bank?

$500. Worcester, July 31, 1848.

Ninety days from date, I promise to pay Jacob Murray, or order, five hundred dollars, for value received.

JAMES WILLSON.

$375. Salem, Oct. 15, 1848.

Thirty days from date, we promise to pay Charles Adams, or order, three hundred and seventy-five dollars, for value received.

MAYNARD & KING.

$187.50. Philadelphia, Aug. 18, 1848.

For value received, I promise to pay Marchant & James, or order, one hundred and eighty-seven $\frac{50}{100}$ dollars, in sixty days after date, without defalcation.

GEORGE WAITE.

$750. Portland, Sept. 5, 1848.

Four months from date, I promise to pay Simeon Martin, or order, seven hundred and fifty dollars, value received.

J. M. LAWSON.

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* *The face of a note* is the amount for which the note is given.

$462.75. Baltimore, Aug. 1, 1848.

Ninety days from date, for value received, we promise to pay Marcus Willis, or order, four hundred and sixty-two $\frac{75}{100}$ dollars.

JACKSON & ELLIS.

377—391. What would be received for each of the above notes, if discounted at 5 per cent.? at 7 per cent.? at 8 per cent.?

392. What is the value of a note at 30 days for $8750, discounting at $5\frac{1}{2}$ per cent.?

393. What is the value of a note at 60 days, for $1200, discounting at $4\frac{3}{4}$ per cent.?

394. What is the value of a note at 90 days for $975, discounting at $4\frac{7}{8}$ per cent.?

395. What is the value of $\frac{1}{7}$ of $\frac{3}{8}$ of a note at 120 days, for $2000, discounting at $5\frac{1}{4}$ per cent.?

396. What is the value of a note at 90 days for £63 17s. 6d., discounting at 4 per cent.?

397. What is the difference between the *equitable* discount, (§ **126**,) and the *bank* discount, on $487.50, for 63 days?

398. How much would be gained by purchasing molasses for $1600 cash, and selling it immediately for $2000, payable in 4 months—the Mechanics' Bank discounting the note at 5 per cent.?

399. What is the value of a note at 60 days, for the amount of the following bill? $163\frac{5}{8}$yd of silk, at $87\frac{1}{2}$ cts. a yd.; $127\frac{1}{2}$yd. India muslin, at $62\frac{1}{2}$ct. a yd.; $97\frac{3}{4}$ yd. Alpacca, at $93\frac{3}{4}$ct. a yd.; 225yd. merino, at 50ct. a yd.; and $115\frac{5}{8}$yd. printed lawn, at $37\frac{1}{2}$ct. a yd.

400. Lowell, July 17, 1848.

George Manning and Brothers,
Bought of William Simpson & Co.

$6\frac{4}{7}$cwt. of sugar	at $5\frac{1}{2}$ct. a pound.
81 barrels of flour	at 4.62\frac{1}{2}$. a barrel.
3.375cwt. of coffee	at $8\frac{5}{8}$ct. a pound.

2cwt. 1qr. 17lb. of rice, at $3.50 a cwt.
13 boxes candles, each 32½lb. at 29¾ct. a pound.
Received payment by note at 90 days.
William Simpson & Co.

How much would be received on the note, if discounted at 7 per cent.?

QUESTIONS ON CHAPTERS VI., VII., VIII., AND IX.

What is a Prime Number? a composite number? Name all the prime numbers less than 100. How can you tell whether any number can be divided by 2, 5, 3, 9? How do you find the prime factors of any number? How do you determine whether any number is a prime? What is meant by a multiple? a common multiple? the least common multiple? How do we find the least common multiple of two or more numbers? Show that this process will give the least common multiple. What is a common divisor? the greatest common divisor? How do we find the greatest common divisor? Why do we proceed in this manner? What is cancelling? How do you cancel? When you have cancelled all the numbers in either the dividend or divisor, what must be put in their place? When the divisor is a composite number, how may we obtain the quotient?

What are Fractions? In how many different ways may they be read? Show why each of these ways may be adopted. What is meant by the numerator? the denominator? the terms of a fraction? What is shown by the numerator? by the denominator? How is the value of a fraction affected by increasing the denominator? the numerator? by diminishing the numerator? the denominator? What is a mixed number? How would you find what part 324 is of 187? Why?

What is Reduction? How do we reduce fractions to whole numbers? to mixed numbers? to decimals? Explain these reductions. What are proper fractions? improper fractions? When is a fraction greater than 1? less than 1? equal to one? How would you write a decimal in the form of a fraction? Explain the rule for reducing whole or mixed numbers, or decimals, to fractions. Explain the method of reducing compound to simple fractions; fractions to their lowest terms; fractions to a common denominator; complex to simple fractions; fractions to others having any given numerator or denominator. Explain the rules for addition, subtraction, multiplication and division of fractions.

What are Infinite Decimals? What is the repetend? How is it distinguished? To what is every repetend equivalent? Show that it is so. How then would you reduce any infinite decimal to a fraction? How may addition and subtraction of infinite decimals, be performed without reducing them to fractions?

What are Compound Numbers? Repeat the table of Federal

Money; English Money; Troy Weight; Apothecaries' Weight; Avoirdupois Weight; Long Measure; Cloth Measure; Square Measure; Cubic Measure; Dry Measure; Liquid Measure; Circular Measure; Time Measure. How many shillings in a guinea? How many dollars in a pound sterling? How do we find the area* of a rectangular surface? How many cubic feet make a foot of wood, or a cord foot? How many cord feet in a cord? How many cubic feet in a ton of round timber? a ton of square timber? a ton of shipping or storage? How do we find the solid contents of any rectangular solid?

For what purpose are each of the weights and measures used? How many cubic inches in a half peck? in a common gallon? in a gallon of milk or of malt liquor? For what purpose is the hogshead (measure) used? How many degrees in a quadrant? in a sign of the zodiac? How many days in a leap year! What years are leap years! Repeat the number of days in each month. In mercantile business how many days are considered as making a month? In Chap. VIII., § 109, solve examples 821, 816, 810, 830, 807, 806, 779, 780, 772, 784, and give your reasons for each step.

Give, in your own language, a rule for reducing higher denominations to lower; lower denominations to higher; for compound addition; compound subtraction; compound multiplication; compound division. Explain each rule. Can fractions and decimals be reduced by the same rule as whole numbers?

What is meant by Per Cent.? How do we compute any required percentage? Why? What is meant by commission? insurance? premium? policy? underwriters? taxes? stocks? the par value? the dividend on a stock? When is a stock said to be above par? below par? at a discount? at an advance? What part of the original cost is stock worth, that sells at 16⅔ per cent. below par? at 12½ per cent. advance? at a discount of 17⅞ per cent.? at 25 per cent. above par? What is meant by duties? specific duties? ad valorem duties? draft? tare? gross weight? net weight? In custom house business, what allowance is made for leakage and breakage? for draft? How do we find what per cent. is gained or lost by any transaction?

What is Interest? simple interest? compound interest? What is the principal? the rate? the amount? Give, in your own language, the General Rule for Interest? the Bank and Business Rule? Show how each rule is obtained. What is a promissory note? an endorsement? What is the usual mode of computing interest on notes that are settled within a year from their date? What is the *Legal* Rule? How do we find the *amount* of any sum at compound interest? How do we find the *compound interest?*

How do you find the rate, when the principal, interest, and time

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* The number of square feet, rods, or acres, &c., in any surface, *is called its area.*

are given? the time, when the principal, interest, and rate are given? the principal, when the time, rate, and interest are given? the principal, when the time, rate, and amount are given? Explain the reasons for each of these methods.

What is DISCOUNT? equitable discount? bank discount? How do you find equitable discount? bank discount? What are days of grace? How many days of grace are usually allowed?

X. ANALYSIS AND RATIO.

128. ANALYSIS is the examination of any proposed question, to determine the proper method for its solution.

In every question, one or more things are *given*, (which are sometimes called the *known quantities*,) and one or more things are *required*. The required quantities are often called *unknown quantities*.

If the value of ONE of each of the unknown quantities can be found, the value of ANY number of them can be readily determined, (§ **35**.) Therefore, whenever there is any difficulty in solving a question, we should *first endeavor to find the value of* ONE *of each unknown quantity*. In most cases, this can be easily done by the method stated in § **55**.

The following may be given as the

GENERAL RULE FOR ANALYSIS.

First endeavor to find the value of ONE *of each unknown quantity, and then by multiplication, find the value of the* REQUIRED NUMBER.

In every question, EXPRESS *the operations that are to be performed, (as far as it is possible to do so,) and shorten your work by* CANCELLING, *whenever you can.*

EXAMPLE. If 10 men can build a wall in 6 days, in what time could it be built by 15 men?

The question is, "in how many days would 15 men build the wall?" and we first find in how many days *ONE man* would build it.

It would take 1 man 10 times as long as 10 men, or

10 × 6 days. It would take 15 men only $\frac{1}{15}$ as long as 1 man, or $\frac{1}{15}$ of 10 × 6 days, or $\frac{\overset{2}{\cancel{10}} \times \overset{2}{\cancel{6}}}{\underset{\cancel{3}}{\cancel{15}}} = 4$ days.

129. EXAMPLES FOR THE PUPIL.

1—10. How long would it take 9 men to consume a stock of provisions, that would last 15 men 18 weeks? How long would it take 10 men? 6 men? 5 men? 30 men? 27 men? 45 men? 54 men? 90 men? 135 men?

11—20. If a stock of provisions is sufficient to supply 36 men for 35 weeks, how many men would consume it in 6 weeks? in 7 weeks? 30 weeks? 14 weeks? 15 weeks? 21 weeks? 10 weeks? 28 weeks? 42 weeks? 63 weeks?

21—30. If 15 men consume 70 pounds of provisions in a day, how many men would consume 112lb. in the same time? How many would consume 42lb.? 140lb.? 28lb.? 84lb.? 126lb.? 56lb.? 98lb.? 840lb.? 196lb.?

31—40. If 19 yards of broadcloth cost $85.50, what will be the cost of 13yd.? 27yd.? 84yd.? 53yd.? $49\frac{3}{4}$yd.? 6.875yd.? 99yd.? $37\frac{1}{2}$yd.? $3\frac{3}{8}$yd.? $\frac{15}{16}$yd.?

41—50. How long will it take 13 men to eat a stock of provisions, that 9 men can eat in 26 days? How long will it take 6 men? 18 men? 26 men? 39 men? 36 men? 24 men? 54 men? 78 men? 117 men?

51—60. If it takes 3 yards of cloth that is 4 quarters wide, to make a coat, how many yards would it take of cloth 3 quarters wide? 2 quarters? 6qr.? 5qr.? 8qr.? $4\frac{3}{4}$qr.? $\frac{15}{16}$qr.? $3\frac{7}{8}$qr.? $5\frac{1}{2}$qr.? $4\frac{1}{2}$qr.?

61—70. If $9\frac{1}{2}$yd. of cloth that is 6qr. wide will make a cloak, how many yards would be required, that is $5\frac{3}{4}$qr. wide? 7qr. wide? 8qr. wide? 4qr. wide? $6\frac{1}{2}$qr. wide? $7\frac{7}{8}$qr. wide? $3\frac{1}{4}$qr. wide? $4\frac{2}{3}$qr. wide? 9qr. wide? $7\frac{1}{2}$qr. wide?

71—80. If $4\frac{1}{2}$ acres of land cost 28.12\frac{1}{2}$, what is the price of 3 acres? 7 acres? $5\frac{1}{2}$A.? $6\frac{1}{4}$A.? $11\frac{3}{4}$A.? 19A.? 95A.? $17\frac{9}{10}$A.? 8.25A.? $\frac{15}{16}$A.?

81—90. A man can do a piece of work in $9\frac{1}{2}$ days, by working 8 hours a day; how many days will it take him, if he works 5 hours a day? 4 hours? 2h.? 6h.? 12h.? 9h.? 3h.? 7h.? 10h.? $12\frac{2}{3}$h.?

91—100. When a post 12 feet high casts a shadow of $9\frac{3}{5}$ feet, how high is a post that casts a shadow of 12 feet? 18ft.? 9ft.? 3ft.? 7ft.? 8.9ft.? $4\frac{7}{8}$ft.? 11ft.? 27ft.? $16\frac{2}{3}$ft.?

130. Ratio is the relation which one quantity bears to another. It is expressed by the quotient of the *first* quantity by the *second.** Thus the ratio of 4 to 5 is $\frac{4}{5}$; of 11 to 15 is $\frac{11}{15}$.

. The ratio of two numbers may be indicated by either of the methods adopted for expressing division, (§ **47**,) as $\frac{3}{4}$, $3 \div 4$, 3 : 4. The latter method is usually employed, thus:

12 : 7, 4 : 9, 15 : 8, 1 : 6,

which may be read, *the ratio of* 12 *to* 7, or 12 *is to* 7; *the ratio of* 4 *to* 9, or 4 *is to* 9, &c.

The first term of any ratio is called the *antecedent*, the second term the *consequent*.

If the ratio of two numbers of different denominations is required, *they must both be reduced to the same denomination.* Thus the ratio of 15lb. to 2cwt., is not $\frac{15}{2}$, but it is the same as the ratio of 15lb. to 224lb., or 15 : 224.

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* I have adopted the English definition of ratio, as more philosophical and more intelligible than the French. If I am asked "what is the ratio of 3 to 4?" 3 is evidently the number concerning which an answer is expected, and the proper answer would be, "3 is $\frac{3}{4}$ of 4."

The common definition of ratio, as the quotient of the second quantity by the first, probably arose from the custom of making the fourth term in proportion the unknown term. But even this custom, when properly regarded, favors the position of the English writers. For example, take the proportion

3 : 4 : : 7 : —

The ratio of the second term to the first, is the same as the ratio of the fourth to the third. But 4 is $\frac{4}{3}$ of 3, therefore the fourth term must be $\frac{4}{3}$ of 7. In such instances it is not the direct ratio of 3 to 4, but the inverse ratio of 4 to 3, that is taken into consideration.

131. EXAMPLES IN DETERMINING RATIO.

101—110. Write the ratio of 7 to 9; 15 to 3; 8 to 11; 1 to 4; 25 to 37; 40 to 35; 100 to 25; 33 to 44; 33 to 13; 19 to 190.

111—120. Write in the form of a fraction, the ratio of 3 to 5; 11 to 2; 1 to 8; 8 to 17; 5 to 4; 13 to 7; 6 to 14; 18 to 19; 4 to 8; 8 to 4.

121—130. Reduce to a decimal, the ratio of 1 to 2; 3 to 4; 4 to 3; 7 to 8; 8 to 5; 8 to 3; 1 to 8; 15 to 16; 19 to 16; 13 to 32.

131—140. Reduce each of the following ratios to its lowest terms. 4 to 8; 2 to 10; 15 to 20; 18 to 30; 42 to 14; 81 to 9; 45 to 27; 117 to 156; 143 to 78; 95 to 171.

141—150. Express in the form of a fraction, the ratio of 3s. to £5; 7s. 6d. to £1; 2s. 6d. to £1 5s.; 2yd. to 7in.; 1ft. 6in. to 5yd.; 1r. to 2ft. 8in.; $9 to 50 cents; $12\frac{1}{2}$ct. to $1; $16\frac{2}{3}$ct. to $1; $4.50 to $2.25.

151—160. What is the ratio of 13s. 11d. to £$2\frac{7}{8}$; £4.625 to .5d.; 13dwt. 12gr. to 10oz.; 2lb. 1oz. to 3oz. 7dwt.; 2℈ to 7ʒ; 19gr. to 7ʒ; 4cwt. to 3qr.; 1.5cwt. to 8T. $2\frac{1}{8}$cwt.; 1bu. to 5qt.; 7bu. 3pt. to 7qt.

161—170. What ratio expresses the part that 8 is of 11 (§ **70**, Ex. 1—50)? 3, of 5? 16, of 10? 2, of 4? 1qr., of 1cwt.? 3qr., of 2cwt.? 1cwt. 2qr., of 7cwt. 1qr.? 14T., of 14lb.? 5ʒ, of 3℔? 2s. 6d., of 9d.?

171—180. What part of 19dy. 7h. 25min. is 3h. 15min.; 9h. 30min.; 2dy.; 11dy. 7h.; 8min. 30sec.; 1mo.; $\frac{5}{8}$mo.; .4375dy.; 2mo. 11dy.; $5\frac{1}{16}$dy.

132. The work of analysis may often be abbreviated by the use of ratio, as in the following example.

If a locomotive goes 57 miles in 3 hours, how far will it go in 8 hours?

The answer may be obtained by the general rule in § **128**. Or, as 8 is $\frac{8}{3}$ of 3, in 8 hours it will go $\frac{8}{3}$ of the distance that it goes in 3 hours, which is $\frac{8 \times 57}{3 \times 1} = 8 \times 19 = 152$ miles. In other words, the ratio *of 8 hours* to 3 hours, is the same as the ratio of *the distance travelled* in 8 hours, to *the distance travelled* in 3 hours.

133. EXAMPLES FOR THE PUPIL.

Solve the following examples, by determining the ratio which the UNKNOWN QUANTITY *will bear to the* GIVEN QUANTITY.

181—190. If a man travels 100 miles in 5 days, how far will he go at the same rate, in 10 days? in 40 days? 2dy.? 11dy.? 9½dy.? 8.7dy.? 7¾dy.? 12.05dy.? .875dy.? $\frac{15}{16}$dy.?

191—200. A wall was built by 36 men in 8 months; how many men could have built it in 6mo? in 4mo.? 9mo.? 2mo.? 3mo.? 12mo.? 18mo.? 16mo.? 72mo.? 36mo.?

201—210. If the freight on a lot of goods is $14 for 45 miles, how much would it be at the same rate, for 18 miles? for 15m.? 40m.? 90m.? 37½m.? $19\frac{2}{7}$m.? $32\frac{1}{7}$m.? $51\frac{3}{7}$m.? 14¾m.? 36.81m.?

211—220. If £9 = $40, what is the value of £6.5? of £5.25? £19¾? £47.8625? £$3\frac{1}{16}$? £10 5s.? £8 3s. 3d.? 10s. 9d. 3qr.? £81? £1 11s. 11d.?

221—230. If $40 = £9, what is the value in English Money of $100? $87.50? $36? $28.64? $\$444\frac{4}{9}$? $97⅞? $350? $625? $98.33? $37½?

231—240. How much wheat at $1.20 a bushel, must be given in exchange for 90 barrels of flour, at $4.75 per barrel? at $4.50 per bbl.? at $5.25? $4.87½? $5⅞? $5⅜? $6? $5.50? $5¾? $5.62½?

241—250. The shadow of a stick, that is 5ft. 6in. high, measures 3ft. 4in. What is the height of a tree, whose shadow, at the same time, measures 75ft.? of a tree whose shadow is 50ft.? 100ft.? 80ft.? 62½ft.? 95ft.? 83.75ft.? 120ft.? 87¾ft.? 99ft.?

134. REDUCTION OF CURRENCIES BY RATIO.

In most parts of the United States, the English denominations of money are still employed to some extent. But the value of the pound differs in different places, as *will be seen by* the following Tables.

ENGLISH, OR STERLING MONEY.

£1 = $\$\frac{40}{9}$. $\$1 = £\frac{9}{40}$ = 4s. 6d.

CANADA CURRENCY.

Used in Canada and Nova Scotia.

£1 = $4 $\$1 = £\frac{1}{4}$ = 5s.

NEW ENGLAND CURRENCY.

Used in N. E., Virginia, Kentucky, and Tennessee.

£1 = $\$3\frac{1}{3}$. $\$1 = £\frac{3}{10}$ = 6s.

NEW YORK CURRENCY.

Used in New York, Ohio, and N. Carolina.

£1 = $\$2\frac{1}{2}$. $\$1 = £\frac{2}{5}$ = 8s.

PENNSYLVANIA CURRENCY.

Used in Pa., New Jersey, Delaware, and Maryland.

£1 = $\$2\frac{2}{3}$. $\$1 = £\frac{3}{8}$ = 7s. 6d.

GEORGIA CURRENCY.

Used in Georgia and South Carolina.

£1 = $\$4\frac{2}{7}$. $\$1 = £\frac{7}{30}$ = 4s. 8d.

Examples. Reduce £5 7s. 6d., Pa. currency, to Federal Money.

£5 7s. 6d. = £5.375. As £1 = $\$\frac{8}{3}$, in any number of pounds there will be $\frac{8}{3}$ as many dollars as pounds, and $\frac{8}{3}$ of 5.375 is $\frac{43}{3}$. The answer is therefore $\$\frac{43}{3} = \$14.33\frac{1}{3}$.

Reduce $87.50 to N. E. currency.

As $1 = £.3, in any number of dollars, there will be $\frac{3}{10}$ as many pounds as dollars, and .3 of 87.50 is 26.25. The answer is therefore £26.25 = £26 5s.

135. Examples for the Pupil.

251—260. Reduce $125 to each of the foregoing currencies; $93.75 to N. E., N. Y., Pa., and Ga. currencies.

261—270. Reduce to Federal Money, £17 9s. 3d. of each of the foregoing currencies; £9 5s. 8d. of N. E., N. Y., Pa., and G. currencies.

271—280. Reduce $72.25 to each of the foregoing currencies; $\$4\frac{1}{8}$ to each of the United States currencies.

281—290. Reduce to Federal Money, 7s. 6d. of each of the foregoing currencies; £1 3s. 9d. of each of the U. S. currencies.

291—300. What part of £1 sterling, is £1 of each of the other currencies? Then what is the value of £9 2s. 11d., in each of the other currencies?

136. REDUCTION OF STERLING MONEY, AT A PREMIUM.

Although the English pound is usually estimated at $\$4\frac{4}{9}$, it is actually worth about $4.87. Therefore, to cancel a debt of £1 in England, an American merchant would be obliged to remit more than $\$4\frac{4}{9}$.

This sum, ($\$4\frac{4}{9}$,) is called the *par value* of £1, and the percentage required above the par value, is called the *premium* of exchange. Thus, when exchange on England is at 10 per cent. premium, (or 10 per cent. above par,) $£1 = 1.10 \times \$4\frac{4}{9}$, and $\$1 = £\frac{9}{40} \div 1.10$, (§ **115**): when at $7\frac{1}{2}$ per cent. premium, $£1 = 1.07\frac{1}{2} \times \$4\frac{4}{9}$, and $\$1 = £\frac{9}{40} \div 1.07\frac{1}{2}$. Hence,

TO REDUCE ENGLISH MONEY TO FEDERAL MONEY.

Reduce the money at its par value, (§ **134—135**,) *and* MULTIPLY *by such percentage of the par value as will include the premium.*

TO REDUCE FEDERAL MONEY TO ENGLISH MONEY.

Reduce the money at its par value, and DIVIDE *by such percentage of the par value as will include the premium.*

137. EXAMPLES FOR THE PUPIL.

301—310. When exchange is at a premium of 9 per cent., what is the value in Federal Money of £7? £15? £24? 11s. 6d.? 9s. 4d.? 5s. 11d.? £13 2s. 9d.? 19s. $7\frac{3}{4}$d.? £9 9s.? £27 13s.?

311—320. When exchange is at a premium of 8 per cent., what is the sterling value of $100? $700? $350? $6.75? $28.50? $19.25? $79.80? $33.89? $1000? $1.33?

321—330. At .08½ premium find the Federal value of 3s. 9d.; 2s. 4d; 5s. 8d.; £2 11s.; £9 3s.; £16 14s.; 19s.; £49; £1000; £4 2s. 7d.

331—340. At 9¾ per cent. premium find the sterling value of $40; $7.25; $19⅜; $27⅞; $63⅝; $142.50; $1.75; $9⅔; $225; $800.

138. PRACTICE, OR ABRIDGMENT BY RATIO.

It is often more convenient to multiply or divide by the *part* of a dollar which is equivalent to a given price, than to use the price itself, as in the following examples.

At 37½cts. a yard, what will be the price of 15yd of cloth? 37½ct. $= \$\frac{3}{8}$, and $15 \times \$\frac{3}{8} = \$\frac{45}{8} = \$5\frac{5}{8} = \$5.62\frac{1}{2}$.

At 31¼cts. a bushel, how many potatoes can be bought for $6? 31¼ct. $= \$\frac{5}{16}$, and $\$6 \div \$\frac{5}{16} = 6 \times \frac{16}{5} = \frac{96}{5} = 19\frac{1}{5}$ bushels.

TABLE OF PARTS OF A DOLLAR.

$		$	$		$	$		$	$		$
.08⅓	=	$\frac{1}{12}$	.31¼	=	$\frac{5}{16}$	.56¼	=	$\frac{9}{16}$	.80	=	$\frac{4}{5}$
.10	=	$\frac{1}{10}$	.33⅓	=	$\frac{1}{3}$	.58⅓	=	$\frac{7}{12}$	.81¼	=	$\frac{13}{16}$
.12½	=	$\frac{1}{8}$	.37½	=	$\frac{3}{8}$	.60	=	$\frac{3}{5}$	.83⅓	=	$\frac{5}{6}$
.16⅔	=	$\frac{1}{6}$	.40	=	$\frac{2}{5}$	.62½	=	$\frac{5}{8}$	.87½	=	$\frac{7}{8}$
.18¾	=	$\frac{3}{16}$	.41⅔	=	$\frac{5}{12}$	.66⅔	=	$\frac{2}{3}$	.91⅔	=	$\frac{11}{12}$
.20	=	$\frac{1}{5}$	.43¾	=	$\frac{7}{16}$	.68¾	=	$\frac{11}{16}$	.93¾	=	$\frac{15}{16}$
.25	=	$\frac{1}{4}$	.50	=	$\frac{1}{2}$	.75	=	$\frac{3}{4}$	.95	=	$\frac{19}{20}$

139. EXAMPLES FOR THE PUPIL.

341—350. What is the cost of 14 yards of flannel, at 12½cts. a yd.? at $.16⅔? 25ct.? 31¼ct.? 18¾ct.? 20ct.? 37½ct.? 50ct.? 33⅓ct.? 40ct.?

351—360. How many potatoes can be bought for

\$25 at $12\frac{1}{2}$ct. a bushel? at $16\frac{2}{3}$ct.? 20ct.? 25ct.? $33\frac{1}{3}$ct.? 40ct.? 50ct.? $37\frac{1}{2}$ct.? $31\frac{1}{4}$ct.? $18\frac{3}{4}$ct.?

361—370. What is the price of 48 yards of linen at 50 cents a yard? at 75ct.? $62\frac{1}{2}$ct.? $56\frac{1}{4}$ct.? $68\frac{3}{4}$ct.? $93\frac{3}{4}$ct.? $37\frac{1}{2}$ct.? $43\frac{3}{4}$ct.? 80ct.? 60ct.?

371—380. In what time will a boy earn \$24, at 50 cents a day? at 75ct.? $62\frac{1}{2}$ct.? $56\frac{1}{4}$ct.? $68\frac{3}{4}$ct.? $93\frac{3}{4}$ct.? $37\frac{1}{2}$ct.? $43\frac{3}{4}$ct.? 80ct.? $87\frac{1}{2}$ct.?

381—390. What must I give for 36 bushels of peaches; at $37\frac{1}{2}$ct. a bushel? at 50ct.? 75ct.? $87\frac{1}{2}$ct.? $62\frac{1}{2}$ct.? $56\frac{1}{4}$ct.? 60ct.? $81\frac{1}{4}$ct.? 80ct.? $68\frac{3}{4}$ct.?

391—400. How many yards of sheeting can be bought for \$27, at $12\frac{1}{2}$ct. a yard? at 10ct.? $16\frac{2}{3}$ct.? 25ct.? 20ct.? $18\frac{3}{4}$ct.? $37\frac{1}{2}$ct.? 40ct.? $31\frac{1}{4}$ct.? $8\frac{1}{3}$ct.?

140. EXAMPLES IN COMPLEX ANALYSIS.

Four men, working 13 hours a day, can do $\frac{5}{7}$ of a job in 8 days; how many hours a day must 3 men work, to do $\frac{1}{2}$ the job in 11 days?

The question is, "how many hours a day must 3 men work, to do $\frac{1}{2}$ the job in 11 days?" We first endeavor to find how many hours a day, or *what part* of 13 hours a day, ONE man must work, to do ONE job in ONE day.

One man must work 4 times as long as 4 men, to do the same amount of labor; to do the whole job, he must work $\frac{7}{5}$ as long as to do $\frac{5}{7}$ of the job, (because $\frac{7}{7} = \frac{7}{5}$ of $\frac{5}{7}$;) and to do it in 1 day, he must work 8 times as many hours per day, as to do it in 8 days. Therefore, *one* man, would do the *one* job in *one* day, if he could work $\frac{4}{1} \times \frac{7}{5} \times \frac{8}{1} \times \frac{13}{1}$ hours per day.

It will take 3 men only $\frac{1}{3}$ as long as 1 man, to do the same amount of labor; to do $\frac{1}{2}$ the job, it will take only $\frac{1}{2}$ as long as to do the whole job; and to do it in 11 days it will take only $\frac{1}{11}$ as many hours per day as to do it in 1 day. Therefore, 3 men can do $\frac{1}{2}$ the job *in* 11 days, by working $\frac{1}{3}$ of $\frac{1}{2}$ of $\frac{1}{11}$ as many hours *per day, as* would be required for 1 man to do the whole job in 1 day; that is, $\frac{1}{3}$ of $\frac{1}{2}$ of $\frac{1}{11}$ of $\frac{4}{1} \times \frac{7}{5} \times \frac{8}{1} \times \frac{13}{1} = \frac{1456}{165} = 8\frac{136}{165}$ hours.

In such questions as the above, a division may be made into *given terms*, and *terms of demand.* In the present instance, the given terms are 4 men, 13 hours a day, $\frac{5}{7}$ of a job, and 8 days. The terms of demand, or the terms about which a question is asked, are — hours a day, (the *unknown term,*) 3 men, $\frac{1}{2}$ of a job, and 11 days. One of the given terms is always of the same kind as the answer, or *unknown term*, and in order to find the answer, we need only find what part the unknown term must be, of the given term that corresponds to it. This can be readily done by the following Rule, which is merely a recapitulation of the process adopted in the analysis we have just made.

RULE FOR COMPLEX ANALYSIS

1. *Observe which are the* GIVEN TERMS *of the question, which are the* TERMS OF DEMAND, *and which is the* REQUIRED TERM.

2. *Write in the numerator of a fraction, the given term which corresponds to the unknown term, (or the term which is of the same kind as the answer.)*

3. *Write the remaining given terms in such manner as to obtain the answer for* ONE *of each of the terms of demand, (placing in the numerator the numbers by which you wish to multiply, and the numbers by which you wish to divide, in the denominator.)*

4. *Having thus obtained a statement for* ONE, *arrange the terms of demand so as to give a statement for the* NUMBER REQUIRED. *Cancel, if possible, and reduce the resulting fraction to its simplest form.**

141. EXAMPLES FOR THE PUPIL.

401. If 29 men can do a piece of work in 11 days by working 10 hours a day, how long will it take 18 men, working 8 hours a day, to do the same work?

The question is, how many days, &c., or *what part* of 11 days? We therefore first write 11, and afterwards proceed with each term in order, as directed in the Rule. The answer is

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* The pupil should be required to practise upon this rule until he can state any example that may be proposed, as rapidly as he can set down the figures.

$$\frac{11 \times 29 \times \overset{5}{\cancel{10}}}{18 \times \underset{4}{\cancel{8}}} = \frac{1595}{72} = 22\tfrac{11}{72}\text{dy.}$$

402. How many men will reap 96 acres in 12 days, if 13 men reap 78 acres in 6 days?

$$\frac{\cancel{13} \times \cancel{6} \times \overset{8}{\cancel{96}}}{\underset{\cancel{13}}{\cancel{78}} \times \cancel{12}} = 8 \text{ men.}$$

403—412. By example 402, how many men would reap 72 acres in 36 days? in 2dy.? 18dy.? 12dy.? 3dy.? 4dy.? 9dy.? 6dy.? 8dy.? 24dy.?

413—423. How many men in 6 months, can build a wall that 36 men will build in 8 months? How many men would build it in 2mo.? in 18mo.? 9mo.? 4mo.? 12mo.? 3mo.? 24mo.? 72mo.? 32mo.? 48mo.?

424—434. How many bushels of meal will serve 36 persons 12 months, if 6 persons consume 8 bushels in 2 months? in 3mo.? 9mo.? 4mo.? 6mo.? 8mo.? 16mo.? 12mo.? 18mo.? 32mo.? 24mo.?

435—445. If $12 gain $3 in 5 months, how much ought $25 to gain in 16 months? in 4mo.? 2mo.? 10mo.? 15mo.? 7mo.? 6mo.? 8mo.? 1mo.? 5mo.? 13mo.?

446—456. If the freight of 900lb. for 56 miles is $2.50, how far should 2 tons be carried for $36? for $8? $15? $10? $4? $9? $7? $40? $11? $3? $6?

457—467. If $1200 support 24 persons 8 months, how long will $900 support 16 persons? 72 persons? 18? 24? 36? 144? 12? 6? 48? 9? 4?

468. If the expenses of a family of 8 persons are $40 in 10 weeks, how many persons can be supported 12½ weeks for $100?

[*Observe in this* example, that $1 \div \frac{25}{2} = \frac{2}{25}$, and in all cases, 1 *divided by any fraction, is the same fraction* REVERSED. (§ 81.)]

469. If $29\frac{1}{3}$ bushels of wheat yield 1760 bushels in 5 years, how much will $15\frac{1}{2}$ bushels yield in 6 years?

470. How many men in 24 days of 16 hours, will do 3 times as much work as 18 men can perform in 32 days of 12 hours?

471—480. How many men would accomplish the work required in Ex. 470, in 12 days of 16 hours? in 3dy.? 18dy.? 9dy.? 2dy.? 4dy.? 8dy.? 6dy.? 81dy.? 27dy.?

481. If I pay $14 for the freight of 3 tons 45 miles, what ought I to pay for carrying 11 tons 17 miles?

482. If 18 men build a cistern 20 feet long, 12 feet wide, and 10 feet high, in 3 weeks, by working 5 days in a week and 9 hours a day, how many men working 6 days in a week and 12 hours a day, will build a cistern 32 feet long, 16 feet wide, and 15 feet high, in 12 weeks?

483—492. How many men could build the above cistern in 3 weeks? in 9w.? 4w.? 6w.? 2w.? 36w.? 27w.? 18w.? 54w.? 108w.?

493. If the rent of 19A. 3R. of land is £4 10s. what should be the rent of $24\frac{1}{2}$A.?

494. If it cost $120 to support 16 persons 15 weeks, how many persons can be supported $6\frac{1}{4}$ weeks for $200?

495—504. How many persons would the above amount support 16 weeks? 5w.? 25w.? 20w.? $11\frac{3}{7}$w.? $3\frac{4}{7}$w.? 10w.? $26\frac{2}{3}$w.? 50w.? 400w.?

505. If $750 gain $$28\frac{3}{7}$ in 90 days, how much ought $$656\frac{1}{4}$ to gain in 60 days?

506. If 4 men in 3 days of 8 hours build 40 rods of wall, how many rods will 18 men build in 5 days of 9 hours?

507. If the freight of 2T. 6cwt. for 28 miles, is $14.50, what will be the freight of 9T. 4cwt. for 96 miles?

508. If a man travels 240 miles in 8 days, when the days are 12 hours long, how far can he travel in 24 days, when the days are 16 hours long?

509. If a man by walking 3 miles an hour for 6 hours a day, can accomplish a journey in 12 days, in how many days would he walk the same distance, at the rate of $2\frac{1}{2}$ miles an hour for 9 hours a day?

510—519. How many days would be required at the same rate, if he walked 12 hours a day? $7\frac{1}{5}$h.? $14\frac{2}{5}$h.? 4h. 48min.? 3h. 36min.? 5h. 24min.? 6h.? 8h.? $13\frac{1}{2}$h.? $10\frac{2}{7}$h.?

520. If $495 will support a family of 7 persons, 11 months and 6 days, how much will maintain a family of 9 persons, 13 months and 10 days?

521. How many persons can be supported for a year by $1014, if $140.25 will support 11 persons $8\frac{1}{2}$ weeks?

522. If $973.16 yields an interest of $91.25 in $13\frac{1}{2}$ months, what will be the interest of $2801 for $17\frac{2}{3}$ months, at the same rate?

523. If $3\frac{1}{3}$ tons are carried $87\frac{1}{2}$ miles for $$15\frac{1}{3}$, how far may $2\frac{7}{8}$ tons be carried for $27.75?

524. If 11 men reap 16 acres of grain in $3\frac{1}{2}$ days, how many men will reap $9\frac{1}{7}$ acres in $5\frac{1}{2}$ days?

525. If 11 men reap 16 acres in $3\frac{1}{2}$ days, how many acres can 7 men reap in 33 days?

526. If 10 compositors, in 4 days of 10 hours, set $66\frac{2}{3}$ pages of types, each page containing 45 lines of 50 letters, how many compositors will set $94\frac{2}{7}$ pages, each page containing 35 lines of 40 letters, in $5\frac{1}{2}$ days of 8 hours?

527. If $14\frac{5}{8}$ yards of cloth, that is $\frac{7}{8}$yd. wide, cost $29.50, what should be the price of $21\frac{1}{2}$yd. of similar quality, that is $1\frac{1}{2}$yd. wide?

528. If 300 tiles that are 9in. long and 6in. wide, will pave a court yard, how many tiles would be required that are 6in. long and 4in. wide?

529. What is the weight of 16 iron bars, each 7ft. long, 6in wide, and $3\frac{1}{2}$in. thick, if a bar 2ft. long, 2in. wide, and 1in. thick, weighs 18 pounds?

530. How many men will build a wall 240yd. long, 6ft. high, and 3ft. thick, in 8 days of 9 hours, if 7 men can build a wall 40yd. long, 4ft. high, and 2ft. thick, in 32 days of 7 hours?

142. DUODECIMALS are compound numbers, in which the ratio of each denomination to the one above it is 1 : 12. They are used in square and cubic measure.

The denominations of Duodecimals are, the foot or unit; the inch or prime, marked ($'$); the second, or twelfth of a prime ($''$); the third, or twelfth of a second ($'''$); the fourth($''''$), and so on. These marks are called *indices*.

Duodecimals are added and subtracted in the same manner as other compound numbers, but multiplication and division are often attended with difficulty.

To the product of any two denominations we must annex as many indices as there are in both factors. For example, $5'' \times 8''''$ is the same as $\frac{1}{12}$ of $\frac{1}{12}$ of 5 $\times$ $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of 8. In one factor the denominator 12 is contained twice, in the other 4 times; and in their product it will be contained 6 times, which will be represented by *six* indices. Thus, $5'' \times 8'''' = 40''''''$.

Required the product of 2 3' 7'' by 9 5' 11''.

$$\begin{array}{rrrrr}
 & 2 & 3' & 7'' & \\
 & 9 & 5' & 11'' & \\
\hline
 & 2' & 1'' & 3''' & 5'''' \\
 & 11 & 5 & 11 & \\
20 & 8 & 3 & & \\
\hline
21 & 9' & 10'' & 2''' & 5''''
\end{array}$$

We commence multiplying by 11''. $11'' \times 7'' = 77'''' = 6''' \; 5''''$. We set down the 5'''', and carry the 6'''. $11'' \times 3' = 33'''$, and 6''' make $39''' = 3'' \; 3'''$. Write down the 3''', and carry the 3''. $11'' \times 2 = 22''$, and 3'' make $25'' = 2' \; 1''$, which we set down. Next, multiplying by 5', we say, $5' \times 7'' = 35''' = 2'' \; 11'''$. Set down 11''', and carry 2''. $5' \times 3' = 15''$, and 2'' make $17'' = 1' \; 5''$. Set down 5'', and carry 1'. $5' \times 2 = 10'$, and 1' make 11', which we set down. Multiplying by 9, we say, $9 \times 7'' = 63'' = 5' \; 3''$. Set down 3'', and carry 5'. $9 \times 3' = 27'$, and 5' make $32' = 2 \; 8'$. Set down 8' and carry 2. $9 \times 2 = 18$, and 2 make 20, which we set down. Finally, adding the partial products, we find 5⁗; 14‴ = 1″ 2‴; 10″; 21′ = 1 9′; 21 units.

143. Examples for the Pupil.

531—540. Multiply 5 7′ 9″ by 4; by 7; 5′; 9′; 3″; 11″; 1 10′; 3 8′; 5 6′; 2 9′ 6″.

541—550. Multiply 4 0′ 7″ by 9 8′; 7′ 6″; 5″ 4‴; 3 2″; 1 11′; 2 10′; 1 7′ 5″; 5′ 9″; 3″ 3‴; 11‴.

551. How many feet in 6 boards, each of which is 13 11′ long, and 1 9′ wide?

552. How many square feet in a floor 18 6′ long, and 12 6′ wide?

553. How many cubic feet in a wall 19ft. 6in. long, 2ft. 3in. thick, and 4ft. 6in. high?

554. How many cubic feet in a pile of wood 16 3′ long, 4 6′ wide, and 8 feet high?

555. What is the price of a pile of wood, 128ft. long, 8 9′ wide, and 6 3′ high, at $6.25 per cord?

556. How many square yards of plastering in the ceiling of a room, that is 21 4′ long, and 18 7′ 6″ wide?

557. How many square feet of glass in 8 windows of 12 panes each, each pane being 1 1′ long, and 9′ wide?

558. How many cord feet in a pile of wood 32ft. long, 7 6′ wide, and 12 3′ high?

559. How many square yards in a grass plat, that is 24 9′ long, and 18 6′ wide?

560. Find the third power of 9 9′ 9″ 9‴.

144. *For the quotient of any two denominations we must subtract from the indices of the dividend as many as are contained in the divisor.* For example, 40′′′′′′ ÷ 5″, is equivalent to $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of 40 ÷ $\frac{1}{12}$ of $\frac{1}{12}$ of 5, or

$$\frac{40 \times \cancel{12} \times \cancel{12}}{\cancel{12} \times \cancel{12} \times 12 \times 12 \times 12 \times 12 \times 5.}$$

Cancelling two of the 12's from both terms, we have *only four 12's* left in the denominator, which will be represented by *four* indices. Then $40'''''' \div 5'' = 8''''$.

Divide 21 9' 10'' 2''' 5'''' by 9 5' 11''.

$$\begin{array}{rrrrrl}
9\ \ 5'\ \ 11'')21 & 9' & 10'' & 2''' & 5''''(2 & 3'\ \ 7'' \\
18 & 11' & 10'' & & & \\
\hline
2 & 10' & 0'' & 2''' & & \\
2 & 4' & 5'' & 9''' & & \\
\hline
 & 5' & 6'' & 5''' & 5'''' & \\
 & 5' & 6'' & 5''' & 5'''' & \\
\hline
\end{array}$$

9 units is contained in 21 units 2 units times. Multiplying the whole divisor by 2, and subtracting, we have a remainder 2 10' 0''. Bringing down the 2''', we find that 2 10' = 34' contains 9, 3' times. Multiplying and subtracting as before, we have a remainder 5' 6'' 5''', to which we annex the 5'''' of the dividend. 5' 6'' = 66'' contains 9, 7'' times. Multiplying by 7'', there is no remainder; the quotient is therefore 2 3' 7''.

145. EXAMPLES FOR THE PUPIL.

561—565. Divide 17 6' 3'' by 2 5'; by 7 3'; 2 7' 10'' 8''' by 9 4'; by 3' 5''; 4 8'.

566—570. Divide 7 0' 6'' by 4 4'; by 8 8'; by 1 7' 6''; by 9' 9''; by 3 3'.

571—575. Divide 8 4' 9'' 6''' by 3 5'; 2 5' 6''; 1 2' 9''; 10 3'; 1 8' 6''.

576—580. Divide 49 8' 2'' by 8 2'; 6 1'; 12 2'; 4 1'; 12 3'.

581—585. Divide 7' 11'' 2''' 2'''' by 12 10'; 7'' 5'''; 6' 5''; 3 2' 6''; 3 8' 6''.

582. The surface of a floor contains 166 9' 6'' 3''' 9'''' and the length of the room is 17 11' 3''. What is the breadth?

146. MISCELLANEOUS ANALYSIS.

The following examples are introduced to test the skill of the pupil in solving questions without formal rules.

583. A father's age is 7 times his son's, and the sum of their ages is 40. What is the age of each? (40 is how many times the son's age?)

584. The greater of two numbers is 5½ times the less, and the sum of the numbers is 13. What are the numbers?

585. The smaller of two numbers is $\frac{3}{7}$ of the greater, and the sum of the numbers is 25. Required the numbers?

586. In a certain school there are 51 pupils, and there are twice as many girls as boys. Required the number of each.

587. A man performed a journey of 144 miles, going three times as far on the second day as on the first, and twice as far on the third day as on the second. How far did he travel each day?

588. A., B., and C., entered into partnership, contributing altogether $2700. B. paid twice as much as A., and C. paid as much as A. and B. What was each man's contribution?

589. An estate of $5000 was so divided that the widow received $600 more than the daughter, and the son $800 more than the widow. What was the share of each? ($5000 is how much more than three times the daughter's share?)

590. Divide the number 194 into four such parts that the second may be twice the first, the third seven more than the second, and the fourth 18 more than the first.

591. A thief, travelling at the rate of 6 miles an hour, has been absent 5½ hours, when a constable starts in pursuit, at the rate of 9 miles an hour. In what time will the thief be overtaken?

592. A farmer bought some cows and calves for $418, giving $22 apiece for the cows, and $8 apiece for the calves, and there were twice as many calves as cows. How many were there of each?

593. If from 4 times a certain number 13 be sub-

tracted, the remainder will be 127. What is the number?

594. One-third of a certain pole is painted green, and $\frac{2}{5}$ of it is painted white, the remainder, which is 8 feet, being in the ground. What is the length of the pole? (8 feet is what part of the entire length?)

595. Five-eighths of a certain number exceeds $\frac{1}{8}$ of it by 21. What is the number?

596. The sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{2}{7}$ of a certain number is 115. Required the number.

597. The sum of three numbers is 96; the second is 2 more than the first, and the third is 4 more than the sum of the other two. What are the numbers?

598. Two stages are travelling toward each other, one at the rate of $5\frac{2}{7}$ miles an hour, the other $6\frac{1}{9}$ miles an hour. In what time will they meet, if they are now $38\frac{2}{3}$ miles apart?

599. Two men start from the same place and travel in opposite directions, one at the rate of $4\frac{1}{2}$ miles, and the other $5\frac{3}{11}$ miles an hour. In what time will they be 100 miles apart?

600. A farmer hired a certain number of boys, and twice as many men, agreeing to pay each man $1.00 a day, and each boy 50 cents. The daily wages of the whole amounted to $12.50. How many were there of each?

601. What number is that from which if we deduct $\frac{2}{7}$ of itself, and $\frac{2}{3}$ of the remainder, there will be 18 left?

602. If A. can do $\frac{1}{2}$ of a piece of work in 5 days, B. can do $\frac{1}{3}$ of it in 4 days, and C. can do $\frac{1}{6}$ of it in 2 days, in what time can they do the whole by working together?

603. There are two girls of the same ages, but if one was 3 years older and the other 5 years younger, the former would be twice as old as the latter. What is their age?

604. $\frac{1}{3}$ and $\frac{2}{5}$ and $\frac{7}{10}$ and $\frac{9}{20}$ of a certain number, make 36. What is the number?

605. If a man can do $\frac{7}{8}$ of a piece of work in 5 days and a boy can do ⅔ of it in 6 days, in what time can they both do the whole?

606. There were 668 sheep in four flocks. The first flock had 33 more than the second, 95 more than the third, and 200 more than the fourth. How many were there in each flock?

607. A teacher being asked the number of his pupils, replied: "If I had as many more, and ¾ and ⅝ and $\frac{7}{20}$ and $\frac{5}{10}$ as many, I should have 338." How many had he?

608. A person, after spending ¼ and ⅜ of his money, had $26⅜ left. How much had he at first?

609. A cistern has three pipes; the first can fill it in ⅓ of an hour, the second can fill it in ¼ of an hour, and the third can empty it in ½ of an hour. In what time will the cistern be filled if all the pipes are open?

610. Four-ninths of a certain number exceeds ⅔ of ⅓ of it by 32. What is the number?

611. A hare starts 5 rods before a greyhound, and runs at the rate of 12 miles an hour. After running 48 seconds, the hound starts in pursuit, and runs 18 miles an hour. In what time will the hare be overtaken?

612. A spendthrift, after squandering $\frac{3}{7}$ of his fortune and ⅔ of the remainder, had $1600 left. How much had he at first?

613. A daughter received $800, which was ⅔ of her brother's share, and 5 times her brother's share was twice as much as her father was worth. What was their father's fortune?

614. Find two numbers in the ratio of 5 to 9, the sum of which shall be 196.

615. Find two numbers in the ratio of 11 to 8, whose difference is 19.

616. If it takes a man 7½ days to perform a piece of work, working 8⅔ hours a day, how long will it take him if he works 9⅞ hours a day?

617. If I lend $175.50 for 5 months, how much ought I to receive in return, to use 2 months?

618. Three men traded in partnership; A. contributed \$230, B. \$215, and C. \$300. What part of the profits should each receive?

619. If the profits in the preceding example were \$149, what would be each man's share?

620. If a bushel of beans cost \$1.25, a bushel of potatoes .6 as much, and a bushel of wheat 2.5 as much as the potatoes, how much do they all cost?

621. A man being asked how far he had travelled, said that if he had travelled as far again, and $\frac{1}{13}$ and $\frac{2}{15}$ as far, he would have gone 119 miles. How far had he travelled?

* * *

XI. PROPORTION.

147. A PROPORTION consists of two equal ratios; as, $2:3 = 4:6$, *the ratio of* 2 *to* 3 *equals the ratio of* 4 *to* 6.

Instead of the sign of equality, four points are usually written; thus,

$$2:3::4:6,$$

which is read, 2 *is to* 3 *as* 4 *is to* 6.

148. It will be readily seen, (§ **47**, **130**, **147**,) that a proportion may be written in either of the following ways:

$$2:3::4:6;\ 2:3 = 4:6;\ 2 \div 3 = 4 \div 6;\ \tfrac{2}{3} = \tfrac{4}{6}.$$

The first and fourth terms of a proportion, are called the *extremes*, and the second and third terms are called the *means*. In every proportion, *the product of the extremes is equal to the product of the means.* Thus, in the proportion $2:7::6:21$, or $\frac{2}{7} = \frac{6}{21}$, if we multiply by both denominators, we find that

$$\frac{21 \times \cancel{7} \times 2}{\cancel{7}} = \frac{\cancel{21} \times 7 \times 6}{\cancel{21}}, \text{ or } 21 \times 2 = 7 \times 6.$$

149. If any term of a proportion is wanting, it may be found by the rule for reducing any fraction, *to an equivalent fraction whose numerator or denominator is given*, (§ **77**.)

It is customary in stating questions by Proportion, to make the fourth term represent the unknown term. The numerator of the second fraction is then given, and the denominator required. The required denominator will be found by the above rule, by multiplying the third term by the ratio of the second term to the first term, which is the same as multiplying the third term by the second, and dividing by the first term.

150. Therefore, when one of the extremes of a proportion is required, *divide the product of the means by the given extreme.* When one of the means is required, *divide the product of the extremes by the given means.* (§ 47, 148, 149.)

151. EXAMPLES FOR THE PUPIL.

1—10. Determine the fourth term in each of the following proportions: 1 : 3 : : 5 : —; 2 : 7 : : 8 :—; 5 : 2 : : 25 : —; 4 : 3 : : 12 : —; 6 : 5 : : 18 : —; 9 : 1 : : 9 : —; 2 : 4 : : 1 : —; 7 : 5 : : 14 : —; 3 : 3 : : 11 : —; 2 : 9 : : 6 : —.

11—20. Find the fourth term in the following proportions: 15 : 2 : : 7½ : —; 5 : 9 : : 2 : —; 3 : 7 : : 4 : —; 6 : 2 : : 4 : —; 9 : 2¼ : : 8 : —; 3 : 5⅓ : : 9 : —; 1½ : 2 : : 6 : —; 7 : 11 : : 8 : —; 2 : 1 : : 3¾ : —; 16 : 19 : : 13 : —.

21—30. Determine the following fourth terms: 3 : 4 : : 5 : —; 6 : 7 : : 8 : —; 9 : 10 : : 11 : —; 1½ : 2½ : : 3½ : —; .5 : 1¾ : : 2⅛ : —; 8 : .75 : : 15⅔ : —; 13 : 3 : : 11 : —; 4 : 7 : : 5 : —; 6 : 1¾ : : 24 : —; 2½ : 3 : : 10 : —.

31—40. Determine the following fourth terms: 1.75 : .6 : : .31 : —; ¾ : ⅘ : : 2½ : —; 6 : 6¼ : : 24 : —; ⅓ : ½ : : 2⅔ : —; 3 : 5 : : $\frac{2}{7}$: —; $\frac{2}{9}$: ⅜ : : 6⅔ : —; 11 : 8 : : 10 : —; 4 : 13½ : : 1⅝ : —; 6 : $\frac{6}{8}$: : ⅜ : —; 14 : $\frac{2}{7}$: : 3½ : —.

41—50. Find the unknown terms in the following expressions: 2 : 5 : : — : 9; — : 4 : : 3 : 7; 1 : — : : 5 : 3; $\frac{2}{—} = \frac{7}{8}$; $\frac{—}{5} = \frac{3}{4}$; $\frac{1}{6} = \frac{—}{2}$; $\frac{3}{11} = \frac{5}{—}$; 5 ÷ — = 3 ÷ 2½; 4 ÷ 9⅔ = — ÷ 5¾; — ÷ 6¼ = 6½ ÷ ⅞.

152. All questions which can be solved by multiplying a given quantity by one term, and dividing by another, (as in § **131—135**, and **139**,) may be *stated in* the form of a proportion, by placing for-

the third term the quantity which is to be multiplied or divided, the multiplier for the second term, and the divisor for the first term. (§ **150.**)

Examples for the Pupil.

Arrange each of the following examples as the first, second, and third terms of a proportion, the fourth term of which would be the product or quotient required.

51—70. $\frac{1}{2} \times 7$; $\frac{2}{3} \times 9$; $\frac{3}{4} \times 8$; $\frac{6}{7} \times 14$; $\frac{5}{9} \times 17$; $13 \times \frac{2}{5}$; $16 \times \frac{8}{3}$; $9 \times \frac{5}{2}$; $\frac{4}{11} \times 12$; $\frac{3}{2} \times 4$; $5\frac{1}{2} \times 2$; $6 \times \frac{13}{7}$; $4 \times 2\frac{4}{5}$; $\frac{16}{19} \times 10$; $9 \times \frac{3}{7}$; $8 \times 2\frac{1}{3}$; $4 \times 7\frac{1}{2}$; $\frac{3}{5} \times 5$; $\frac{6}{8} \times 9$; $\frac{1}{3} \times 5$.

71—90. $\frac{3}{4}$ of 5; $\frac{6}{7}$ of 8; $\frac{9}{10}$ of 11; $\frac{5}{6}$ of 7; $\frac{8}{9}$ of 12; $\frac{6}{5}$ of 3; $\frac{10}{9}$ of 2; $\frac{7}{6}$ of 1; $\frac{4}{3}$ of 8; $\frac{3}{11}$ of 11; $\frac{5}{7}$ of 16; $\frac{7}{5}$ of 19; $\frac{4}{2}$ of 8; $\frac{10}{3}$ of 24; $\frac{7}{2}$ of 16; $\frac{3}{10}$ of 9; $\frac{2}{13}$ of 15; $\frac{16}{5}$ of 25; $\frac{14}{3}$ of 7; $2\frac{1}{2} : 7\frac{7}{8}$ of $9\frac{2}{3}$.

91—110. $16 : 17 \times 4$; $2 : 8 \times 3$; $5 : 4 \times 6$; $7 : 1 \times 19$; $9 : 8 \times 2$; $1\frac{3}{7} : \frac{4}{9} \times 13$; $\frac{3}{5} : 2\frac{1}{2} \times 5\frac{7}{11}$; $2 : 1 \times 4$; $5 : 3 \times 16$; $4 : 7 \times 8$; $1 : 2 \times 3$; $4 : 5 \times 6$; $7 : 8 \times 9$; $\frac{1}{4} : \frac{2}{3} \times \frac{5}{7}$; $\frac{6}{5} : \frac{3}{11} \times 2\frac{3}{4}$; $\frac{6}{9} \times 8$; $\frac{2}{5} \times \frac{1}{2}$; $\frac{4}{7} \times 9$; $3 : 11 \times 2$; $6 \times 1 : 3$.

111—130. $\frac{5}{8} \div 4$; $\frac{2}{7} \div 9$; $\frac{1}{3} \div 2\frac{1}{2}$; $\frac{4}{7} \div 11$; $\frac{11}{4} \div 7$; $\frac{7}{11} \div 4$; $\frac{7}{4} \div 11$; $2 : 3 \div 8$; $1 : 9 \div 5$; $6 : 2 \div 1$; $8 : 4 \div 2$; $\frac{1}{3} \div 2$; $\frac{3}{5} \div 8$; $\frac{2}{7} \div 1\frac{1}{2}$; $\frac{6}{8} \div 6$; $\frac{13}{8} \div 12$; $\frac{2}{5} \div 9$; $\frac{5}{19} \div 27$; $\frac{4}{13} \div 8\frac{7}{11}$; $\frac{3}{4} \div 5$.

131—150. Complete the proportions in Ex. 51—70, by finding the fourth terms of each.

153. All questions in which the value of any number of things is given, and either the value of some other number of things or the number of things corresponding to some other value is required, can be solved by multiplication and division, as stated in the last paragraph. (See Examples in Chapter X.)

(1.) If 9 men build 10 rods of wall in a day, how much will 7 men build in the same time?

We must divide 10 by 9 to find how much one man would build, and multiply by 7 to find how much 7 men would build.

(2.) If 8 men build a wall in 12 days, in what time would 3 men build it?

We must multiply 12 by 8 to find how long it would take one man, and divide by 3 to find how long it would take 3 men.

The ratio of the given quantity to the required quantity in Ex. (1), is said to be *direct*, because, (§ 132, 133,)—

men. men. the no. of rods. the no. of rods.
9 : 7 : : that 9 men build : that 7 men build.

The ratio of the given time to the required time in Ex. (2,) is said to be *inverse*, because

men. men. the time the time
3 : 8 : : 8 men require : 3 men require.

154. The number to be multiplied and divided in all questions of proportion, will be of the same kind as the answer, (§ **140**,) and therefore, (§ **152**,) we always *write the number which is of the same kind as the answer, for the third term.*

If the answer is *greater* than the third term, the multiplier, (or second term,) must be *greater* than the divisor, (or first term.) If the answer is *less* than the third term, the multiplier must be *less* than the divisor, (§ **132**, **68**, **69**.) Hence, the

RULE FOR SIMPLE PROPORTION.*

Write the given number which is of the same kind as the answer, for the third term of a proportion, and consider whether the answer will be GREATER *or* LESS *than the third term. If greater, write the greater of the two remaining numbers for the second term, and the less for the first term. If less, write the smaller number for the second term, and the greater for the first term.*

Multiply the third term by the second, and divide the product by the first term, (cancelling if possible before PERFORMING *the operation,) and the result will be the answer sought.*

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* *This rule* is sometimes called "THE SINGLE RULE OF THREE."

155. EXAMPLES FOR THE PUPIL.

Solve the following examples both by Analysis and by Proportion, and endeavor to obtain the same statement by each method.

151. If 5 horses draw 20 tons on a railroad, how much could 11 horses draw?

$$\text{Statement};\ 5 : 11 :: 20 : \text{—};\ \frac{11 \times \overset{4}{\cancel{20}}}{\cancel{5}} = 44.$$

152. If 5 horses draw 20 tons, how many horses could draw 44 tons?

153. If 11 horses draw 44 tons, how many horses could draw 20 tons?

154. If 11 horses draw 44 tons, how much could 5 horses draw?

155. If 5 men can do a piece of work in 14 days, in what time would 7 men do it?

156. If 5 men can build a wall in 14 days, how many men would build it in 10 days?

157. If a stock of provisions would supply 7 men for 10 days, how long would it supply 5 men?

158. How many men in 14 days, could accomplish as much as 7 men in 10 days?

159. If a cistern discharge $83\frac{1}{3}$ gallons of water in 1.3 hours, how much will it discharge in $6\frac{5}{7}$ hours?

160. If a cistern discharge 49 gallons in 15 minutes, in what time will it discharge 37 gallons?

161. The price of 17 yards of cloth being $\$2.12\frac{1}{2}$, what would be the price of $29\frac{1}{2}$ yards?

162. When 7 barrels of flour cost $31.50, what is $\frac{3}{4}$ of a barrel worth?

163. If $\frac{3}{4}$ of a barrel of flour is worth $3.75, what is the value of $19\frac{2}{3}$ barrels?

164. The interest on a note at 6 per cent. is $137.50; what would it have been at $7\frac{1}{2}$ per cent.?

165. At what per cent. would a note yield $204, if at 5 per cent. it yields $170?

166. The interest on a note is $219.75 a year. In what time would the interest amount to $351.60?

167. At 6 per cent. per annum, what would be the rate for 7 years and 9 months?

168. At 5 per cent. a year, in what time would the rate amount to 29 per cent.?

169. If 16 apples are worth 18 pears, how many pears are equivalent to 104 apples?

170. If the freight of 90 tons is $160, what should be the freight of 22T. 10cwt. for the same distance?

171. If a man earn $10.50 a week, in how many working days will he earn $70?

172. If 12 horses eat 11.375 bushels of oats in a week, how much will 74 horses eat in the same time?

173. How long will it take a carrier dove to fly 75 miles at the rate of 62½ miles in 2.5 hours?

174. How many yards of serge that is ¾ wide, will line a cloak, containing 9¾ yards of broadcloth that is 1.5 yards wide?

175. A family of 9 persons use a barrel of flour in 70 days. At the same rate, how many persons would use a barrel in 17½ days?

176. The provisions of a garrison are sufficient to supply 380 men 15 days. How long would they last 570 men?

177. How much carpeting that is ⅞yd. wide, will cover a floor 4½yd. long and 3½yd wide?

178. If the interest of $1.50 is $.1125, what should be the interest of $492.75 for the same time?

179. How many dollars are equivalent to £9 11s. 8d., if $1.00 = 4s. 6d.?

180. If 9lb. of sugar cost 62½ cents, how many pounds could be bought for $10?

181. If I give 6 cents a year for the use of $1, how much per year must I pay for the use of $950.375?

182. If a silver cup at $1.25 per ounce cost $45.93¾, what is its weight?

183. If at noon to-day the shadow of a post 6ft. 3in. high, measures 4ft. how high is a spire whose shadow measures 82ft. 6in.?

184. If 340 men can build a wall in 5 months, how many men would build it in 17 months?

185. If I lend $115 for 5 months, how long may I keep $138 in return for the favor?

186. If I can perform a journey in 14 days, when the days are 9 hours long, how many days will it take when the days are 12 hours long?

187. How many cows will consume as much pasturage in 35 days as 28 cows in 15 days?

188. If 1cwt. of sugar cost £1$\frac{3}{16}$, what will 24$\frac{1}{2}$lb. cost?

189. What is a vessel worth, if $\frac{5}{8}$ of it can be bought for 850\frac{3}{8}$?

190. If $\frac{3}{8}$ of a yd. of cloth cost $\frac{3}{16}$ of a dollar, what would $\frac{7}{8}$yd. cost?

156. COMPOUND PROPORTION.

The ratio of the required quantity to the third term, often depends on the product of two or more other ratios; the proportion existing in such cases, is called *Compound Proportion.*

Four men, working 13 hours a day, can do $\frac{5}{7}$ of a job in 8 days; how many hours a day must 3 men work, to do $\frac{1}{2}$ the job in 11 days?

We wish to find *what part* of 13 hours a day the answer will be, and we therefore place 13 for the third term. (§ 152, 154.)

If we leave the number of days and amount of work out of the question, it would take 3 men $\frac{4}{3}$ as many hours per day as 4 men, (§ 132, 133,) to do the *same* work in the *same* number of days; or,

$$4 : 3 \times 13 \text{ hours.}$$

But to do only $\frac{1}{2}$ of the job, it will take only $\frac{1}{2} : \frac{5}{7}$ as long as to do $\frac{5}{7}$ of it; or,

$$\frac{1}{2} : \frac{5}{7} \times 4 : 3 \times 13 \text{ hours.}$$

And to do it in 11 days, it will take only $\frac{8}{11}$ as long as to do it in 8 days; or,

$$8 : 11 \times \frac{1}{2} : \frac{5}{7} \times 4 : 3 \times 13 \text{ hours.}$$

The third term must therefore be multiplied by the product of all the ratios which can be formed, by considering the different circumstances on which the answer will depend.

As in simple proportion, the multiplying terms or *numerators of* the ratios, are written for the second terms, and the denominators or dividing terms, for the first terms, (§ **150**, **152**,) the preceding question may be stated as follows:

$$\left.\begin{array}{r}3 : 4\\ \frac{2}{7} : \frac{1}{5}\\ 11 : 8\end{array}\right\} :: 13 : \textit{Ans.}$$

Then multiplying the third term by the product of the second terms, and dividing by the product of the first terms, we obtain for the answer

$$\frac{13 \times \overset{2}{\cancel{4}} \times 1 \times 7 \times 8}{3 \times \cancel{2} \times 5 \times 11} = \frac{1456}{165} = 8\tfrac{136}{165} \text{ hours,}$$

the same result as was obtained by Analysis, (§ **140**.)

RULE FOR COMPOUND PROPORTION.*

Write the given number which is of the same kind as the answer, for the third term. Of the remaining quantities, compare any two of a kind, and arrange them as directed in Simple Proportion. In the same way compare any other two of a kind, and so proceed until all the numbers are employed.

Multiply the third term by the product of all the second terms, and divide by the product of all the first terms, (cancelling, if possible, before PERFORMING *the operation,) and the result will be the answer sought.*

157. EXAMPLES FOR THE PUPIL.

Solve the following examples, both by Analysis and by Proportion.

191. If 9 men build 63 rods of wall in 7 days, how much would 13 men build in 17 days?

192. If 7 men mow 56 acres of grass in 4 days, how many men would mow 70 acres in 7 days?

193. If 30 reams of paper will print 1500 pamphlets of 10 sheets each, how many reams will print 740 pamphlets of 12½ sheets each?

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* *This rule is sometimes called* "THE DOUBLE RULE OF THREE."

194. If 15 men build a wall 45ft. long 8ft. high and 4ft. thick, in 16 days of 9 hours, how many days of 8 hours will 60 men require, to build a wall 100ft. long 9ft. high and 3ft. thick?

195. If the wages of 14 workmen for 2 days are $42.00, what should 19 men receive for 5 days' work?

196. If the wages of 7 men for 6 days are $52.50, how many men can be hired 14 days for $157.50?

197. If the interest of $1.00 is 6 cents a year, what would be the interest of $496.50 for 8mo. 24dy.?

198. If the interest of $496.50 for 8.8mo. is $21.846, what is the rate?

199. What principal at 6 per cent. will gain $21.846 in $8\frac{4}{5}$mo.?

200. In what time will $496.50 at 6 per cent., gain $21.846?

Solve by Proportion the examples in § **141.**

158. ARBITRATION OF EXCHANGE; OR, THE CHAIN RULE.

Debts are often paid, and sales are sometimes effected, through the medium of a number of persons besides the principal debtor or creditor. The method by which we determine the result of the several transactions, is called Arbitration of Exchange.

A French merchant wishes to pay in London, a bill of £1500. How many francs must he remit for that purpose, through brokers in St. Petersburg, Hamburg, and Madrid, if £13 = 75 roubles; 5 roubles = 9 marcs of Hamburg; 3 marcs = 1 Spanish dollar; and 9 dollars = 50 francs?

$$\left.\begin{array}{r}13:75\\5:\ 9\\3:\ 1\\9:50\end{array}\right\}::1500:\mathit{Ans.}$$

Since £13 = 75 roubles, there will be $\frac{75}{13}$ as many roubles as pounds. As 5 roubles = 9 marcs, there will be $\frac{9}{5}$ as many marcs as roubles. In like manner there will be $\frac{1}{3}$ as many dollars as marcs, and $\frac{50}{9}$ as many francs as dollars. The number of francs required

BY PROPORTION. *The whole stock : each man's share of the stock : : the whole gain or loss : his share of the gain or loss.*

1400 : 500 : : 350 : 125 A.'s share.
1400 : 300 : : 350 : 75 B.'s share.
1400 : 600 : : 350 : 150 C.'s share.

161. EXAMPLES IN FELLOWSHIP.

Solve the following examples both by Analysis, and by Proportion.

211. A. contributes to an adventure $200, and B. $500. Their gain was $350, what was the share of each?

212. A.'s stock in trade is $450, B.'s $350, and C.'s $500. How must a loss of $169 be divided between them?

213. Divide $936 dollars, so as to give A. 1 share, B. 2 shares, and C. 3 shares.

214. Divide 196 into four parts which may be in the proportion of 5, 3, 4, and 2, respectively.

215. A. pastured 6 horses, B. 3, and C. 9. How much must each pay; the whole expense of pasturing being $72?

216. A bankrupt wishes to pay a dividend of $500 among three of his creditors. How much must he pay to each, supposing him to owe A. $236, B. $980, and C. $884?

217. A bankrupt owes $15600, and his property is worth only $10600. How much can he pay for every dollar of his debts?

218. A. owns $\frac{4}{7}$ of a house, B. owns $\frac{2}{7}$, and C. owns $\frac{1}{7}$. How much of the rent, which is $280, should each receive?

219. A man left by will $500 to his widow, $600 to his eldest son, $500 to his youngest son, and $400 to his daughter, but his whole estate proved to be worth only $1600. How much should each receive?

220. Divide 650 into four parts, which shall be to *each other in* the proportion of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{7}{12}$.

162. COMPOUND FELLOWSHIP is Fellowship in which two or more circumstances are to be considered, in determining each share of the gain or loss.

A. contributes $5000 for 4 months, B. $6000 for 3 months, and C. $8000 for 2 months. How should the loss, which is $4320, be divided among the partners?

The use of $5000 for 4 months, is worth as much as the use of $20000 for one month. $6000 for 3 months = $18000 for 1 month. $8000 for 2 months = $16000 for 1 month. The gain or loss should therefore be divided by Simple Fellowship, in the same manner as if A. had contributed $20000, B. $18000, and C. $16000, making the whole stock $54000.

54000 : 20000 : : 4320 : 1600 *A.'s share.*
54000 : 18000 : : 4320 : 1440 *B.'s share.*
54000 : 16000 : : 4320 : 1280 *C.'s share.*

Hence, when time and money are both to be considered in Fellowship, *multiply each stock by the time for which it is employed, and proceed with the* PRODUCTS, *as with the* ORIGINAL STOCKS *in Simple Fellowship.* (§ **160**.)

163. EXAMPLES IN COMPOUND FELLOWSHIP.

Solve the following examples, both by Analysis and by Proportion.

221. A. contributed $200 for 4 months, and B. $500 for 2 months. How should $500 gain be divided between them?

222. Divide a debt of $276 among three partners; A. having invested in the firm $500 for 10 months, B. $600 for 8 months, and C. $800 for 5 months.

223. A. contributed $\frac{2}{9}$, B. $\frac{3}{9}$, and C. $\frac{4}{9}$ of the entire stock of a firm. A.'s money was in trade 5 months, B.'s 4 months, and C.'s 2 months, and they gained $234. How should the gain be divided?

224. A. invests a certain sum for a certain time, B. twice as much for $\frac{1}{3}$ of the time, and C. three times as much for $\frac{2}{3}$ of the time. How should they share the gain, which was $559?

225. A. commenced business with $800 capital, on May 1; he received B. as a partner, with a capital of $1000, on Aug. 1, and on Dec. 1, they admit C. into the firm, with a capital of $1500. At the end of a year from A.'s commencement, they had gained $1592.10; how should it be divided?

(Although time and money are not involved in the remaining examples of this section, the mode of solution is the same.)

226. Five men hire a pasture for $294. A. puts in 10 cows for 3 months; B. 5 cows for 2 months; C. 8 cows for 1½ months; D. 16 cows for 2¼ months; and E. 20 cows for 4 months. How much should each pay?

227. Three men hire a pasture for $40. A. puts in 4 horses for 8 months, and 6 for 2 months, B. puts in 5 for 4 months, and 10 for 1 month, and C. puts in 7 for 4 months and 3 for 2 months. How much should each pay?

228. Three carters were hired for $5.72. A. hauled 6 loads of 15cwt., B. 8 loads of 12cwt., and C. 10 loads of 10cwt. How much should each receive?

229. A. has in a granary 300 bushels of grain worth 60 cents a bushel, B. has 400 bushels worth 50 cents a bushel, and C. has 600 bushels worth 45 cents a bushel. The granary takes fire, and the grain is so damaged that the whole is sold for $162½. How much should be paid to each of the owners?

230. A. pastures 10 cows for 15 weeks, B. 9 cows for 20 weeks, C. 11 cows for 10 weeks, and D. 15 cows for 14 weeks. How much of the whole cost of pasturage, which is $40⅜, must each pay?

XII. AVERAGE.

164. THE AVERAGE of a series of quantities in any arithmetical operation, is a number which may be substituted for each of the given series, without altering the result of the operation.

1.50	1.50
1.25	1.50
1.37½	1.50
1.87½	1.50
6.00	6.00

Thus the average cost of four barrels of apples, which cost respectively $1.50, $1.25, $1.37½, and $1.87½, is $1.50. For if each of the four barrels cost $1.50, the entire cost would be the same as if one of the barrels cost $1.50, one $1.25, one $1.37½, and one $1.87½. Since the average must be repeated as many times as are equivalent to the number of quantities whose average is required, we have the following rule for determining average.

Divide the SUM *of the quantities whose average is required, by the* NUMBER *of quantities.*

165. EXAMPLES FOR THE PUPIL.

1—10. Find the average of 1, 2, and 3; 3, 5, and 7; 1, 4, 6, and 9; 2, 7, 3, and 4; 1, 5, 11, and 3; 8, 2, 7, and 7; 3, 5, 9, and 15; 1, 2, 3, 4, and 5; 7, 2, 6, 9, and 1; 13, 3, and 29.

11—20. What is the average of 13 and 17; 99 and 25; 64 and 82; 3, 19, and 29; 17, 54, and 49; 1, 14, and 27; 13, 15, and 32; 5, 9, 14, and 12; 3, 7, 11, and 15; 2, 8, 12, 17, and 11?

21—30. What is the average of 16 and 17; 28 and 31; 43 and 12; 95 and 84; 22, 33, and 43; 16, 28, and 32; 42, 27, and 14; 4, 8, 1, and 2; 31, 16, 25, and 19; 2, 18, 81, and 5?

31—40. What is the average of 1¾ and 2⅞; 13½ and 9⅜; 7½ and 5¾; 2.25 and 6½; 4.875 and 19¾; 17.6, 2.87, and 5.12; 4.09, 91⅜, and 11⅗; 1½, 1⅓, 1¼, and 1⅕; 372, 183, and 241.5; 62½, 31¼, and 75?

41. What is the average length of 5 pieces of sheeting, measuring respectively 30yd., 33yd., 32yd., 31¼yd., and 32½yd.?

42. What is the average weight of 4 bundles of hay, No. 1 weighing 156lb.; No. 2, 225lb.; No. 3, 175lb.; and No. 4, 139lb.?

43. What are the average contents of three barns, the first of which contains 40 tons of hay, and 30

bushels of grain, the second 63 tons of hay, and 25 bushels of grain, and the third 80 tons of hay, and 50 bushels of grain?

44. What is the average cost of four equal lots of wheat,—the first lot being worth $1.10 a bushel, the second $1.15 a bushel, the third $1.25 a bushel, and the fourth $1.20 a bushel?

45. What is the average age of 5 boys, supposing the age of the first to be 10 years; of the second, 15 years; of the third, 12 years; of the fourth, 9 years; and of the fifth, 17 years?

166. ALLIGATION is a term sometimes applied to the process by which we find the average of a given series of quantities, or the quantities which will yield a given average.

Alligation Medial is the process by which we find the average or *mean* value of a number of given quantities.

A grocer mixes 10 pounds of tea worth 50 cents a pound, 12 pounds at 75 cents, and 16 pounds at 62½ cents. What is the value of the mixture per pound?

10 pounds at 50 cents, cost $5.00
12 pounds at 75 cents, cost $9.00
16 pounds at 62½ cents, cost $10.00

38 pounds cost $24.00, therefore
1 pound would cost $\$24 \div 38 = \$\,.63\frac{3}{19}$.

The same result may be obtained by the rule for simple average, (§ 164,) as follows:

We wish to obtain the average of 50, ten times repeated,—75, twelve times repeated,—and 62½, 16 times repeated. The SUM of the quantities whose average is required, is 2400, and the NUMBER of quantities is 38; then the average is $2400 \div 38 = 63\frac{3}{19}$.

If we adopt the first of these methods, the course to be pursued in Alligation Medial may be stated as follows:

Find the value of the given quantity of each ingredient, add all the values, and divide their sum by the sum of the quantities.

167. Examples for the Pupil.

Solve each of the following examples, both by Alligation Medial, and by the general rule for Average.

46. A grocer mixed 18 gallons of molasses at 25 cents a gallon, with 20 gallons at 30 cents, 24 gallons at 40 cents, and 5 gallons of water. What was the mixture worth per gallon?

47. A goldsmith mixed 3lb. of gold 22 carats fine, 5lb. 20 carats fine, and 8lb. 24 carats fine. What was the fineness of the mixture? (A *carat* is $\frac{1}{24}$ of an ounce. By gold 22 carats fine, we mean that $\frac{22}{24}$ of the whole weight is pure gold.)

48. If 1lb. of silver that is 8oz. fine, 2lb. that is 9oz. fine, 3lb. that is 10oz. fine, and 4lb. that is 11oz. fine, be melted together, what will be the fineness of the mixture? (11oz. fine, denotes that 11oz. in the lb. are pure silver, and the rest alloy.)

49. If the thermometer stands for 3 hours at 68°, 2 hours at 70°, 3 hours at 72°, 1 hour at 75°, and 3 hours at 74°, what is the mean temperature of the 12 hours?

50. If I mix 10lb. of sugar at 7 cents, 9lb. at 5 cents, 8lb. at 6 cents, and 26lb. at 8 cents, what will be the value of the whole per pound?

168. Equation of Payments is the process by which we find the *average time* at which several debts may be paid, without loss to either the debtor or creditor. It differs from Alligation Medial, only by the substitution of the *debts* and *times*, for the *quantities* and *values*.

A man owes $40 payable in 3 months, $65 payable in 6 months, and $25 payable in a year. At what time may he pay the whole, so that neither party shall lose any interest?

The int. of $40 for 3mo. = the int. of $120 for 1mo.
The int. of $65 for 6mo. = the int. of $390 for 1mo.
The int. of $25 for 12mo. = the int. of $300 for 1mo.

The int. of $130 for —— = the int. of $810 for 1mo.

The interest on all the debts, is therefore equivalen to the interest of $810 for 1mo., and we wish to find in what time $130 will yield the same amount of interest. This may be done either by analysis, or by proportion.

By analysis; $130 may be kept $\frac{810}{130}$ as long as $810. $\frac{810}{130}$mo. = $6\frac{3}{13}$mo.

By proportion; 130 : 810 : : 1 : $6\frac{3}{13}$. Hence, the following

RULE FOR EQUATION OF PAYMENTS.

Multiply each debt by the time in which it becomes due, add all the products, and divide their sum by the sum of the debts.

This rule is founded on the supposition that the interest of the money which is not paid until after it is due, is equal to the discount of that which is paid before it is due. This is not strictly correct, but the difference is so trifling that it is never regarded in ordinary business transactions.

169. EXAMPLES FOR THE PUPIL.*

51. Find the equated time for the payment of $500 due in 3 months, and $700 due in 5 months.

52. What is the equated time for the payment of $75 due in 30 days, $50 due in 60 days, and $500 due in 90 days?

53. When may three debts be cancelled at once, one of $100 being due in 8mo., one of $50 due in 1¼yr., and one of $200 due in 2yr.?

54. If I owe $40 payable now, $40 in 30 days, $50 in 60 days, and 30 dollars in 120 days, when may the whole be paid without loss of interest?

55. When will the following account become due? $150 due Feb. 16; $200 due Feb. 29; $130 due March 18; $175 due April 4.

In averaging accounts when the date of each entry

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[*] Any fraction not greater than ½ of a day, is disregarded; if greater than ½ a day, it is counted as a whole day.

is given, it is customary to estimate the times from the first mentioned date. The present example is averaged as if there were $150 due in 0 days, $200 due in 13 days, $130 due in 31 days, and $175 due in 48 days. The result obtained by equation, will be the number of days *after* Feb. 16.

55. Average the following account. $200 due March 4; $150 due Apl. 3; $200 due Apl. 30, and $300 due May 15.

56. The following account was settled Oct. 9. $300 due Jan. 7, $400 due Jan. 18, $200 due Jan. 30, $500 due Feb. 11. How much interest was due upon the whole?

(Interest should of course be estimated from the time the account becomes due.)

57. How much interest was due Dec. 11, upon the following charges? $100 due Mar. 19, $100 due Mar. 24, $500 due Mar. 25, and $300 due Apl. 10.

58. Find the equated time for $6.25 due Oct. 10, $19 due Oct. 19, $11.50 due Oct. 31, $25 due Nov. 2.

59. Average the following account. $92.75 due June 5, $8.69 due June 24, $49.27 due Aug. 5.

60. When may I settle by a single payment, for $300 due now, $200 due in 10 days, $100 due in 15 days, $100 due in 29 days, and $250 due in 45 days?

61. Find the equated time for $500 due May 1, $300 due June 1, and $400 due July 1.

62—68. Average the above account, supposing the amounts due Jan. 1, Jan. 15, and March 1; Jan. 3, Jan. 27, and Feb. 5; Jan. 19, Feb. 11, and Apl. 3; May 9, May 18, and June 10; Aug. 11, Sept. 5, and Oct. 3; Feb. 13, Apl. 10, and June 4; May 5, May 7, and May 25.

170. ALLIGATION ALTERNATE is the process by which we determine the proportional quantities of several ingredients, which must be employed to form a mixture of a given value.

In what proportion should I mix sugars at 5cts., 7cts., 8cts., 10cts., and 12cts., in order that the mixture may be worth 9 cents a pound?

3lb. at 5ct. = .15
1lb. at 7ct. = .07
2lb. at 8ct. = .16
4lb. at 10ct. = .40
5lb. at 12ct. = .60

15lb. " cost " 1.38
15lb. at 9ct. = 1.35

Excess .03

We may commence by taking any quantity we please, of each ingredient, and finding the cost of the whole. If, for example, we take 3lb. at 5ct., 1lb. at 7ct., 2lb. at 8ct., 4lb. at 10ct., and 5lb. at 12ct.,(making 15lb in all,) the whole cost will be $1.38. But 15lb. at 9ct. would cost only $1.35, therefore our estimated quantities give an excess of 3 cents above the required cost. To balance this excess, we must either add some of the sugar that costs less than the average price proposed, or take out some of that which costs more than the average. For every pound that we add at 5 cents, there will be a deficiency of 4 cents from the mean rate; for every pound at 7ct., a deficiency of 2ct.; for every pound at 8ct., a deficiency of 1ct. Then we may either add ¾ of a lb. more at 5ct., or 1½lb. more at 7ct., or 3lb. more at 8ct., or 1lb. more at 8ct. and 1lb. at 7ct., or any other quantity that will make a deficiency of 3ct.

If the quantities first assumed had given a *deficiency* instead of an *excess*, we should have been obliged to take some additional quantity of one or more of the ingredients whose value is *greater* than the proposed average. or to take out some of the ingredients whose value is less than the average. The cost of the quantities assumed in the margin, would be only 91ct., but 12lb. at 9ct. would cost $1.08. There is therefore a deficiency of 17ct., which may be balanced by taking enough of the sugar that costs more than 9cts., to make an excess of 17ct. Thus 5lb. additional at 12ct. and 2lb. at 10ct. would answer the required conditions. The pupil may determine other values for himself.

4lb. at 5ct. = .20
3lb. at 7ct. = .21
2lb. at 8ct. = .16
1lb. at 10ct. = .10
2lb. at 12ct. = .24

12lb. " cost " .91
12lb. at 9ct. = 1.08

Deficiency .17

The following are therefore some of the answers to the proposed question. (1.) 3¾lb. at 5ct., 1lb. at 7ct., 2lb. at 8ct., 4lb. at 10ct., and 5lb. at 12ct. (2.) 3lb. at 5ct., 2½lb. at 7ct., 2lb. at 8ct., 4lb. at 10ct., and 5lb. at 12ct. (3.) 3lb. at 5ct., 1lb. at 7ct., 5lb. at 8ct., 4lb. at 10ct., and 5lb. at 12ct. (4.) 3lb. at 5ct., 2lb. at 7ct., 3lb. at 8ct., 4lb. at 10ct., and 5lb. at 12ct. (5.) 4lb. at 5ct., 3lb. at 7ct., 2lb. at 8ct., 3lb. at 10ct., 7lb. at 12ct.

Let the pupil perform the above examples by *taking out* some of the quantities originally assumed.

RULE FOR ALLIGATION ALTERNATE.

Assume any quantity you please of each ingredient, find the cost of the whole, and also the cost of the same quantity at the mean rate proposed. If the assumed quantities cost TOO MUCH, *take such additional quantities of the lower priced, or such diminished quantities of the higher priced ingredients, as will exactly counterbalance the excess. If* TOO LITTLE, *take such additional quantities of the higher priced, or such diminished quantities of the lower priced ingredients, as will exactly counterbalance the deficiency.*

If the quantity of either ingredient is limited, we must of course assume additional quantities of the ingredients that are unlimited.

171. Examples for the Pupil.

69—75. In what proportion must I take sugars at 5ct. and 8ct. a pound, to form a mixture worth 7ct. a pound? 6ct. a pound? 5½ct.? 6½ct.? 7½ct.? 6¾ct.? 7¼ct.?

76—80. In what proportion must I mix oils at 70ct. and $1 per gallon, so that the mixture may be worth 80ct. a gallon? 90ct. a gallon? 75ct.? 95ct.? 85ct.?

81—85. Mix oats at 40 cents a bushel with corn at 70 cents, so that the whole may be worth 50 cents a bushel; 45ct. a bushel; 60ct.; 65ct.; 55ct.

86—90. A grocer wishes to mix two kinds of tea, worth 30ct. and 75ct. a pound. How must he do it to make the mixture worth 50ct. a pound? 62½ct. a lb.? 37½ct.? 45ct.? 40ct.?

91—95. How many ounces of gold, 12, 18, and 20 carats fine, may be melted together to form a mixture

16 carats fine? 15 carats fine? 17 carats? 19 carats? 14 carats?

96—100. What quantities of vinegar at 20ct. a gallon, cider at 25ct. a gallon, and water, may be taken to form a mixture worth 22ct. a gallon? 18ct. a gallon? 16ct.? 21ct.? 12½ct.?

101—105. How much water must be mixed with 50 gallons of vinegar, at 28ct. a gallon, to make it worth but 25ct. a gallon? 20ct. a gal.? 22ct.? 18¾ct.? 16ct.?

50gal.	at 28ct.	=	$14.00
1gal.	at 0ct.	=	0.00
51gal.		cost	$14.00
51gal.	at 25ct.	=	$12.75
		Excess	1.25

On every gallon of water added, there is a gain of 25ct. To gain $1.25, we must therefore add 5 gallons more of water, making 6 gallons in all.

106—110. How much water must be mixed with 40 gallons of syrup at 50ct. a gallon, to make the whole worth 37½ct. a gallon? 25ct. a gal.? 12½ct.? 33⅓ct.? 30ct.?

111—115. A grocer has 10 gallons of vinegar at 37½ct. a gallon, 12gal. at 25ct., and 8gal. at 30ct., with which he wishes to mix cider at 32ct. and water. How much of each must he take, to make the whole worth 20ct. a gallon? 28ct.? 19ct.? 15ct.? 23ct.?

116—120. What quantity of raisins, at 10ct., 18ct., and 20ct. per lb., may be mixed together, to fill a cask of 200lb. and to be worth 19ct. a lb.? 14ct. a lb.? 17ct.? 16ct.? 12ct.? (After obtaining the *proportions* by Alligation, find the exact quantities by Fellowship.)

121—125. A grocer has an order for 150lb. of tea at 90ct., but having none at that price, he would mix some at 75ct., some at 87½ct., and some at $1.00 per pound. How much of each sort must he take? How much to make the whole worth 85ct. a lb? 95ct.? 80ct.? 78ct.?

126—130. How much syrup at 25ct. must be mixed with 30gal. at 40ct., that the whole may be worth 30ct. a gal.? 28ct.? 35ct.? 37½ct.? 32ct.?

XIII. INVOLUTION AND EVOLUTION.

172. Involution is the repeated multiplication of a number by itself.

The product obtained by Involution is called a *power*. The *root* is the number involved, or the *first power*. If the root be multiplied by itself, or employed twice as a factor, the product is the second power. If the root is employed three times as a factor, it is raised to the 3d power; if 5 times, to the 5th power, &c. Thus, 2 is the 1st power of 2, or 2^1. 2×2 or 4, is the 2nd power of 2, or 2^2. $2 \times 2 \times 2$ or 8, is the 3d power of 2, or 2^3. $2 \times 2 \times 2 \times 2 \times 2$ or 32, is the 5th power of 2, or 2^5. The power is usually denoted by a small figure over the right of the root, called the *exponent*, or *index*. When there is no exponent, the number is regarded as the 1st power.

The second power is often called the square, because the number of square feet in any square surface, is obtained by multiplying the number of feet in one side by itself.

The third power is often called the cube, because the number of cubic feet in any cubical block, may be obtained by raising the number of feet in one side to the 3d power.

173. Examples for the Pupil.

1—10. Find the value of each of the following expressions; 16^2; 4^3; 2^4; 1^9; $.5^3$; 1.25^2; $.08^4$; 2.7^3; 19^4; 1.9^4.

11—50. Find the second, third, fourth, and fifth powers of each of the following numbers; 2; 3; $\frac{2}{3}$; $1\frac{1}{2}$; 2.5; 40; $3\frac{1}{3}$; $2\frac{3}{4}$; $\frac{3}{10}$; .09.

174. Evolution is the process by which we discover the root of any given power.

The *root* of any number is a factor, which if repeatedly multiplied by itself will produce the number. Thus, 2 is the 2d root of 4, or the 3d root of 8, or the 5th root of 32, because $2^2 = 4$, $2^3 = 8$, $2^5 = 32$. So the 2d or square root of 49 is 7; the 3d or cube root of 125 is 5; the 4th root of 16 is 2; the 5th root of 1024 is 4.

The radical sign, ($\surd$ or $\sqrt{}$,) when placed before a number, indicates the square root. If we wish to denote any other root, we write the index of the root over the radical sign, as $\sqrt[3]{}$, $\sqrt[7]{}$, $\sqrt[12]{}$.

175. EXAMPLES FOR THE PUPIL.

51—60. Read the following expressions; $\sqrt{5}$; $\sqrt[5]{3}$; $\sqrt[3]{9.7}$; $\sqrt{.08}$; $\sqrt[4]{17}$; $\sqrt[3]{1}$; $\sqrt[11]{2.44}$; $\sqrt{169}$; $\sqrt[7]{9000}$; $\sqrt{6}$.

61—70. Extract the roots indicated in the following examples. $\surd 4$; $\surd 9$; $\surd 81$; $\surd 16$; $\surd 49$; $\surd 25$; $\surd 36$; $\surd 1$; $\surd 64$; $\surd 100$.

71—80. What is the value of $\sqrt[3]{8}$? $\sqrt[3]{125}$? $\sqrt[3]{27}$? $\sqrt[3]{343}$? $\sqrt[3]{1000}$? $\sqrt[3]{64}$? $\sqrt[3]{512}$? $\sqrt[3]{216}$? $\sqrt[3]{729}$? $\sqrt[3]{1}$?

176. EXTRACTION OF THE SQUARE ROOT.

The square root of a number is *the number, which, if multiplied by itself, will produce the given number.*

In the following Table are the numbers from 1 to 10 inclusive, and beneath them are their squares; therefore the numbers in the lower line have for their square roots the corresponding numbers of the upper line.

Roots,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
Squares,	1,	4,	9,	16,	25,	36,	49,	64,	81,	100.

From the ordinary process of multiplication, it may be seen that the square of any number will have twice as many figures as the root, or one less than twice as many.*

If there are decimals in the root, there will be twice as many in the square: because any product contains as many decimals as both factors. And conversely, there will be half as many decimals in the root as in the square.

---

* Let the pupil perform a few examples, and discover the principle on which this fact depends; that the square of units gives units and tens; the square of tens gives hundreds and thousands, &c.

Therefore, if we divide a square number into periods of two figures, by placing a point over units, and one over every second figure to the left, the number of periods will denote the number of integral figures in the root. Thus the square root of $\dot{1}4\dot{4}$ contains two figures, and is 12. The square root of $1\dot{6}0\dot{0}$ contains 2 figures, and is 40.

177. To discover a rule for finding the square root, we will multiply 28 by itself, writing each product separately, instead of carrying the tens as in ordinary multiplication. Commencing with the units' figure, we say 8 × 8 units = 64 units; 8 × 2 tens = 16 tens; then taking the tens' figure, we say 2 tens × 8 = 16 tens; 2 tens × 2 tens = 4 hundreds. Taking care to write each product in its proper place, and adding the several products together, we find that the square of 28 is 784. But the four partial products which compose the entire square, are (commencing with the largest product,) (1) *the square of the tens;* (2) & (3) *twice the product of the tens by the units;* (4) *the square of the units.*

$$\begin{array}{rl}
28 & \\
28 & \\
\hline
64 & = 8 \times 8\ (4) \\
16 & = 2 \times 8\ (3) \\
16 & = 2 \times 8\ (2) \\
4 & = 2 \times 2\ (1) \\
\hline
784 &
\end{array}$$

178. Let us now reverse the operation, and extract the square root of 784. Pointing the periods, (§ **176**,) we find that the root will consist of two figures, and the square of the tens must therefore be contained in the 7 hundreds. The greatest square contained in 7, (Table, § **176**,) is 4, the root of which, 2, is written as the *tens'* figure of the required root.

$$\begin{array}{r}
\dot{7}8\dot{4}(28 \\
4 \\
\hline
48)384 \\
384 \\
\hline
\end{array}$$

Now, by the preceding paragraph, if we subtract from the entire square (1) *the square of the tens*, the remainder 384 will contain (2) & (3) *twice the product of the tens by the units*, and (4) *the square of the units.*

We do not yet know the units' figure, but the 38 tens of 384 must contain the product of twice the tens by the units, added to the tens which are contained in the square of the units. Therefore, to obtain the units' figure, (§ **47**,) we divide the 38 tens by 2 × 2 = 4 tens, which gives a quotient 9. But if we call the units' figure 9, we find that it gives too large a square. We therefore try 8, which we place in the root, and also at the right hand of the trial divisor 4. 48 × 8 = 384; but 8 × 4 tens = *twice the product of the tens by the units*, and 8 × 8 = *the square of the units.* 8 is therefore the true units' figure. Hence we obtain the following

RULE FOR THE EXTRACTION OF THE SQUARE ROOT.

Separate the number into periods of two figures each, by placing a point over the units' figure, and another over every second figure to the left, (and also to the right, if decimals are desired in the root.)

Write as a quotient figure the ROOT *of the greatest square contained in the left hand period, subtract the square from the period, and to the remainder annex the second period for a dividend.*

Double the root already found, for a trial divisor. Find how many times the trial divisor is contained in the TENS *of the dividend, placing the result in the root, and also at the right hand of the trial divisor. Multiply the completed divisor by the new root figure, subtract the product from the dividend, and annex the next period to the remainder, for a new dividend.*

Double the root already found for a new trial divisor, and proceed as before. Repeat this operation for each succeeding root figure, until all the periods are brought down.

When any trial divisor is not contained in the tens of the dividend, place a naught in the root, and also at the right of the divisor, and bring down the next period.

If any figure obtained for the root proves too large, substitute the next lower figure and repeat the work.

Approximate roots may be obtained, by annexing decimal periods of two figures each.

179. Examples for the Pupil.

81—100. Extract the square root of 121; 225; 400; 361; 144; 289; 196; 256; 169; 324; 625; 900; 441; 729; 484; 841; 529; 576; 676; 784.

101—120. Find the square root of 9.61; .1024; 13.69; 1600; 2500; 10.89; .1521; 1296; 11.56; 1444; $20\frac{1}{4}$; 1681; 24.01; .1849; 22.00; 1764; 1936; 21.16; 23.04; $12\frac{1}{4}$.

121—140. Find the square root of $\frac{4}{9}$; $\frac{16}{25}$; $\frac{1}{4}$; $\frac{9}{49}$; $\frac{64}{9}$; $7\frac{1}{9}$; $6\frac{1}{4}$; $\frac{81}{144}$; $13\frac{4}{9}$; $10\frac{9}{16}$; $5\frac{19}{25}$; $10\frac{1}{36}$; $2\frac{46}{49}$; $4\frac{33}{64}$; $69\frac{4}{9}$; $240\frac{1}{4}$; $32\frac{32}{49}$; $\frac{1}{289}$; $\frac{81}{2304}$; $165\frac{15}{49}$.

141—160. Find the *approximate* square root, to two decimal places, of 2; 7; 13; 29; 5.8; 62.9; 140; $7\frac{2}{3}$; $99\frac{1}{2}$; 600; 24; 1.062; 480.9; 37.7; 377; .377; 11; 44; 176; 30.

161—170. What is the side of a square field, that contains 144 square rods? 169sq. r.? 81sq. r.? 1681sq. r.? 23.04sq. r.? 196sq. r.? $12\frac{1}{4}$sq. r.? $69\frac{4}{9}$sq. r.? $240\frac{1}{4}$ sq. r.? 1444sq. r.?

171—180. Find the square root of 2.2201; 6; .893025; 72853; 2570.49; 11380; 998001; 99; 1512900; 65157184.

181—200. Find the approximate square root, to three decimal places, of .3; $\frac{1}{5}$; $\frac{4}{5}$; 1.2; 1.8; 2.7; $\frac{1}{8}$; $\frac{1}{2}$; 1.125; $\frac{1}{32}$; $\frac{9}{32}$; $\frac{1}{72}$; $\frac{1}{18}$; $\frac{2}{9}$; $\frac{7}{8}$; $\frac{25}{72}$; $\frac{8}{9}$; $1\frac{7}{18}$; $\frac{25}{32}$; $\frac{49}{72}$.

180. Extraction of the Cube Root.

The cube root of a number, is *the number which, if raised to the third power, will produce the given number.*

In the following table are the numbers from 1 to 10 inclusive, and beneath them are their cubes; therefore the numbers of the lower line have for their cube roots the corresponding numbers of the upper line.

Roots,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
Cubes,	1,	8,	27,	64,	125,	216,	343,	512,	729,	1000.

All perfect cubes from 1 to 1000, evidently have but one integral figure in their cube root. All numbers between 1000 or 10^3, and 1000000 or 100^3, will have two figures in their root. And generally, if we divide a cube into periods of three figures each, by placing a point over units, and one over every third figure from units, the number of points will show the number of figures in the root.

181. To discover a rule for finding the cube root, we will raise 28 to the third power, writing each product separately, as in § **177**.

$$\begin{array}{rl}
28 & \\
28 & \\
\hline
64 & = 8 \times 8 \\
16 & = 2 \times 8 \\
16 & = 2 \times 8 \\
4 & = 2 \times 2 \\
28 & \\
\hline
\dot{5}12 & = 8 \times 8 \times 8 \ (8) \\
128 & = 2 \times 8 \times 8 \ (7) \\
128 & = 2 \times 8 \times 8 \ (6) \\
32 & = 2 \times 2 \times 8 \ (5) \\
128 & = 2 \times 8 \times 8 \ (4) \\
32 & = 2 \times 2 \times 8 \ (3) \\
32 & = 2 \times 2 \times 8 \ (2) \\
8 & = 2 \times 2 \times 2 \ (1) \\
\hline
21952 &
\end{array}$$

After obtaining the several partial products which constitute the square, we multiply them separately by 8 and by 2, writing each product so that it will occupy the proper place, as follows: 8 × 64 = 512; 8 × 16 tens = 128 tens; 8 × 16 tens = 128 tens; 8 × 4 hundreds = 32 hundreds; 2 tens × 64 = 128 tens; 2 tens × 16 tens = 32 hundreds; 2 tens × 16 tens = 32 hundreds; 2 tens × 4 hundreds = 8 thousands. Therefore, the eight partial products which compose the entire cube, are (1) *the cube of the tens;* (2) (3) & (5) 3 *times the square of the tens multiplied by the units;* (4) (6) & (7) 3 *times the pro-*

duct of the tens by the square of the units ; (8) *the cube of the units.*

Let us reverse the operation, and extract the cube root of 21952. Pointing the periods, (§ **180**,) we find that the root will consist of two figures, and the cube of the tens must therefore be contained in the 21 thousands. The greatest cube contained in 21, (Table, § **180**,) is 8, the cube root of which, 2, we write as the tens' figure of the required root. By the preceding paragraph it will be seen that if we subtract from the entire cube (1) *the cube of the tens*, the remainder 13952 will contain (2) (3) & (5) 3 *times the square of the tens multiplied by the units;* (4) (6) & (7) 3 *times the product of the tens by the square of the units*, and (8) *the cube of the units.*

```
         .   .
         21952(28
          8
        ------
12      )13952
 48
  64
----
1744     13952
         -----
```

The 139 hundreds of 13952 must then contain 3 times the square of the tens multiplied by the units, added to the hundreds which are contained in the other portions of the remainder. Therefore to obtain the units' figure, we divide the 139 hundreds by 3 times the square of the tens, (3 × 2 tens × 2 tens = 12 hundreds,) which gives a quotient of 11. But the units' figure cannot be greater than 9, and if we try 9, we shall find that it gives too large a cube. We therefore try 8, which we place in the root, and we complete our trial divisor as follows.

If the dividend 13952 were divided by the units, the quotient would be 3 times the square of the tens *plus* 3 times the tens multiplied by the units *plus* the square of the units. And conversely, if the dividend were divided by this quotient, the resulting quotient would be the units.

Our trial divisor already contains 3 times the square of the tens, (12 hundreds,) therefore to complete it we must add 3 times the tens multiplied by the units, (48 tens,) and the square of the units, (64 units.) Multiplying the completed divisor 1744 by the units' figure 8, we obtain exactly the remainder of the cube. Hence we may deduce the following

RULE FOR THE EXTRACTION OF THE CUBE ROOT.

Separate the number into periods of three figures, by placing a point over the units' figure, and one over every third figure to the left of units, (and also to the right, if decimals are desired in the root.)

Write as a quotient figure the ROOT *of the greatest cube contained in the left hand period, subtract the cube from the period, and to the remainder annex the second period for a dividend.*

Take three times the square of the root already found, for a trial divisor. Find how many times the trial divisor is contained in the HUNDREDS *of the dividend. Place the result in the root, and its square at the right of the trial divisor, (supplying the place of tens with a naught if the square is less than ten.)*

Multiply 30 *times** *the root already found, by the last root figure; add the product to the trial divisor, and you will obtain the completed divisor. Multiply the complete divisor by the last root figure, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.*

Take three times the square of the root already found, for a new trial divisor, and proceed as before. Repeat this operation for each succeeding root figure, until all the periods are brought down.

Whenever the trial divisor is not contained in the hundreds of the dividend, place a naught in the root, and two naughts at the right of the trial divisor, and bring down the next period for a new dividend.

If any figure obtained for the root proves too large, substitute the next lower figure and repeat the work.

Approximate roots may be obtained, by annexing decimal periods of three figures each.

---

* We say "30 times," because 3 × the tens × the units, would give tens, and a naught would be annexed for units.

182. ILLUSTRATION OF THE RULE.

What is the cube root of 226802438.843904?

$22\dot{6}80\dot{2}43\dot{8}.84\dot{3}90\dot{4}(609.84$

$6^3 = 216$

tr. div.

$3 \times 6^2 = 108,01$ 108,02

$30 \times 6 \times 1 = 1\,80$

10981 c. div.

which is not contained in the dividend.

tr. div.

$3 \times 60^2 = 10800,81$ 108024,38

$30 \times 60 \times 9 = 16200$

1096281 com. div. 9866529

tr. div.

$3 \times 609^2 = 1112643,64$ 9359098,43

$30 \times 609 \times 8 = 146160$

111410524 com.div. 891284192

tr. div.

$3 \times 6098^2 = 111556812,16$ 446256519,04

$30 \times 6098 \times 4 = 731760$

44625651904

11156412976 com.div.

The trial divisors, after the first, may be more conveniently found, *by adding to the last complete divisor the last number used to complete it, and twice the square of the last root figure.* Thus in the foregoing example, our third trial divisor =

$$\begin{array}{r} 1096281 \\ 16200 \\ 2 \times 9^2 = \quad 162 \\ \hline \end{array}$$

trial divisor, 1112643

183. EXAMPLES FOR THE PUPIL.

201—220. Extract the cube root of 1331; 8000; 27000; 1728; 6859; 64000; 2197; 5832; 125000; 2744; 4913; 9261; 4096; 10648; 3375; 12167; 24389; 13824; 29791; 21952.

221—240. Find the cube root of 15625; 35937; 50.653; 68921; 17576; 42875; 85.184: 97336; 19683; .110592; 74.088; 59319; 32768; 117.649; 46656; .091125; 54872; .103823; 79.507; .039304.

241—260. Find the cube root of $\frac{8}{27}$; $\frac{1}{64}$; $\frac{125}{216}$; $42\frac{7}{8}$; $18\frac{26}{27}$; $\frac{125}{729}$; $\frac{1}{1000}$; $20\frac{51}{64}$; $\frac{729}{8000}$; $13\frac{103}{125}$; $\frac{8}{27000}$; $1\frac{91}{125}$; $91\frac{1}{8}$; $166\frac{3}{8}$; $81\frac{10}{27}$; $21\frac{119}{125}$; $296\frac{8}{27}$; $1157\frac{5}{8}$; $857\frac{3}{8}$; $74\frac{11}{125}$.

261—280. Find the *approximate* cube root, to two decimal places, of 2; 7; 13; 29; 5.8; 62.9; 140; $7\frac{3}{8}$, $99\frac{1}{2}$; 600; 24; 1.062; 480.9; 37.7; 377; .377; 11; 88; 704; 30.

281—290. What is the side of a cubical box that contains 1728 cubic inches? 1331c. in.? 6859c. in.? $1157\frac{5}{8}$c. in.? 91.125c. in.? 21.952c. in.? 79507c. in.? 117649c. in.? $74\frac{11}{125}$c. in.? 132651c. in.?

291—300. Find the cube root of 2299.968; 13; 571787000000; 796.597983; 49; 65; .082; .73537; 1027.243729; 475.647962616; 953.

301—320. Find the approximate cube root, to three decimal places, of .864; .5; .256; .108; .032; .004; .001715; .001372; .005; .04; .135; .32; .625; $\frac{1}{8}$; $\frac{1}{2}$; 1.125; $3\frac{3}{8}$; 8; $15\frac{5}{8}$; 42875.

XIV. PROGRESSION, OR SERIES.

184. A SERIES, in Arithmetic, is a succession of numbers governed by some uniform law.

The numbers 1, 2, 3, 4, 5, 6, 7, &c., constitute the *natural series*, in which the difference between any two consecutive terms is 1.

An *arithmetical series*, is one in which the successive terms increase or decrease uniformly by the same number, as 2, 4, 6, 8, 10, 12; 20, 16, 12, 8, 4, 0.

A *geometrical series*, is one in which the successive terms increase or diminish uniformly in the same ratio, as 1, 2, 4, 8, 16, 32; 81, 27, 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$.

A *harmonical series*, is one that can be represented by a series of fractions, of which the numerators are all alike, and the denominators form an arithmetical series, as 15, 21, 35, 105, a series which is equivalent to $\frac{105}{7}$, $\frac{105}{5}$, $\frac{105}{3}$, $\frac{105}{1}$.

185. Arithmetical Progression.

An arithmetical series, (sometimes called an *equidifferent series,* because the difference between every two consecutive terms is the same,) is said to be in *Arithmetical Progression,*—or Progression by Difference.

The difference between the consecutive terms, is called the *common difference.* The first and last terms of the series are called the *extremes;* the other terms the *means.*

Thus, in the *ascending,* or *increasing* series,—

1, 4, 7, 10, 13, 16, 19;

the extremes are 1, and 19, and the common difference is 3.

In the *descending,* or *decreasing* series,—

50, 40, 30, 20, 10, 0;

the extremes are 50, and 0, and the common difference is 10.

Any three of the five following things being given, the other two may be found:

1. The first term.
2. The last term.
3. The number of terms.
4. The common difference.
5. The sum of all the terms.

PRELIMINARY EXERCISES.

1—10. Find the first 9 terms of an ascending series, the first term being 1, and the common difference 2; 1st t. 5, c. d. 3; 1st t. 2, c. d. 4; 1st t. 2, c. d. ½; 1st t. 4, c. d. 1½; 1st t. 8, c. d. ¾; 1st t. 10, c. d. 5; 1st t. 17, c. d. 10; 1st t. 4, c. d. 100; 1st t. 11, c. d. 5⅔.

11—30. If the first term of a decreasing series is 100, what are the 5 succeeding terms,—supposing the common difference to be 5? 10? 2? 6? 20? 15? 4? 11? ½? ⅔? 7½? 6¼? 9⅔? 8¾? 17? 12½? 3? 3½? 3⅞? 19?

31—50. The fifth term of an ascending series b

75, determine the four preceding, and the five succeeding terms, if the common difference is 5, 3, 10, 15, 11, 2, 8, 4, 14, 7½, 12½, ⅖, 3⅔, 16, 7, 6, 13, 6¼, 2¼, ¾.

186. CASE I. TO FIND ONE OF THE EXTREMES, AND THE SUM OF ALL THE TERMS.

If the smaller extreme is given, to find the second term, we must *add* the common difference *once;* for the 3d term, we must add it *twice;* for the 5th term, 4 times; for the 10th term, 9 times, &c. If the greater extreme is given, we must *subtract* the common difference in a similar manner.

If we take any two terms that are equally distant from the extremes, the less term is as much greater than the less extreme, as the greater term is less than the greater extreme. Therefore, *the average of the extremes is equal to the average of any two terms equally distant from the extremes, or to the middle term when the number of terms is odd.* In other words, the average of the extremes is the average of the whole series, and this average, multiplied by the number of terms, will give the sum of the terms. Therefore,

TO FIND EITHER EXTREME, *Multiply the common difference by the number of terms, less* 1, *and add the product to the less extreme,—or subtract it from the greater extreme.*

TO FIND THE SUM OF THE TERMS, *Multiply the number of terms by the sum of the extremes, and divide the product by* 2.

187. EXAMPLES FOR THE PUPIL.

For answers to § **187**, see § **193**.

51. The first term of an arithmetical series is 5, the common difference 2, and the number of terms 11. Find the last term, and the sum of all the terms.

52. What is the sum of 15 terms of the natural series, 1, 2, 3, 4, &c.?

53. Find the 9th term, and the sum of 9 terms, of the series 30, 27, 24, 21, &c.

54. How many times does a clock strike in 12 hours?

55. The first term of a descending series is 10, the common difference ½, and the number of terms 12. Required the last term, and the sum of the series.

56. If I invest $100 a year at 6 per cent. simple interest, to what will the whole amount in 20 years? (The less extreme is $100, the common difference $6, and the number of terms 20.)

188. CASE II. TO FIND THE COMMON DIFFERENCE, AND THE SUM OF THE TERMS.

The difference of the extremes, (§ **186**,) is equal to the common difference multiplied by the number of terms less 1. Then the difference of the extremes divided by the number of terms less 1, must give the common difference. Therefore,

TO FIND THE COMMON DIFFERENCE, *divide the difference of the extremes, by the number of terms less* 1.

THE SUM OF THE TERMS is found by Case I.

189. EXAMPLES FOR THE PUPIL.

For answers to § **189**, see § **187**, **193**.

57. The extremes of an equidifferent series are 5 and 25, and the number of terms 11. Find the common difference, and the sum of the terms.

58. The extremes are 1 and 15, and the number of terms 15. Find the common difference, and sum of the terms.

59. The extremes are 30 and 6, and the number of terms 9. What are the common difference, and the sum of the terms?

60. Required the sum of the terms and common difference, the extremes being 1 and 12, and the number of terms 12.

61. The extremes are 10 and 4½, and the number of terms 12. Find the sum of the terms and the common difference.

62. At what rate per cent. must I invest $100 per

annum, that the several investments may form an arithmetical series, of which the extremes are $100 and $214, and the number of terms 20?

190. CASE III. TO FIND THE NUMBER OF TERMS, AND THE SUM OF THE TERMS.

The difference of the extremes divided by the common difference, (§ **186, 188,**) gives the number of terms, less 1. Therefore,

To FIND THE NUMBER OF TERMS, *divide the difference of the extremes by the common difference, and add* 1 *to the quotient.*

THE SUM OF THE TERMS is found by Case I.

191. EXAMPLES FOR THE PUPIL.

For answers to § **191**, see § **195**.

63. The extremes are 5 and 25, and the common difference 2. Find the number of terms, and the sum of the terms.

64. The extremes 1 and 15, the common difference 1, what are the number of terms, and the sum of the series?

65. The extremes are 30 and 6, and the common difference 3. Required the number and the sum, of the terms.

66. Find the number of terms and the sum of the series, the extremes being 1 and 12, and the common difference 1.

67. Required the number and sum of the terms, the extremes being 10 and 4½, and the common difference ½.

68. The extremes of a series of investments at 6 per cent., simple interest, are $100 and $214. For how many years have the investments been made, and what is the amount of the whole?

192. CASE IV. TO FIND THE NUMBER OF TERMS, AND THE COMMON DIFFERENCE.

The number of terms and the sum of the extremes, (§ **186**,) are two factors whose product is twice the sum of the terms. Therefore,

To find the number of terms, *divide twice the sum of the terms by the sum of the extremes.*

The common difference is found by Case II.

193. Examples for the Pupil.

For answers to § **193**, see § **187**.

69. The extremes are 5 and 25, and the sum of the terms 165. Required the number of terms and the common difference.

70. The extremes are 1 and 15, and the sum of the terms 120. Find the number of terms and common difference.

71. The extremes are 30 and 6, and the sum of the terms 162. What are the number of terms and the common difference.

72. Find the number of terms and common difference, the extremes being 1 and 12, and the sum of the terms 78.

73. Required the number of terms and common difference, if the extremes are 10 and 4½, and the sum of the terms 87.

74. For how many years, and at what rate per cent. must $100 be invested annually, to amount at simple interest to $3140, the amount of the first year's investment being $214?

194. CASE V. TO FIND ONE OF THE EXTREMES AND THE COMMON DIFFERENCE.

The *sum* of the extremes may be found, (§ **186**, **192**,) by dividing twice the sum of the terms by the number of terms. Therefore,

To find the required extreme, *divide twice th* *sum of the terms by the number of terms, and subtract the given extreme from the quotient.*

THE COMMON DIFFERENCE is found by Case II.

195. EXAMPLES FOR THE PUPIL.

For answers to § **195**, see § **191**.

75. One extreme is 5, the sum of the terms 165, and the number of terms 11. Required the other extreme and the common difference.

76. One extreme is 15, the sum of the terms 120, and the number of terms 15. Find the other extreme and common difference.

77. The sum of the series is 162, the less extreme 6, and the number of terms 9. What are the greater extreme and the common difference?

78. Find the first term and common difference, the last term being 12, the number of terms 12, and the sum of the terms 78.

79. Required the last term and common difference, the sum of the terms being 87, the number of terms 12, and the first term 10.

80. What sum must be invested annually for 20 years, and at what rate of simple interest, that the first year's investment may amount to $214, and the amount of the whole may be $3140.

196. GEOMETRICAL PROGRESSION.

A geometrical series, (sometimes called a series of *continual proportionals*, because a proportion may be formed of any four consecutive terms,) is said to be in *Geometrical Progression*, or Progression by Quotient.

The quotient of any term by the term immediately preceding, is called the *ratio.*[*] The first and last terms are called the *extremes*, the other terms the *means*.

---

[*] The inverse ratio is used, because it is more convenient to *multiply* than to divide by the ratio.

Thus in the ascending series,—

1, 3, 9, 27, 81, 243,

the extremes are 1 and 243, and the ratio is 3.

In the descending series,—

24, 12, 6, 3, $\frac{3}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{3}{16}$,

the extremes are 24 and $\frac{3}{16}$, and the ratio is $\frac{1}{2}$.

From the nature of the series, it is evident that any four successive terms of a Geometrical Progression, constitute a proportion; as 1 : 3 :: 9 : 27, 3 : 9 :: 27 : 81, &c., in the first of the above series; 12 : 6 :: 3 : $\frac{3}{2}$, 6 : 3 :: $\frac{3}{2}$: $\frac{3}{4}$, &c., in the second series.

When any three of the five following things are given, the other two may be found, viz.:

1. The first term.
2. The last term.
3. The number of terms.
4. The ratio.
5. The sum of all the terms.

PRELIMINARY EXERCISES.

81—90. Find the first 9 terms of an increasing geometrical series, the first term being 1, and the ratio 2; 1st t. 2, r. 3; 1st t. 1, r. 4; 1st t. 3, r. 5; 1st t. 2, r. 6; 1st t. $\frac{1}{343}$, r. 7; 1st t. $\frac{3}{32}$, r. 2; 1st t. .0001, r. 10; 1st t. 2, r. $2\frac{1}{2}$.

91—110. If the first term of a decreasing series is 420, what are the 5 succeeding terms,—supposing the ratio to be $\frac{1}{2}$? $\frac{1}{3}$? $\frac{2}{3}$? $\frac{1}{4}$? $\frac{3}{4}$? $\frac{2}{5}$? $\frac{5}{6}$? $\frac{3}{7}$? $\frac{7}{8}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{3}{5}$? $\frac{4}{5}$? $\frac{2}{7}$? $\frac{4}{7}$? $\frac{5}{7}$? $\frac{6}{7}$? $\frac{3}{8}$? $\frac{1}{8}$?

111—130. The third term of an increasing series being 360, find the two preceding and the five succeeding terms, if the ratio is 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, $1\frac{1}{3}$, $2\frac{1}{4}$, $4\frac{1}{2}$, $3\frac{3}{4}$, $3\frac{3}{5}$, $1\frac{1}{4}$, $1\frac{3}{5}$, $1\frac{1}{6}$, $3\frac{1}{3}$, 20.

197. CASE I. TO FIND THE LAST TERM, THE FIRST TERM, RATIO, AND NUMBER OF TERMS BEING GIVEN.

To find the second term, we must multiply the first term by the ratio; for the third term, we must multiply by the ratio twice, or by the *2d power* of the ratio; for the 5th term, by the *4th power* of the ratio; for the

10th term, by the 9*th power* of the ratio, &c. Therefore,—

TO FIND THE LAST TERM, *Raise the ratio to a power, the index of which is equal to the number of terms less* 1, *and multiply the result by the first term.*

The process of raising the ratio to the required power, may sometimes be abbreviated. For example, $2^{13} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$, the factor 2 being 13 times repeated. But this product can be resolved into a number of larger factors,—as $2^3 \times 2^3 \times 2^3 \times 2^3 \times 2$; or $2^5 \times 2^5 \times 2^3$; or $2^4 \times 2^4 \times 2^5$; or into any other combination of the powers of 2, in which the sum of the several indices is 13. We may, then, if we choose, 1. *Involve the ratio to the fourth or fifth power, and write the index over each power.* 2. *Add together such of the indices as will make the index of the required power.* 3. *Multiply together the powers corresponding to those indices.*

EXAMPLES. To find 3^{11}. $\overset{1}{3}\ \overset{2}{9}\ \overset{3}{27}\ \overset{4}{81}$; $3^{11} = 3^4 \times 3^4 \times 3^3 = 81 \times 81 \times 27 = 177147$.

To find $(\frac{2}{3})^{17}$. $\overset{1}{\frac{2}{3}}\ \overset{2}{\frac{4}{9}}\ \overset{3}{\frac{8}{27}}\ \overset{4}{\frac{16}{81}}\ \overset{5}{\frac{32}{243}}$; $(\frac{2}{3})^{17} = (\frac{2}{3})^4 \times (\frac{2}{3})^5 \times (\frac{2}{3})^4 \times (\frac{2}{3})^4 = \frac{16}{81} \times \frac{32}{243} \times \frac{16}{81} \times \frac{16}{81} = \frac{131072}{129140163}$.

198. EXAMPLES FOR THE PUPIL.

131. The first term of a geometrical series is 5, the ratio 2, and the number of terms 7. What is the last term?

132. If the first pane of glass in a window cost 1 mill, the second 3 mills, the third 9 mills, and so on, what would be the price of the twelfth pane?

133. If a farmer plants a grain of wheat, and each year plants the product of the preceding harvest, how much will he harvest in the 11th year, the annual yield being 12 fold?

134. The first term is 9, the ratio $\frac{2}{3}$, and the number of terms 9. What is the last term?

135. A farmer inquiring the price of a drove of 20 oxen, was told that he might have the whole for the price of the 13th ox, valuing the first at 1 cent, the second at 3 cents, and trebling the price of each ox for the price of the next. What would be the price per head at that rate?

199. CASE II. TO FIND THE SUM OF ALL THE TERMS.

The first-term of a series is 162, the last term is 2, and the ratio $\frac{1}{3}$. What is the sum of the series?

The series is 162, 54, 18, 6, 2, the sum of which is 242. 54, 18, 6, 2, $\frac{2}{3}$, is another series, obtained by multiplying each term of the first series by the ratio. Subtracting the second series from the first, we have $161\frac{1}{3}$, which is the difference between the first term and the last term multiplied by the ratio. It is also $\frac{2}{3}$ of the sum of the first series, since it is obtained by subtracting $\frac{1}{3}$ of the series from the whole series. Then if we divide by $\frac{2}{3}$, or the difference between the ratio and 1, we shall obtain the desired sum. Therefore,—

TO FIND THE SUM OF THE TERMS, *Multiply the last term by the ratio, and divide the difference between the product and the first term, by the difference between the ratio and* 1.

200. EXAMPLES FOR THE PUPIL.

136. What is the sum of 9 terms of the series 2, 6, 18, 54, &c.? (The last term is found by Problem I.)

137. The first term 9, the ratio $\frac{1}{3}$, and the last term $\frac{1}{243}$, are given. What is the sum of the series?

138. What is the sum of the series 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, &c., to infinity? (The last term, in an infinite decreasing series, is 0.)

139. Required the sum of the infinite series 5, $\frac{5}{4}$, $\frac{5}{16}$, &c.

140. The first term is 8, the ratio $\frac{2}{3}$, and the number of terms 5: What is the sum of all the terms?

201. COMPOUND INTEREST BY SERIES.

What would be the amount of $25 in 7 years, at 6 per cent. compound interest?

$1 in 1 year would amount to $1.06, in 2 years to $\$1.06 \times 1.06 = \$1.06^2$; in 3 years to $\$1.06^3$; in 7 years to $\$1.06^7$. $25 would amount to 25 times as much as $1, or 25 × $1.06^7 = $37.59. Compound Interest may therefore be computed by Case I., (§ **197**,) the *first term* being the principal; the *ratio*, the amount of $1 for 1 year; and the *number of terms*, the number of years *plus* 1.

202. EXAMPLES FOR THE PUPIL.

141. Find the amount of $10 for 5 years, at 5 per cent. compound interest. (First term $10; ratio 1.05; number of terms 6.)

142. Find the amount of $3.25 for 4 years, at 7 per cent. compound interest.

143. At 6 per cent. compound interest, to what will $19.50 amount in 6 years?

144. Find the amount of $400 for 5 years, at 5 per cent. compound interest.

203. The labor of computing compound interest, may be abridged by a table in which the amount of $1 is computed at different rates, and for a number of years. (See Table I., page 216.)

I. To find the amount of any sum by the Table, *multiply the given sum by the amount of $1 for the given time.*

EXAMPLE. What will be the amount at 7 per cent. compound interest, of $200 for 15yr.?

$1 in 15yr. at 7 per cent. amounts to 2.759031, and 2.759031 × $200 = $551.8062.

II. To compute compound discount, or to find the present worth at compound interest, of any sum due at a future time, *divide the given sum by the amount of $1 for the given time.*

EXAMPLE. When money is worth 5 per cent. compound interest, what is the present worth of $5000 due in 19yr. 4mo. 24dy.?

$1 at 5 per cent. would amount in 19yr. 4mo. 24dy. to $2.577489, and $5000 ÷ 2.577489 = $1939.87.

III. To find the time in which any principal will amount to a given sum, *divide the amount by the principal, and look for the quotient in the Table, under the given rate.*

EXAMPLE. In what time at 6 per cent. compound interest, will $25 amount to $48?

$\frac{48}{25} = 1.92$. $1 would amount to 1.898299 in 11 years, or to 2.012196 in 12yr.

1.92 exceeds 1.898299 by .021701, and 2.012196 exceeds 1.898299 by .113897. Then if the gain in 12 months is .113897, in what time would there be a gain of .021701.

.113897 : .021701 : : 12mo. : 2mo. 8dy. *very nearly.*

IV. To find the rate at which any principal will amount to a given sum in a given time, *divide the amount by the principal, and look for the quotient in the Table, opposite the given time.*

EXAMPLE. At what rate of compound interest, will $250 amount to $550 in 18 years?

$\frac{550}{250} = 2.2$. In the line of 18 years, we find 2.2 under 4½ per cent.

204. EXAMPLES FOR THE PUPIL.

145—159. Find the amounts at 4½, 6, and 7 per cent. of $630 for 3 years; 7yr.; 13yr.; 5yr. 8mo.; 10yr. 6mo. 15dy. (After finding the amount for the years, we add the interest for the months and days, as in simple interest.)

160—174. Find the present worth, at 5, 6, and 7 per cent. compound interest, of $1000 due in 29 years; 50yr.; 38yr. 3mo.; 17yr.; 11yr. 10mo. 24dy.

175—189. In what times, at 3, 5, and 6 per cent. compound interest, will $150 amount to $300? $450? $230? $620? $180?

190—194. At what rate per cent. will $400 amount to $2297.40 in 30 years? in 35yr. 9mo. 27dy.? 44yr. 6mo. 23dy.? 25yr. 10mo.? 39yr. 8mo. 15dy.?

195—200. At what rate per cent. will $300 amount to $1315.17 in 50 years? in 21yr. 10mo. 2dy.? 33yr. 6mo. 10dy.? 25yr. 4mo. 9dy.? 37yr. 8mo. 4dy.? 30yr. 3mo. 13dy.?

205. ANNUITIES, AT COMPOUND INTEREST.

Any sum of money to be paid regularly, at stated periods, is called an ANNUITY. The payment may be stipulated for a given number of years, in which case it is called an *annuity certain*, or it may be dependent upon some particular circumstance, as the life of one or more individuals. The latter is called a *contingent annuity*. A *perpetual annuity*, is one which can only be terminated by the grantor, on the payment of a sum whose interest will be equivalent to the annuity. Of this character is the consolidated debt of England.

An *annuity in possession,* is one on which there is a present claim; an *annuity in reversion,* or deferred annuity, is one that does not commence until the lapse of a stated time, or the occurrence of some uncertain event, as the death of an individual.

The *present worth* of an annuity, is the sum which, at compound interest for the time of its duration, would amount to the sum of all the payments, each being placed at compound interest as it became due.

CASE I. TO FIND THE AMOUNT DUE ON AN ANNUITY WHICH HAS BEEN UNPAID FOR A GIVEN TIME.

On the last year's payment there will be no interest due; the last payment but one will be worth the amount of the annuity for 1 year; the last but two will be worth the amount of the annuity at compound interest for 2 years, and the sum of the several amounts will be the amount due on the annuity. We must, therefore,

Find the sum of a Geometrical Progression, in which the FIRST TERM *is the annuity, the* RATIO *is the amount of* $1 *for the time that should elapse from one payment to another, and the* NUMBER OF TERMS *is the number of payments due.*

BY THE TABLE OF AMOUNTS, on page 217, the amount may be found *by multiplying the annuity by the amount of an annuity of* $1 *for the given time.*

206. EXAMPLES FOR THE PUPIL.

201—210. If a person saves $100 per annum, and invests it at 6 per cent. compound interest, how much will he be worth in 30 years? in 10yr.? 29yr.? 15yr.? 40yr.? 50yr.? 38yr.? 13yr.? 27yr.? 45yr.?

211—220. What is the amount of an annual pension of $450 at 5 per cent. for 7yr.? 3yr.? 11yr.? 20yr.? 15yr.? 18yr.? 13yr.? 5yr.? 9yr.? 39yr.?

221—230. What is the amount of a salary of $875 for 16 years at 5½ per cent. compound interest? For

7yr.? 20yr.? 30yr.? 10yr.? 28yr.? 33yr.? 17yr.? 13yr.? 19yr.?

231—240. At 6 per cent. compound interest, what is the amount of an annuity of $500, forborn for 7 years? 11yr.? 50yr.? 30yr.? 29yr.? 27yr.? 35yr.? 31yr.? 23yr.? 43yr.?

207. CASE II. TO FIND THE PRESENT WORTH OF AN ANNUITY CERTAIN.

Find the AMOUNT *of the annuity, as in Case I., and find the* PRESENT WORTH *of that amount, as in* § **203**, II.

BY THE TABLE, page 218, the present worth may be found *by multiplying the annuity by the present worth of an annuity of* $1 *for the given time.*

208. EXAMPLES FOR THE PUPIL.

241—250. Find the present worth, allowing 5 per cent. compound interest, of an annual salary of $700 to continue 5 years; 8yr.; 11yr.; 20yr.; 7yr.; 10yr.; 3yr.; 15yr.; 6yr.; 9yr.

251—260. What is the present worth of $5 annuity at 5½ per cent. compound interest, for 7 years? 11yr.? 5yr.? 2yr.? 8yr.? 16yr.? 9yr.? 6yr.? 4yr.? 45yr.?

261—270. What sum invested at 6 per cent., would yield me an income of $1600 per annum, for 25 years? for 10yr.? 3yr.? 18yr.? 20yr.? 8yr.? 15yr.? 30yr.? 11yr.? 21yr.?

271—276. What sum must be invested at 4 per cent. to yield an annual income of $300 for 18 years? What sum at 3 per cent.? at 5 per cent.? 3½ per cent.? 4½ per cent.? 6 per cent.?

277—280. A man wishes to give most of his estate to his children. If money is worth 5½ per cent., how much must he reserve to secure an annuity of $700 for 15 years? How much for an annuity of $500? $300? $650?

TABLE 1.

SHOWING THE AMOUNT OF $1.00, AT COMPOUND INTEREST, FROM 1 YEAR TO 50.

Year.	3 p. cent.	3½ p. cent.	4 p. cent.	4½ p. cent.	5 p. cent.	6 p. cent.	7 p. cent.
1	1.030000	1.035000	1.040000	1.045000	1.050000	1.060000	1.070000
2	1.060900	1.071225	1.081600	1.092025	1.102500	1.123600	1.144900
3	1.092727	1.108718	1.124864	1.141166	1.157625	1.191016	1.225043
4	1.125509	1.147523	1.169859	1.192519	1.215506	1.262477	1.310796
5	1.159274	1.187686	1.216653	1.246182	1.276282	1.338226	1.402552
6	1.194052	1.229255	1.265319	1.302260	1.340096	1.418519	1.500730
7	1.229874	1.272279	1.315932	1.360862	1.407100	1.503630	1.605781
8	1.266770	1.316809	1.368569	1.422101	1.477455	1.593848	1.718186
9	1.304773	1.362897	1.423312	1.486095	1.551328	1.689479	1.838459
10	1.343916	1.410599	1.480244	1.552969	1.628895	1.790848	1.967151
11	1.384234	1.459970	1.539454	1.622853	1.710339	1.898299	2.104852
12	1.425761	1.511069	1.601032	1.695881	1.795856	2.012196	2.252192
13	1.468534	1.563956	1.665073	1.772196	1.885649	2.132928	2.409845
14	1.512590	1.618694	1.731676	1.851945	1.979932	2.260904	2.578534
15	1.557967	1.675349	1.800943	1.935282	2.078928	2.396558	2.759031
16	1.604706	1.733986	1.872981	2.022370	2.182875	2.540352	2.952164
17	1.652848	1.794675	1 947900	2.113377	2.292018	2.692773	3.158815
18	1.702433	1.857489	2.025816	2.208479	2.406619	2.854339	3.379931
19	1.753506	1.922501	2.106849	2.307860	2.526950	3.025599	3.616526
20	1.806111	1.989789	2.191123	2.411714	2.653298	3.207135	3.869683
21	1.860295	2.059431	2.278768	2.520241	2.785963	3.399564	4.140561
22	1.916103	2.131512	2.369919	2.633652	2.925261	3.603537	4.430400
23	1.973586	2.206114	2.464715	2.752166	3.071524	3.819750	4.740528
24	2.032794	2.283328	2.563304	2.876014	3.225100	4.048935	5.072365
25	2.093778	2.363245	2.665836	3.005434	3.386355	4.291871	5.427431
26	2.156591	2.445959	2.772470	3.140679	3.555673	4.549383	5.807351
27	2.221289	2.531567	2.883369	3.282009	3.733456	4.822346	6.213866
28	2.287928	2.620177	2.998703	3.429700	3.920129	5.111687	6.648836
29	2.356565	2.711878	3.118651	3.584036	4.116136	5.418388	7.114255
30	2.427262	2.806794	3.243397	3.745318	4.321942	5.743491	7.612253
31	2.500080	2.905031	3.373133	3.913857	4.538039	6.088101	8.145110
32	2.575083	3.006708	3.508059	4.089981	4.764941	6.453387	8.715268
33	2.652335	3.111942	3.648381	4.274030	5.003188	6.840590	9.325337
34	2.731905	3.220860	3.794316	4.466361	5.253348	7.251025	9.978110
35	2.813862	3.333590	3.940089	4.667348	5.516015	7.686087	10.676578
36	2.898278	3.450266	4.103932	4.877378	5.791816	8.147252	11.423939
37	2.985227	3.571025	4.268090	5.096860	6.081407	8.636087	12.223614
38	3.074783	3.696011	4.438813	5.326219	6.385477	9.154252	13.079277
39	3.167027	3.825372	4.616366	5.565899	6.704751	9.703507	13.994827
40	3.262038	3.959260	4.801021	5.816364	7.039989	10.285718	14.974465
41	3.359899	4.097834	4.993061	6.078101	7.391988	10.902861	16.022677
42	3.460696	4.241258	5.192784	6.351615	7.761587	11.557033	17.144265
43	3.564517	4.389702	5.400495	6.637438	8.149667	12.250455	18.344363
44	3.671452	4.543342	5.616515	6.936123	8.557150	12.985482	19.628469
45	3.781596	4.702358	5.841176	7.248248	8.985008	13.764611	21.002461
46	3.895044	4.866941	6.074823	7.574420	9.434258	14.590487	22.472634
47	4.011895	5.037284	6.317816	7.915268	9.905971	15.465917	24.045718
48	4.132252	5.213589	6.570528	8.271455	10.401270	16.393872	25.728918
49	4.256219	5.396065	6.833349	8.643671	10.921333	17.377504	27.529943
50	4.383906	5.584927	7.106683	9.032636	11.467400	18 420154	29.457039

TABLE II.

The amount of an annuity of $1.00 from 1 year to 50.

Year.	3 p. cent.	3½ p. cent.	4 p. cent.	4½ p. cent.	5 p. cent.	5½ p. cent.	6 p. cent.
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	2.030000	2.035000	2.040000	2.045000	2.050000	2.055000	2.060000
3	3.090900	3.106225	3.121600	3.137025	3.152500	3.168025	3.183600
4	4.183627	4.214943	4.246464	4.278191	4.310125	4.342266	4.374616
5	5.309136	5.362466	5.416322	5.470710	5.525631	5.581091	5.637093
6	6.468410	6.550152	6.632975	6.716892	6.801913	6.888051	6.975319
7	7.662462	7.779408	7.898294	8.019152	8.142008	8.266894	8.393838
8	8.892336	9.051687	9.214226	9.380014	9.549109	9.721573	9.897468
9	10.159106	10.368496	10.582795	10.802114	11.026564	11.256259	11.491316
10	11.463879	11.731393	12.006107	12.288210	12.577893	12.875354	13.180795
11	12.807796	13.141992	13.486351	13.841179	14.206787	14.583498	14.971643
12	14.192029	14.601962	15.025805	15.464032	15.917127	16.385590	16.869941
13	15.617790	16.113030	16.626838	17.159913	17.712983	18.286798	18.882138
14	17.086324	17.676986	18.291911	18.932109	19.598632	20.292572	21.015066
15	18.598914	19.295681	20.023588	20.784054	21.578564	22.408663	23.275971
16	20.156881	20.971030	21.824531	22.719337	23.657492	24.641139	25.672528
17	21.761588	22.705016	23.697512	24.741707	25.840366	26.996402	28.212880
18	23.414436	24.499691	25.645413	26.855084	28.132385	29.481905	30.905653
19	25.116868	26.357180	27.671229	29.063562	30.539004	32.102671	33.759992
20	26.870374	28.279682	29.778078	31.371423	33.065954	34.868318	36.785591
21	28.676486	30.269471	31.969202	33.783137	35.719252	37.786075	39.992727
22	30.536780	32.328902	34.247970	36.303378	38.505214	40.864309	43.392290
23	32.452884	34.460414	36.617888	38.937030	41.430475	44.111846	46.995828
24	34.426470	36.666528	39.082604	41.689196	44.501999	47.537998	50.815577
25	36.459264	38.949857	41.645908	44.565210	47.727099	51.152588	54.864512
26	38.553042	41.313102	44.311745	47.570645	51.113454	54.965979	59.156383
27	40.709634	43.759060	47.084214	50.711324	54.669126	58.989109	63.705766
28	42.930923	46.290627	49.967583	53.993333	58.402583	63.233510	68.528112
29	45.218850	48.910799	52.966286	57.423033	62.322712	67.711353	73.639798
30	47.575416	51.622677	56.084938	61.007070	66.438847	72.435478	79.058186
31	50.002678	54.429471	59.328335	64.752388	70.760790	77.419429	84.801677
32	52.502759	57.334502	62.701469	68.666245	75.298829	82.677498	90.889778
33	55.077841	60.341210	66.209527	72.756226	80.063770	88.224760	97.343165
34	57.730177	63.453152	69.857909	77.030256	85.066959	94.077122	104.183755
35	60.462082	66.674013	73.652225	81.496618	90.320307	100.251363	111.434780
36	63.275944	70.007603	77.598314	86.163966	95.836323	106.765188	119.120867
37	66.174223	73.457869	81.702246	91.041344	101.628139	113.637274	127.268119
38	69.159449	77.028895	85.970336	96.138205	107.709546	120.887324	135.904206
39	72.234233	80.724906	90.409150	101.464424	114.095023	128.536127	145.058458
40	75.401260	84.550278	95.025516	107.030323	120.799774	136.605614	154.761966
41	78.663296	88.509537	99.826536	112.846688	127.839763	145.118923	165.047684
42	82.023196	92.607371	104.819598	118.924789	135.231751	154.100464	175.950545
43	85.483892	96.848629	110.012382	125.276404	142.993339	163.575989	187.507577
44	89.048409	101.238331	115.412877	131.913842	151.143006	173.572669	199.758032
45	92.719861	105.781673	121.029392	138.849965	159.700156	184.119165	212.743514
46	96.501457	110.484031	126.870568	146.098214	168.685164	195.245720	226.508125
47	100.396501	115.350973	132.945390	153.672633	178.119422	206.984234	241.098612
48	104.408396	120.388257	139.263206	161.587902	188.025393	219.368367	256.564529
49	108.540648	125.601846	145.833734	169.859357	198.426663	232.433627	272.958401
50	112.796867	130.997910	152.667084	178.503028	209.347996	246.217477	290.335905

TABLE III.

THE PRESENT WORTH OF AN ANNUITY OF $1.00 FROM 1 YEAR TO 50.

Year.	3 p. cent.	3½ p. cent.	4 p. cent.	4½ p. cent.	5 p. cent.	5½ p. cent.	6 p. cent.	Year.
1	0.97087	0.96618	0.96154	0.95694	0.95238	0.94786	0.94339	1
2	1.91347	1.89969	1.88609	1.87267	1.85941	1.84631	1.83339	2
3	2.82861	2.80164	2.77509	2.74896	2.72325	2.69793	2.67301	3
4	3.71710	3.67308	3.62990	3.58753	3.54595	3.50514	3.46511	4
5	4.57971	4.51505	4.45182	4.38998	4.32948	4.27028	4.21236	5
6	5.41719	5.32855	5.24214	5.15787	5.07569	4.99553	4.91732	6
7	6.23028	6.11454	6.00205	5.89270	5.78637	5.68297	5.58238	7
8	7.01969	6.87396	6.73274	6.59589	6.46321	6.33457	6.20979	8
9	7.78611	7.60769	7.43533	7.26879	7.10782	6.95220	6.80169	9
10	8.53020	8.31661	8.11090	7.91272	7.72173	7.53762	7.36009	10
11	9.25262	9.00155	8.76048	8.52892	8.30641	8.09254	7.88687	11
12	9.95400	9.66333	9.38507	9.11858	8.86325	8.61852	8.38384	12
13	10.63495	10.30274	9.68565	9.68285	9.39357	9.11708	8.85268	13
14	11.29607	10.92052	10.56312	10.22283	9.89864	9.58965	9.29498	14
15	11.93794	11.51741	11.11839	10.73955	10.37966	10.03759	9.71225	15
16	12.56110	12.09412	11.65230	11.23401	10.83777	10.46216	10.10589	16
17	13.16612	12.65132	12.16567	11.70749	11.27407	10.86461	10.47726	17
18	13.75351	13.18968	12.65930	12.15999	11.68959	11.24607	10.82760	18
19	14.32380	13.70984	13.13394	12.59329	12.08532	11.60765	11.15812	19
20	14.87747	14.21240	13.59033	13.00794	12.46221	11.95034	11.46992	20
21	15.41502	14.69797	14.02916	13.40472	12.82115	12.27524	11.76408	21
22	15.93692	15.16712	14.45112	13.78442	13.16300	12.58317	12.04158	22
23	16.44361	15.62041	14.85684	14.14777	13.48857	12.87504	12.30338	23
24	16.93554	16.05837	15.24696	14.49548	13.79864	13.15170	12.55036	24
25	17.41315	16.48151	15.62208	14.82821	14.09394	13.41391	12.78336	25
26	17.87684	16.89035	15.98277	15.14661	14.37518	13.66250	13.00317	26
27	18.32703	17.28536	16.32959	15.45130	14.64303	13.89810	13.21053	27
28	18.76411	17.66702	16.66306	15.74287	14.89813	14.12142	13.40616	28
29	19.18845	18.03577	16.98371	16.02189	15.14107	14.33310	13.59072	29
30	19.60044	18.39205	17.29203	16.28889	15.37245	14.53375	13.76483	30
31	20.00043	18.73628	17.58849	16.54439	15.59281	14.72393	13.92909	31
32	20.38877	19.06887	17.87355	16.78889	15.80268	14.90420	14.08404	32
33	20.76579	19.39021	18.14765	17.02286	16.00255	15.07507	14.23023	33
34	21.13184	19.70068	18.41120	17.24676	16.19290	15.23703	14.36814	34
35	21.48722	20.00066	18.66461	17.46101	16.37419	15.39055	14.49825	35
36	21.83225	20.29049	18.90828	17.66604	16.54685	15.53607	14.62099	36
37	22.16724	20.57053	19.14258	17.86224	16.71129	15.67400	14.73678	37
38	22.49246	20.84109	19.36786	18.04999	16.86789	15.80474	14.84602	38
39	22.80822	21.10250	19.58448	18.22965	17.01704	15.92866	14.94907	39
40	23.11477	21.35507	19.79277	18.40158	17.15909	16.04612	15.04630	40
41	23.41240	21.59910	19.99305	18.56611	17.29437	16.15746	15.13802	41
42	23.70136	21.83488	20.18563	18.72355	17.42321	16.26299	15.22454	42
43	23.98190	22.06269	20.37079	18.87421	17.54591	16.36303	15.30617	43
44	24.25427	22.28279	20.54884	19.01838	17.66277	16.45785	15.38318	44
45	24.51871	22.49545	20.72004	19.15635	17.77407	16.54772	15.45583	45
46	24.77515	22.70092	20.88465	19.28837	17.88007	16.63283	15.52437	46
47	25.02471	22.89943	21.04294	19.41471	17.98102	16.71357	15.58903	47
48	25.26671	23.09124	21.19513	19.53561	18.07716	19.79011	15.65003	48
49	25.50166	23.27656	21.34147	19.65130	18.16872	16.86266	15.70757	49
50	25.72976	23.45562	21.48218	19.76201	18.25593	16.93143	15.76186	50

209. CASE III. TO FIND THE PRESENT WORTH OF A PERPETUAL ANNUITY.

The present worth of a perpetual annuity, is a sum which would yield an interest equivalent to the annuity. As the interest is found by multiplying the principal by the rate, the principal may be found (§ **47**,) by dividing the interest by the rate. Therefore,

Divide the annuity by the rate per cent.

210. EXAMPLES FOR THE PUPIL.

281—290. What sum of money at 4½ per cent. will yield an annual interest of $150? of $320? $50? $62.50? $93.75? $200? $1000? $30? $175? $125?

291—300. What sum, invested in an estate that rents for $450 per annum, will yield an annual interest of 8 per cent.? of .04? .07? .05½? .09? .12½? .06? .04¾? .04½? .10?

301—310. A farm rents for $750 per annum. For what price should it be sold when money is worth 6 per cent. a year? when it is worth 7, 8, 9, 5, 5½, 7½, 4½, 4, 6½ per cent.?

311—320. What is the par value in the 4 per cent. consols,* of an annual income of £500? £150? £225? £125? £50? £10? £63? £19? £31? £47?

211. CASE IV. TO FIND THE PRESENT WORTH OF AN ANNUITY IN REVERSION.

Find the value of the reversion at the time of its commencement, (§ **207**,) *and find the present worth of that value.* (§ **203**, II.)

321—330. What is the present worth in reversion, to commence in 3 years and continue for 8 years, of an annuity of $1000, interest at 6 per cent.? of an annuity of $500? $300? $150? $250? $169? $85? $430? $477? $12?

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* *Consols,* is an abbreviation for the consolidated annuities of the British National Debt.

331—340. At 6 per cent., compound interest, what is the present worth of a reversion of $500 per annum, to commence in 20 years, and continue 30 years thereafter? to continue 25yr.? 10yr.? 7yr.? 5yr.? 2yr.? 13yr.? 1yr.? 17yr.? 29yr.?

341—350. If money is worth 5 per cent., what is the value of a perpetual annuity of $300, to commence in 2 years? 5yr.? 8yr.? 7yr.? 15yr.? 11yr.? 9yr.? 3yr.? 6yr.? 17yr.?

212. CASE V. TO FIND AN ANNUITY, THE PRESENT WORTH, OR THE AMOUNT BEING GIVEN.

Divide the* PRESENT WORTH, *by the present worth of an annuity of $1 for the same time. Or divide the* AMOUNT, *by the amount of an annuity of $1 for the same time.

213. EXAMPLES FOR THE PUPIL.

351—360. What annual investment at 5 per cent., will extinguish a debt of $40000 in 20 years? 15yr.? 17yr.? 29yr.? 40yr.? 50yr.? 30yr.? 37yr.? 47yr.? 9yr.?

361—370. What sum of money must a man annually invest at 6 per cent., to be worth $20000 in 20 years? 25yr.? 29yr.? 23yr.? 13yr.? 30yr.? 40yr.? 37yr.? 31yr.? 39yr.?

371—380. What annual investment at 5 per cent., would amount to $37500 in 20 years? in 25,—29,—23,—13,—30,—40,—37,—31,—39yr.?

214. PERMUTATION.

PERMUTATION shows the number of changes that can be made in the order of a given number of things.

If there are but two things, a and b, they can be arranged in but two positions, ab and ba. If there were three, the third could be placed as 1st, 2d, or 3d, in each of these two changes, and there would then be $1 \times 2 \times 3 = 6$ changes, viz. : cab, acb, abc;—cba, bca, bac. If there were fc th could be placed as

1st, 2d, 3d, or 4th, in each of these 6 changes, and there would then be 1 × 2 × 3 × 4 = 24 changes. Therefore, to find the number of changes that can be made of any given number of things, all different from each other,

Multiply together the natural series of numbers, (1, 2, 3, &c.,) *up to the given number of things, and the product will be the required number of changes.*

215. EXAMPLES FOR THE PUPIL.

381—390. How many changes may be made in the position of 5 persons at table? of 8 persons? 10,—6, —9,—11,—7,—14,—12,—13 persons?

391—395. In how many different orders can 8 bells be rung? 12 bells? 6,—10,—3 bells?

396. In how many different positions might the letters of the alphabet be arranged?

397—400. In how many different ways can a class of 3 boys be arranged? 7 boys? 11 boys? 8 boys?

NOTE.

By Case V. (§ **212**,) we can determine the amount that must annually be reserved as a SINKING FUND, to extinguish any debt, or to yield any required amount in a given number of years.

It is often desirable to make an investment that will yield a stated income, at regular intervals of time greater than a year.

EXAMPLE FOR ILLUSTRATION.

What sum invested at 5 per cent., will found a school worth $5000, and pay all the expenses for repairs, &c., estimating the amount of expenses for 5 years at $12000?

By Table I., page 213, we find that the compound interest of $1 for 5 years is $ .276282, and $12000 ÷ .276282 = $43433.88, which is the amount that would yield the required interest at the end of 5 years. Adding $5000, the original cost of the school, we obtain $48433.88, for the whole investment required.

QUESTIONS ON CHAPTERS X., XI., XII., XIII., AND XIV.

What is ANALYSIS? What is meant by known and unknown quantities? What should we endeavor to do, in solving difficult questions? How may this usually be done? Give a General Rule for analysis.

What is RATIO? How is it expressed? How may it be indicated? Which is the usual mode, and how is it read? What is the antecedent?—the consequent? How do you find the ratio between different denominations?

What do you understand by reduction of currencies? Do the people of the United States ever have occasion to make such a reduction? Why? Name the currencies in common use? How may each be reduced to Federal Money? How may Federal Money be reduced to each currency? How do you reduce English Money to Federal Money, including the premium of exchange? Federal Money to English Money? Give your reasons. What is the cause of the premium?

What do you understand by Practice? Repeat the table of parts of a dollar. Give some examples illustrating the use of the table. What is meant by given terms? by terms of demand? Give the rule for complex analysis, and illustrate it by an example of your own.

What are Duodecimals? In what are they used? What are the denominations of duodecimals? How are they marked? What are those marks called? How do you add or subtract duodecimals? How do you find the product of any two denominations? Why? Multiply 4 7′ 10″ by 3″ 5‴. How do you find the quotient of any two denominations? Why? Divide 8′ 2″ 6‴ 10⁗ 4′′′′′ by 2′ 8″ 4‴.

What is a PROPORTION? How is it usually written, and how read? In what other ways may it be written? Why? What are the extremes of a proportion? the means? Prove that the product of the extremes is equal to the product of the means. If any three terms of a proportion are given, how may the other be found? Which term usually represents the unknown term? How do you find the fourth term? What class of questions may be stated in the form of a proportion? How would you arrange the first and second terms, if you wished the fourth term to represent the multiplication of the third term by a ratio? the division of the third term by a ratio? What do you understand by direct, and by inverse ratio?

What is placed as the third term in a proportion? State the rule for simple proportion. What is this rule sometimes called? Is the value of the third term ever affected by more than one ratio? Illustrate your answer by an example, and state that example in the form of a compound proportion. Why do you invert the several ratios by which the third term is to be multiplied? State the rule for compound proportion.

What is the origin of Arbitration of Exchange? Give an ex ample. In what different ways may arbitration be effected? Which is the usual method? Repeat the Chain Rule.

What is Fellowship? How may it be solved by Proportion? What is Compound Fellowship? Give the rule. Explain examples of your own, both by analysis and by proportion, in Simple and Compound Fellowship.

What is meant by AVERAGE? Give an example. How do you find the average of a series of quantities? What is Alligation? alligation alternate? What is the rule for alligation medial? What is Equation of Payments? How does it differ from alligation medial? Give the rule. On what supposition is this rule founded? What is done with fractions of a day? Give the rule for alligation alternate. Explain the rule by an example of your own.

What is INVOLUTION? What is the first power of a number? the second power? the 5th power? the 11th power? What is the exponent? What is the second power often called, and why? the third power?

What is EVOLUTION? What is the root of a number? What is the radical sign, and how is it used? What is the square root? How may we determine the number of figures that the square root will contain? Explain the method by which the square root is found, by multiplying 39 by 39 and extracting the square root of the product. What must be done when any trial divisor is not contained in the dividend? when any figure obtained for the root proves too large? How may approximate roots be obtained?

What is the cube root of a number? How may we determine the number of figures that the cube root will contain? Explain the method by which the cube root is found, by raising 39 to the third power, and extracting the cube root of the result. What must be done when the trial divisor is not contained in the hundreds of the dividend? when any figure obtained for the root is too large? How may approximate roots be obtained? How may the trial divisors, after the first, be conveniently found?

What is a SERIES? a natural series? an arithmetical series? a geometrical series? a harmonical series? What is meant by Arithmetical Progression? What are the extremes? the means? the common difference? Explain the rule for finding one of the extremes and the sum of all the terms; for finding the common difference and sum of the terms; the number of the terms and sum of the terms; the number of terms and common difference; one extreme and the common difference.

What is meant by Geometrical Progression? by the ratio? the extremes? the means? Explain the rule for finding the last term, when the first term, ratio, and number of terms are given; for finding the sum of all the terms. How may compound interest be found by series? by the table? How may we find by the table, the present worth of any sum at compound interest? the time in which any principal will amount to a given sum? the rate at which the principal will amount to a given sum in a given *time*?

[illegible] a contingent an- [illegible] an an- [illegible] How do [illegible] the present worth of an [illegible] annuity? the [illegible] the present [illegible] Explain [illegible]

[illegible]TION.

216. A parallelogram is a figure bounded by four sides, the opposite sides of which are equal.

The altitude of a parallelogram is the perpendicular distance between its opposite sides. The side on which the perpendicular is erected, is called the base of the parallelogram.

The area of a parallelogram is *equivalent to the product of the base by the altitude.*

FACTORS OF THE AREA: **Base × Altitude = Area.**

Any two of the above terms being given, the other may be found (§ [illegible]).

1. The base of a parallelogram is 40 rods, and its altitude 27 rods. What is the area?

2. What is the area of a square, the length of each side being 35 rods?

3. The area of a rectangular piece of ground is 1080 square rods, and the base is 40 rods. What is the altitude?

4. What is the side of a square field, the area of which is 3025 rods? [As the area $= (\text{side})^2$; the side $= \sqrt{\text{area}}$.]

217. An ANGLE is the opening between two sides that meet.

A TRIANGLE is a figure bounded by three sides, and containing three angles. The *altitude* of a triangle is the perpendicular distance between one of the angles *and the* opposite side.

The area of a triangle is equivalent to ONE HALF *of the product of the base by the altitude.*

FACTORS OF THE AREA: ½ × Base × Altitude = Area.

Any three of the above terms being given, the other may be found (§ **47**).

5. The base of a triangle is 23 rods, and its altitude 18 rods. Required the area.

6. The area of a triangular field is 207 rods, and its altitude 18 rods. What is the base?

7. The area of a triangle is 625 square feet, and the base 50 feet. Required the altitude.

8. An irregular lot of ground can be divided into four triangles. The base and altitude of the first, are 16 and 9 rods; of the second, 11 and 8 rods; of the third, 19 and 12 rods; and of the fourth, 14 and 13 rods. Required the area of the entire field.

218. THE AREA OF ANY FIGURE THAT IS BOUNDED BY STRAIGHT SIDES, *may be found by dividing into triangles, and measuring each triangle separately.* (§ **217**, Ex. 8.)

219. A CIRCLE is a surface enclosed by a curved line, all points of which are equally distant from the centre.

The *circumference* of a circle is the curved line by which it is enclosed. A *diameter* is a straight line passing through the centre, and terminated by the circumference. A *radius* is a line extending from the centre to the circumference, and is half of a diameter.

The circumference of a circle is equivalent to 3.1416 *times the diameter.*

FACTORS OF THE CIRCUMFERENCE: 3.1416 × Diameter = Circumference.

The area of a circle is equivalent to .7854 *times the square of the diameter*, or to .0796 *times the square of the circumference.*

FACTORS OF THE AREA: $.7854 \times (\text{Diameter})^2 =$ Area.

Or, $.0796 \times (\text{Circumference})^2$ = Area.

diameter of a circle is 40 rods. What is the
cumference?—the area? What is the side of a
re having an equal area?

0. The circumference of a circle is 150 rods.
at is the diameter?—the area? What is the side
n equivalent square?

11. The area of a circle is 500 square rods. What is
diameter?—the circumference? [Since .7854 ×
am.)^2 = Area,—(Diam.)^2 = Area ÷ .7854, (§ **47**.)
Diameter = 854, (§ **173**, **177**.)
er we cumference = $\sqrt{\text{Area}}$

2. area of a eadow is 2150 square
ls. quired the l e diameter, and of the
umference.

220. A RIGHT-ANGLED TRIANGLE is one which has wo of its sides perpendicular to each other.

The *hypothenuse* is the longest side of a right-angled triangle.

The square of the hypothenuse is equivalent to the sum of the squares of the other two sides. Therefore (§ **173**, **177**,)

$$\text{Hypothenuse} = \sqrt{(\text{Base})^2 + (\text{Altitude})^2}.$$

$$\text{Base} = \sqrt{(\text{Hypothenuse})^2 - (\text{Altitude})^2}.$$

$$\text{Altitude} = \sqrt{(\text{Hypothenuse})^2 - (\text{Base})^2}.$$

13. The base of a right-angled triangle is 40 feet, and the altitude 20 feet. What is the hypothenuse?—the area?

14. The hypothenuse is 90 feet, and the altitude 27 feet. What is the base?—the area?

15. The hypothenuse is 100 feet, and the base 30 feet. What is the altitude?—the area?

16. A ladder 75 feet long, rests against the limb of a tree that is 50 feet from the ground. How far is the *foot* of the ladder from the root of the tree?

17. Two ships left the same port. How far were

they apart, when one had sailed 125 miles north, the other 100 miles east?

18. A kite lodged in the top of a tree, but the line breaking, I find that its length is 210 feet. What is the height of the tree, the base being 189 feet from my standing place?

221. A SPHERE is a ball, all the points of the surface being equally distant from the centre.

The area of the surface of a sphere is equivalent to 3.1416 *times the square of the diameter, or to* .3183 *times the square of the circumference.*

FACTORS OF THE AREA: 3.1416 × $(\text{Diam.})^2$ = Surface.

Or, .3183 × $(\text{Circum.})^2$ = Surface.

19. The circumference of a globe is 25 inches. What is the area of its surface?

20. The diameter of a globe is 12 feet. What is the area of its surface?

21. What is the area of the earth's surface, estimating its diameter at 8000 miles?

22. The surface of a sphere contains 7000 square feet. What is its diameter?—its circumference?

222. A CYLINDER is a column of uniform thickness, terminated at each extremity by a circle.

The *convex surface* of a cylinder is the curved portion of the surface.

The area of the convex surface of a cylinder is equivalent to the circumference of the base (§ **219**) *multiplied by the height of the cylinder.*

FACTORS OF THE AREA: Circum. of base × Height = Area.

23. The diameter of a cylindrical column is 4 feet, and the height 40 feet. What is the area of the convex surface?

24. What is the area of the whole surface of a cylinder, the diameter being 7 feet, and the height 50

[To the area of the convex surface add the of the two bases (§ **219**).]

3. A PRISM is a solid whose bases are equal l planes, and whose sides are parallelograms.

A CONE is a solid body, the base of which is a circle, d which tapers uniformly to a point.

A PYRAMID is a solid body, the base of which is nded by straight lines, and which tapers uniformly a point.

A frustum of a cone, or pyramid, is the part is left after the top has been removed.

area of the convex surface of a cone is equi-ent to ½ of the product of the circumference of base, by the length of the convex surface.

FACTORS OF THE AREA: ½ × circum. × length = area.

surface of a frustum of a cone may be multiplying the m of the circumferes bases, by half the ength of the frustum.

FACTORS OF THE AREA: ½ × sum of circum. × length = area.

224. *To find the solid contents of a sphere, multiply the cube of the diameter by* .5236.

FACTORS OF THE CONTENTS: .5236 × $(\text{diameter})^3$ = solidity.

25. The diameter of a hollow sphere is 24 inches. How many gallons will it hold? (§ **95**.)

26. A hollow sphere contains 9 gallons. What is its diameter?

225. *To find the solid contents of a cylinder, or prism, multiply the area of the base by the height.*

Factors of the contents: area of base × height = solidity.

27. The height of a cylindrical pail is 18 inches, and the diameter of its base 10 inches. How many gallons will it hold?

28. A pail 20 inches high, holds 5 gallons. What is the diameter of its base?

29. The height of a prism is 14 inches, and its solid

contents 250 cubic inches. Required the area of its base.

30. The diameter of a cylinder is 9 inches, and its solid contents 500 inches. Required its height.

226. *To find the solid contents of a pyramid, or cone, multiply ⅓ of the area of the base by the height.*

Factors of the contents: ⅓ × area of base × height = solidity.

31. A pyramid has a square base, each side of which is 36 inches. What are the solid contents, the height being 30 inches?

32. Required the contents of a conical vessel, the diameter being 22 inches, and the depth 40 inches.

227. *To find the solid contents of a frustum of a pyramid or cone, multiply together the areas of the two bases, and to the square root of their product add the areas of the bases. Multiply this sum by ⅓ of the height, and the result will be the solid contents.*

33. What are the solid contents of a ship's mast, the length being 35 feet, the diameter of the base 24 inches, and the smaller diameter 18 inches?

34. How many gallons will fill a churn that is 18 inches in diameter at the bottom, 12 inches at the top, and 3 feet in height?

228. *To find the solid contents of any irregular body, immerse it in some vessel large enough to contain it, fill the vessel with water, then remove the body, and find how much water would be required to fill the vessel.*

35. On removing a stone from a pail of water, I find that 2½ gallons are required to fill the pail. Required the contents of the stone.

36. A piece of iron was immersed in a pail 12 inches in diameter. On removing the iron, the water was found to have fallen 4 inches. What were the solid contents of the iron?

229. To GAUGE, OR FIND THE DIMENSIONS OF A CASK. *Find the diameter at the bung, the diameter at the head, and the length of the cask, all in inches. Subtract the head diameter from the bung diameter, and note the difference. Add ⅔ of the difference to the head diameter, and the sum will be the* MEAN DIAMETER.

Multiply the square of the mean diameter by the length, and divide the product by 359 for the contents in beer gallons, or by 294 for the contents in wine gallons.

37. How many wine gallons will fill a cask whose bung diameter is 40 inches, the head diameter 30 inches, and the length 50 inches?

38. How many beer gallons will a cask contain, which measures 31 inches at the head, 33 inches at the bung, and 47 inches in length?

230. To FIND THE CARPENTERS' TONNAGE OF A VESSEL. *Multiply the breadth at the main beam, half the breadth, and the length, together. Divide the product by* 95 *and the quotient is the tonnage.*

This is probably the best general rule for forming estimates; but no rule can be given that will produce a perfectly accurate result. The rule employed by government, in the collection of revenue, gives about ⅗ of the tonnage thus obtained.

39. What is the tonnage of a vessel, whose length is 70 feet, and breadth 25 feet?

40. The length of a vessel is 163 feet, and the breadth 31 feet. Required the tonnage.

41. Find the tonnage of a vessel that is 113½ft. long, and 24¾ft. wide.

42. What is the tonnage of a ship that is 116 feet long, and 31 feet wide?

231. USEFUL PROBLEMS, FOR MAKING ESTIMATES.

PROBLEM I. TO FIND THE CONTENTS OF ANY CYLINDRICAL VESSEL, IN PINTS.

(1.) For Wine Measure. Measure the diameter

and height of the vessel in inches; multiply the square of the diameter by the height, and divide the product by 37.

Example. Required the number of pints contained in a pail 8 inches in diameter, and 11 inches high.

$$8 \times 8 \times 11 \div 37 = 19\tfrac{1}{37}\ \textit{pints}.$$

(2.) *For Dry Measure. Multiply the square of the diameter (measured in inches) by* 4 *times the height, and divide the product by* 171.

Example. Required the contents of a tub, 25 inches in diameter, and 13 inches deep.

$25 \times 25 \times 4 \times 13 \div 171 = 219\tfrac{17}{57}$ *pints*, or 3bu. 1pk. 5qt. $1\tfrac{17}{57}$pt.

PROBLEM II. TO DETERMINE THE CONTENTS, IN BEER MEASURE, OF ANY CYLINDRICAL VESSEL.

Multiply the square of four times the diameter, by twice the height, (measured in inches,) and divide the product by 359. *The result will be the contents in gills.*

Example. How many gills are contained in a pail 8 inches in diameter, and $12\tfrac{1}{2}$ inches high?

Four times the diameter is 32 inches, and $32 \times 32 \times 25 \div 359 = 71\tfrac{111}{359}$ gills, or 2gal. 1pt. $3\tfrac{111}{359}$gills.

PROBLEM III. TO FIND THE CONTENTS OF ANY CYLINDRICAL VESSEL, IN QUARTS, WINE MEASURE.

Measure the diameter and height in inches; multiply the square of twice the diameter by the height, and divide the product by 300.

A more accurate rule. Multiply the square of the diameter by twice the height, and divide the product by 147.

Example. How many quarts of vinegar in a pail 10 inches in diameter,—the vinegar being 8 inches deep?

By the first method. $16 \times 16 \times 10 \div 300 = 8\tfrac{160}{300}$ *quarts*, or 8qt. 1pt.

By the second method. $8 \times 8 \times 20 \div 147 = 8\tfrac{104}{147}$ *quarts, or 8qt. 1pt.* 1 gill.

PROBLEM IV. TO FIND THE CONTENTS OF ANY CYLINDRICAL VESSEL, IN GALLONS, WINE MEASURE.

Measure the diameter and height in inches; multiply the square of the diameter by the height, and divide the product by 294.

OR, *Measure the diameter and height in feet, and multiply the square of the diameter by six times the height.* For simple estimates, this rule will be sufficient, but if great accuracy is desired, *subtract one forty-eighth of the result thus obtained.*

Examples. How many gallons are contained in a boiler 23 inches in diameter, and 138 inches long?

By the first rule. 23 × 23 × 138 ÷ 294 = $248\frac{90}{294}$ gallons.

How many gallons will fill a cistern 8 feet in diameter, and 7 feet deep?

By the second rule. 8 × 8 × 42 = 2688.

More accurately, $\frac{1}{48}$ of 2688 is 56, and 2688 — 56 = 2632 gallons.

By the first rule, the dimensions being reduced to inches,

96 × 96 × 84 ÷ 294 = 2629[illegible] gallons. Of these three answers, the second, 2632 gallons, is the most nearly correct, and differs from the true answer by less than three gills.

PROBLEM V. TO FIND THE CONTENTS, IN HOGSHEADS, OF A WELL, OR CYLINDRICAL CISTERN.

Measure the diameter and depth in feet; multiply the square of the diameter by the depth, and divide by 11.

A more accurate method. Measure the dimensions in inches; multiply the square of the diameter by the depth, multiply the product by 54, *and cut off six figures from the right hand.*

Examples. How many hogsheads of water are there in a well 5 feet in diameter, the depth of water being 10 feet?

By method first: 5 × 5 × 10 ÷ 11 = $22\frac{8}{11}$ hogsheads.

How many hogsheads of water will fill a cistern, that is 9ft. 6in. in diameter, and 8 feet deep?

By method second: 114 × 114 × 96 × 54 = 67371264, and cutting off six figures from the right hand, we obtain 67 hogsheads for the answer.

PROBLEM VI. TO FIND THE CONTENTS OF ANY VESSEL WITH TAPERING SIDES, LIKE THE FRUSTUM OF A CONE.

(1.) *For Wine Measure, proceed as in either of the preceding problems, substituting for "the diameter," "one half of the sum of the greater and less diameters."*

A more correct rule. For "the square of the diameter," in each of the first four Problems, substitute "the product of the greater diameter by the less, added to one third of the square of their difference."

1. Required the number of quarts that a water pail will hold,—the diameter of the bottom being 8 inches, the diameter of the top 10 inches, and the height 10 inches.

By the second method of Problem III., substituting for the diameter one half of 8 + 10, which is 9.

$$9 \times 9 \times 20 \div 147 = 11\tfrac{1}{49}\text{qt.}$$

2. How many quarts will fill a churn that is 6 inches in diameter at the top, 12 inches at the bottom, and 36 inches high?

By the same method as before, substituting for the square of the diameter, $6 \times 12 + \frac{1}{3}$ of 6×6, which is equal to 84; $84 \times 20 \div 147 = 11\frac{3}{7}$qt.

(2.) *For Beer Measure. Multiply the greater diameter by the less, and to the product add one third of the square of the difference of the two diameters. Multiply this amount by 32 times the height, and divide by 359 for the contents, in gills.*

Example. Required the contents of a tin pan that is 12 inches in diameter at the bottom, 15 inches at the top, and 4 inches deep.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] milk [illegible]

[illegible] gals. [illegible]

[illegible]

[illegible] measured [illegible]

[illegible] [illegible] inches [illegible] diameter [illegible] inches [illegible]

[illegible] very nearly.

PROBLEM IX. TO DETERMINE THE HEIGHT TO WHICH ANY CYLINDRICAL VESSEL WOULD BE FILLED BY ONE BUSHEL.

Divide 2738 *by the square of the diameter, measured in inches.*

Example. I wish to mark a tub in such a manner as to furnish me with a set of dry measures. The diameter of the tub being 18 inches, at what height must the several marks be placed?

$18 \times 18 = 324$ and $2738 \div 324 = 8\frac{1}{2}$ inches, *very nearly*, the height of the mark for 1 bushel.

The mark for a half bushel must stand one half as high, or $4\frac{1}{4}$ inches.

The mark for a peck will be one half as high as for a half bushel, or $2\frac{1}{8}$ inches.

The mark for a half peck will be $1\frac{1}{16}$ inches high.

The mark for a quart will be $\frac{1}{4}$ of an inch high.

PROBLEM X. TO FIND THE CONTENTS OF ANY RECTANGULAR BOX IN GILLS, WINE MEASURE,

Multiply together the length, the breadth, and 4 *times the height, and divide the product by* 29.

Example. How many gills in a box 6 inches long, 4 inches wide, and two inches high?

$6 \times 4 \times 8 \div 29 = 6\frac{18}{29}$ gills.

If the contents in pints, quarts, or gallons, are desired, they may easily be found by reducing the gills to the measure desired.

Example. How many gallons in a box 27 inches long, 9 inches wide, and 8 inches high?

$27 \times 9 \times 32 \div 29 = 261\frac{7}{29}$ gi., or 8gall. 1pt. $1\frac{7}{29}$gi.

PROBLEM XI. TO FIND THE NUMBER OF GALLONS CONTAINED IN A RECTANGULAR VESSEL.

Take the dimensions of the vessel in inches; multiply the length by the breadth, multiply the product by the height, and divide by 231 *for the answer in wine gallons, or by* 282 *for the answer in beer gallons.*

Example 1. How many wine gallons in a box 39 inches long, 8 inches broad, and 9 inches high?

$39 \times 8 \times 9 \div 231 = 12\frac{12}{77}$ gallons, or 12gal. 1 pint, 1 gill.

Example 2. How many beer gallons in a box 48 inches long, 36 inches broad, and 15 inches high?

$48 \times 36 \times 15 \div 282 = 91\frac{43}{47}$ gallons, or 91gall. 3qts. 1pt. $1\frac{13}{47}$ gills.

PROBLEM XII. TO DETERMINE THE NUMBER OF BUSHELS CONTAINED IN A RECTANGULAR BOX OR BIN.

Take the dimensions in inches; multiply the length, breadth, and height together, multiply the product by 5 *and divide by* 10752.

Examples. How many bushels will fill a bin 6ft. 6in. long, 4ft. 2in. wide, and 5 feet deep?

$78 \times 50 \times 60 \times 5 \div 10752 = 188\frac{8}{45}$ bushels.

PROBLEM XIII. TO DETERMINE THE CONTENTS OF A CONCAVE VESSEL, (LIKE A BOWL,) IN PINTS, WINE MEASURE.

To three times the square of half the top diameter, (measured in inches,) add the square of the depth; multiply this sum by the depth, and divide the product by 55.

Example. The diameter of a bowl is 12 inches, and its depth is 4 inches. How many pints will it hold.

$3 \times 6 \times 6 + 4 \times 4 = 124$, and $124 \times 4 \div 55 = 9\frac{1}{55}$ pints.

A more accurate rule. Multiply the sum of the squares, (obtained as above,) by 7 times the depth, and divide the product by 386.

PROBLEM XIV. TO DETERMINE THE CONTENTS OF A CONCAVE BOILER, IN GALLONS.

(1.) *For Wine Measure. To three times the square of half the top diameter (measured in inches,) add the square of* the depth; multiply this *sum by the depth, and divide* the product by 441.

Example. How many gallons in a boiler 64 inches in diameter, and 27 inches deep?

$3 \times 32 \times 32 + 27 \times 27 = 3801$, and $3801 \times 27 \div 441 = 232\frac{5}{7}$ gallons.

(2.) *For Beer Measure. To three times the square of half the top diameter (measured in inches,) add the square of the depth; multiply the sum by the depth, and divide by* 539.

Example. How many gallons in a boiler 70 inches in diameter, and 30 inches deep?

$3 \times 35 \times 35 + 30 \times 30 = 4575$, and $4575 \times 30 \div 539 = 255$ gallons, *nearly.*

PROBLEM XV. TO MAKE A SET OF RECTANGULAR MEASURES.

(1.) Wine Measure.

Any vessel $1\frac{3}{4}$ inches long $1\frac{1}{2}$ inches broad, and $2\frac{3}{4}$ inches high, will hold 1 gill.

A vessel $2\frac{3}{4}$ inches long, 3 inches broad, and $3\frac{1}{2}$ inches high, will hold 1 pint.

A vessel $4\frac{1}{8}$ inches long, 4 inches broad, and $3\frac{1}{2}$ inches high, will hold 1 quart.

A vessel 6 inches long, $5\frac{1}{2}$ inches broad, and 7 inches high, will hold 1 gallon.

(2.) Beer Measure.

A box $1\frac{3}{4}$ inches square and $2\frac{7}{8}$ inches high, will hold 1 gill.

A box 3 inches long, $2\frac{15}{16}$ inches broad, and 4 inches high, will hold 1 pint.

A box 4 inches long, 3 inches broad, and $5\frac{7}{8}$ inches high, will hold 1 quart.

A box 6 inches long, $5\frac{7}{8}$ inches broad, and 8 inches high, will hold 1 gallon.

(3.) Dry Measure.

A box $3\frac{1}{2}$ inches square and $2\frac{3}{4}$ inches deep, will hold 1 pint.

A box 4½ inches square and 3$\frac{5}{16}$ inches deep, will hold 1 quart.

A box 6⅝ inches square and 6⅛ inches deep, will hold 1 half peck.

A box 9 inches square and 6⅝ inches deep, will hold 1 peck.

A box 10 inches square and 10¾ inches deep, will hold 1 half bushel.

A box 13¼ inches square and 12¼ inches deep, will hold 1 bushel.

PROBLEM XVI. TO MAKE A SET OF CYLINDRICAL MEASURES.

(1.) BEER MEASURE.

A cylindrical cup 2¼ inches in diameter, and 2¼ inches high, will hold 1 gill.

A cup 3 inches in diameter, and 5 inches high, will hold 1 pint.

A cup 4 inches in diameter, and 5⅝ inches high, will hold 1 quart.

A cup 8 inches in diameter, and 5⅝ inches high, will hold 1 gallon.

(2.) WINE MEASURE.

A cylindrical cup 1$\frac{15}{16}$ inches in diameter, and 2$\frac{7}{16}$ inches high, will hold 1 gill.

A cup 3$\frac{3}{16}$ inches in diameter, and 3⅝ inches high, will hold 1 pint.

A cup 3$\frac{13}{16}$ inches in diameter, and 5$\frac{1}{16}$ inches high, will hold 1 quart.

A cup 7 inches in diameter, and 6 inches high, will hold 1 gallon.

(3.) DRY MEASURE.

A measure 4⅝ inches in diameter, and 2 inches deep, will hold 1 pint.

A measure 5⅞ inches in diameter, and 2½ inches deep, will hold 1 quart.

A measure $9\frac{1}{4}$ inches in diameter, and 4 inches deep, will hold 1 half-peck.

A measure $11\frac{3}{4}$ inches in diameter, and 5 inches deep, will hold 1 peck.

A measure $14\frac{3}{4}$ inches in diameter, and $6\frac{1}{4}$ inches deep, will hold 1 half-bushel.

A measure $18\frac{1}{2}$ inches in diameter, and 8 inches deep, will hold 1 bushel.

232. MISCELLANEOUS EXAMPLES.

1—3. What is the multiplier, if the multiplicand is 9, and the product 4?—multiplicand $3\frac{1}{2}$, product $\frac{5}{7}$?—multiplicand 25, product $3\frac{2}{3}$?

4—6. What is the divisor, if the dividend is 2, and the quotient 7?—dividend $1\frac{3}{8}$, quotient $11\frac{1}{2}$?—dividend $5\frac{5}{7}$, quotient $26\frac{5}{9}$?

7—9. If the sum of two numbers is $16\frac{1}{2}$, and one of the numbers is 3.7, what is the other?—if the sum is $29\frac{3}{8}$, and one number $14\frac{3}{11}$?—sum 9.89, and one number $7\frac{12}{13}$?

10—12. If the difference of two numbers is $4\frac{1}{2}$, and the greater number is $19\frac{1}{3}$, what is the less?—difference 16.7, greater number $41\frac{3}{11}$? — difference $4\frac{5}{9}$, greater number $23\frac{3}{17}$?

13—15. The greater of two numbers is as much above their average, as the less is below it, (§ **164.**) Then what are the two numbers whose sum is $9\frac{1}{2}$, and their difference 6?—sum $14.\dot{7}$, difference $11\frac{2}{3}$?—sum $27.\dot{6}\dot{2}$, difference $9.0\dot{7}\dot{5}$?

16—18. If the product of three factors is 16, and two of the factors are 18 and 9, what is the third factor?—if two of the factors are $6\frac{5}{11}$ and $1\frac{4}{7}$?—$4\frac{4}{9}$ and $\frac{27}{40}$?

19—21. What number must be subtracted from $62\frac{7}{8}$ to leave $9\frac{5}{8}$? $4\frac{2}{3}$? 16.875?

22—24. What number must be added to $64\frac{2}{7}$ to make $65\frac{1}{4}$? 80.3? $95.\dot{0}9\dot{5}$?

25—27. What number must be multiplied by $3\frac{1}{2}$, and the product divided by 4.875, to give for a quotient 4? 6⅔? 16⁵⁄₇?

26—30. What is the difference between ¾ and ⅘ of 19? of $7.50? of £9 11s.?

31—40. How many farthings in £2 4s.? in ⅓ of £1 11s.? in $\frac{5}{6}$ of £7 6s. 9d.? £5? $7\frac{2}{3}$s.? £$2\frac{1}{4}$? £$\frac{9}{16}$? £$4\frac{4}{17}$? $3\frac{1}{2}$s.? 10.75d.?

41—55. Find the sum, the difference, and the product of 5.7 and $13\frac{3}{4}$; 9 and $7\frac{3}{8}$; $4.\dot{3}$ and $\frac{1}{3}$; .75 and $\frac{1}{4}$; $1\frac{4}{5}$ and $\frac{3}{4}$.

56—60. If 18 men can build 40 rods of wall in 6 days, how much can 15 men build in 9 days? 18 men in 12 days? 45 men in 21 days? 12 men in 4 days? 16 men in 3 days?

61—70. How many raisins at 9 cents a pound, must be given in exchange for 36 bushels of potatoes, at 20 cents a bushel? How many at 4ct. a lb.? at 12ct. a lb.? at 8ct.? 6ct.? 6¼ct.? 12½ct.? 10ct.? 7½ct.?

71—80. What is the interest of $159 for 4yr. 7mo., at 7 per cent.? at 6 per cent.? at 8½ per cent.? at .04? .03? .05? .07½? .09? .06¼? .05¾?

81—90. If 9 men mow 10 acres of grass in a day, how much will 7 men mow in 4½ days? 18 men in 1¾dy.? 27 men in 11dy.? 3 men in 6dy.? 5 men in 9dy.? 11 men in 13½dy.? 4 men in 6¾dy.? 3 men in 3dy.? 2 men in 13.5dy.? 6 men in 4dy.?

91—100. How much money that is 7 per cent. below par, will pay a debt of $300? How much that is at a premium of 2 per cent.? of 3 per cent.? of 7½ per cent.? of .06? of .01? at a discount of 5 per cent.? of .02? of .025? of .06¼?

101—110. Sold 5¾ tons of hay at $4 per bundle, each bundle containing 5cwt. How many bundles were there, and what was the cost of the whole? What would have been the cost at $5 per bundle? $4.50? $3.00? $3.50? $4.75? $16 per ton? $18? $27? $14 50?

111—120. A.'s capital was $550, B.'s $625, and C.'s *$325.* How would you divide among them a gain of

$150? $75? $225? $300? $1125? $37.50? $630? $255? $1965? $375?

121—130. How many potatoes at 25ct. a bushel, must be given in exchange for 504 pounds of sugar, at $ .06¼ per pound? at 5, 8⅓, 4½, 9, 3¾, 7½, 8, 7, 10 cents per pound?

131—140. A man can do a piece of work in 10 days. In what time can it be done if he is assisted by a man who could do it in 8 days? 12dy.? 10dy.? 5dy.? 7½dy.? 4dy.? 6dy.? 20dy.? 15dy.? 14dy.?

141—150. What would be the cost of 16 bales of sheeting, each bale containing 18 pieces, and averaging 32½ yards per piece, at 10 cents a yard? at 8ct.? 6¼ct.? 9ct.? 7½ct.? 8⅓ct.? 12½ct.? 11ct.? 7ct.? 10¾ct.?

151—160. How much would it cost to carpet a floor 18ft. long, and 13ft. 6in. wide, at $1.50 per yd.? at $ .87½? $ .75? $ .625? $1? $$\frac{9}{8}$? $1⅓? $1.25? $⅔? $ .95?

161—170. Supposing the earth to move in its orbit at the rate of 1553400 miles per day, how far will it go in 1 second? 5sec.? 49sec.? 1min.? 19min.? 1h.? 11h.? 1dy.? 1mo.? 9dy. 11⅓h.?

171—180. If 25 men can build a house in 108 days, how many men could build it in 3 days? in 25dy.? 27dy.? 9dy.? 4dy.? 54dy.? 20dy.? 60dy.? 36dy.? 300dy.?

181—190. A laborer's wages are $1.50 per day. How many days must he work to pay for pasturing his cow 72 weeks, at 37½ct. a week? at $ .50 per w.? $ .62½? $ .56¼? $ .43¾? $ .25? $ .18¾? $ .30? $ .41⅔? $ .58⅓?

191—200. How many bottles, each holding 1½ pints, may be filled from a cask that contains 120 gallons? 69gal.? 87¾gal.? 100gal. 2qt.? $117\frac{9}{16}$gal.? 45gal.? 162gal.? 58½gal.? 99gal.? 212¼gal.?

201—210. How many times will a wheel, that is 7⅓ft. in circumference, revolve in going 10 miles? 4 miles? $1\frac{4}{5}$m.? 8.5m.? 2m.? 6½m.? 1¾m.? $\frac{5}{37}$m.? .25m.? *3.8m.*?

211—220. What must I give for 18 boxes of sugar, each weighing 2cwt. 3qr. 14lb., at $6 per cwt.? at $5? $8? $6.50? $7.25? $5.50? $\$4\frac{7}{8}$? $9? $4? $\$6\frac{2}{3}$?

221—230. What is the worth of a vessel, if $375 is asked for $\frac{1}{2}$ of $\frac{3}{5}$ of her? for $\frac{3}{4}$ of $\frac{1}{4}$? $\frac{1}{3}$ of $\frac{5}{4}$? $\frac{4}{5}$ of $\frac{2}{3}$? $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{3}{4}$? $\frac{2}{3}$ of $\frac{3}{5}$? $\frac{2}{3}$ of $\frac{4}{5}$? $\frac{2}{3}$ of $\frac{3}{4}$? $\frac{1}{3}$ of $\frac{4}{5}$? $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$.

231—240. If a staff, 7ft. 6in. long, cast a shadow of 10ft., what is the height of a tree whose shadow measures 99ft.? 42ft.? 84.3ft.? 87ft.? 30.75ft.? $84\frac{1}{4}$ft.? 90ft. 6in.? 81ft. 9in.? 100ft.? 140ft.?

241—250. How much can a man save from an annual salary of $730, if his expenses are $1 per day? $1.50 per day? $1.20? $1.40? $.80? $.75? $\$.87\frac{1}{2}$? $\$.03\frac{3}{4}$? $\$1\frac{3}{8}$? $1.25?

251—260. Bought a house lot for $900. What is the value of $\frac{1}{2}$ of $\frac{3}{4}$ of the lot? $\frac{13}{25}$ of $\frac{2}{3}$ of the lot? $\frac{7}{15}$ of $\frac{3}{8}$? $\frac{7}{8}$ of $\frac{4}{5}$? $\frac{1}{2}$ of $\frac{7}{9}$? $\frac{5}{16}$ of $\frac{3}{5}$ of $\frac{1}{2}$? $\frac{7}{8}$ of $\frac{7}{9}$? $\frac{3}{8}$ of $\frac{3}{4}$? $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{4}{3}$? .9 of $\frac{5}{16}$?

261—270. How much cloth would there be in a piece worth $\$137\frac{1}{2}$, at $\$4\frac{1}{6}$ per yd.? at $\$3\frac{1}{8}$? $\$8\frac{1}{3}$? $5? $\$12\frac{1}{2}$? $\$6\frac{1}{4}$? $\$5\frac{1}{2}$? $\$13\frac{3}{4}$? $\$6\frac{7}{8}$? $\$17.18\frac{3}{4}$?

271—280. A merchant bought 50 barrels of flour for $185, paying for freight $14, storage $7.50, drayage and other expenses $6. At what price per barrel must he sell it, to gain $25 on the whole? to gain $6.25? $30? $12? $10? $12.50? $\$18\frac{3}{4}$? $\$9\frac{3}{8}$? $\$37\frac{1}{2}$? $50?

281—290. If $\frac{4}{5}$ of a ton of hay cost $\$9\frac{3}{5}$, what will 5 tons cost? $1\frac{1}{4}$T.? 10T.? 3T.? $7\frac{1}{2}$T.? $13\frac{2}{3}$T.? $\frac{5}{8}$T.? $2\frac{7}{8}$T.? 11T.? 6.3T.?

291—300. If a tax of $13125 is assessed on property valued at $1750000, how much should be paid by a man whose property is worth $2000? $700? $1900? $350? $1000? $500? $250? $875? $750? $580?

301—310. How many horses at $130, cows at $25, and oxen at $95, of each a like number, can be bought for $500? $2000? $750? $1250? $5000? $250? $1500? $3000? $2500? $2250?

311—320. What is the value of 14lb. of flour, at

\$7 per barrel of 196lb.? at \$3.50 per bbl.? \$5.25? \$$6\frac{3}{8}$? \$5.60? \$10.50? \$9? \$4? \$6.50? \$8.75?

321—330. What will 2.875cwt. of rice cost, at $4\frac{1}{2}$ct. a lb.? at 5ct.? 3ct.? 4ct.? $3\frac{1}{4}$ct.? $3\frac{3}{4}$ct.? $2\frac{1}{2}$ct.? $4\frac{7}{8}$ct.? $5\frac{5}{8}$ct.? $3\frac{1}{2}$ct.?

331—340. If when wheat is \$1 per bushel, the 6ct. loaf weighs 24oz., what should the 5 cent loaf weigh? What should be the weight of each loaf when wheat is \$1.25 per bushel? \$.75 per bu.? \$$\frac{7}{8}$? \$.9? \$$1\frac{1}{8}$?

341—350. What must be the length of a board that is $13\frac{1}{2}$ inches long, to measure 3 square feet? 12ft.? $1\frac{1}{2}$ft.? $7\frac{1}{2}$ft.? $3\frac{3}{4}$ft.? $5\frac{5}{8}$ft.? 15ft.? 7ft.? 2ft.? $1\frac{7}{8}$ft.?

351—360. When apples are \$$1.62\frac{1}{2}$ per barrel, how much wool at \$$\frac{3}{8}$ a lb. will buy 5 barrels? 13bbl.? 2bbl.? $7\frac{1}{2}$bbl.? 10bbl.? 25bbl.? $3\frac{7}{8}$bbl.? $2\frac{2}{3}$bbl.? 4bbl.? 20bbl.?

361—370. At \$13.75 per ounce Avoirdupois, what would be the value of 5oz. of gold? of $\frac{7}{8}$lb.? $\frac{3}{16}$cwt.? $\frac{1}{8}$qr.? $\frac{2}{11}$lb.? $\frac{3}{4}$oz.? $\frac{1}{28}$cwt.? .4oz.? .3lb.? $4\frac{1}{2}$lb.?

371—380. If a man walks $3\frac{11}{12}$ miles an hour, and each step measures $2\frac{1}{2}$feet, how many steps will he take in 4 hours? in 3h.? 7h.? $\frac{3}{4}$h.? .9h.? 1.3h.? $2\frac{1}{2}$h.? $\frac{1}{2}$h.? .75h.? 10h.?

381—390. To what number must you add $\frac{1}{3}$ of itself, to give 180? 120? 98? 32? 5? $2\frac{1}{2}$? 85? 40? 27? 33?

391—400. Find a number to which if you add 9 more than $\frac{1}{3}$ and $\frac{1}{4}$ of itself, the sum will be 28; 199; 66; 161; 47; 389; 18.5; $21\frac{2}{3}$; $13\frac{3}{4}$; 769.

401—410. If 36 men can perform a piece of work in 25 days, how many men could accomplish twice as much in 18 days? in 12dy.? 4dy.? 9dy.? 5dy.? 20dy.? 15dy.? 200dy.? 225dy.? 45dy.?

411—420. At 6 per cent. what is the interest on \$1 for 2 months? 4mo.? 12mo.? 26mo.? 44mo.? 13mo.? 39mo.? 57mo.? $8\frac{2}{3}$mo.? $11\frac{1}{2}$mo.?

421—430. At 6 per cent., what is the interest on \$1 for 6 days? 12dy.? 18dy.? 24dy.? 3dy? 5dy? 7dy? 19dy.? 29dy.? 11dy.?

431—440. At what time between 1 and 2 o'cl are the hour and minute hands exactly together? between 3 and 4? 7 and 8? 2 and 3? 5 and 6? 6 and 7? 9 and 10? 8 and 9? 4 and 5? 10 and 11?

441—450. At 6 per cent. compound interest, what annual payment will extinguish a debt of $40000000 in 10 years? In 19yr.? 31yr.? 50yr.? 2yr.? 17yr.? 40yr.? 30yr.? 45yr.? 15yr.?

451. A man has property yielding $300 per annum, which he desires to exchange for a life annuity. If money is worth 6 per cent. a year, what annuity must he receive, it being probable that he will live 20 years?

452—454. How much cambric that is ¾yd. wide, will line 10yd. of silk that is 1yd. wide? 9½yd. that is ⅞yd. wide? 11¾yd. that is 1¼yd. wide?

455—457. What would be the price of 5 bales of muslin, each bale containing 10 pieces, and each piece measuring 30½yd., if 25yd. cost $3.12½? if 13yd. cost $1.43? if 17yd. cost $1.78½.

458—460. A. lent B. $500 for 4⅔ months; what sum ought A. to receive in return, to use 7½mo.? 11¾mo.? 2⅝mo.?

461—463. A bankrupt can pay but 75 per cent. of his debts. How much will he pay a creditor for $525? for $62.43? for $9.875?

464—466. If a laborer receive $1.75 for every working day, how much can he spend in a year, and save $100? $150? $225?

467—469. At £1 11s. per yd., what is the price of 2qr. 3na. of cloth? of 5yd. 3qr.? of 17yd. 1na.?

470—472. What part of 95gal. 2qt. is 5gal. 1pt.? 147gal. 3qt.? 8gal. 1qt. 1pt.?

473—475. How much cotton at 15½ct. a pound, can be bought for $13⅝? $19.75? $41⅔?

476—478. Reduce to a common denominator 4.7 and ⅖; .83 and ⅓; .875 and $\frac{11}{16}$.

479—481. What is the value of an estate, if ½ of $\frac{5}{7}$ of it is worth $250? $875? $328.40?

482—484. At what price per yard must I sell 94yd. of calico, in order to gain $4.70, the prime cost being $7.52? to gain $4.23? $3.52½?

485—487. At $18\frac{1}{2}$ cents per franc, what is the value of 169.79 francs? 248.43 francs? 19.87 francs?

488—490. What is the value in Federal Money of £137 5s. 6d., when exchange on London is at a premium of 10 per cent.? $7\frac{1}{2}$ per cent.? $9\frac{3}{4}$ per cent.?

491—493. What is the compound interest of $1125.50 for 5yr. 8mo. 10dy., at 7 per cent.? $4\frac{1}{2}$ per cent.? 6 per cent.?

494—496. What number is that, whose square, if multiplied by $2\frac{7}{9}$, would give 100? 225? 4900?

497—499. If 25 men can build 75 rods of wall in 6 days, by working 10 hours a day, how many men could build 120 rods of similar wall in 12 days of 8 hours? in 25 days of 12 hours? in 30 days of 10 hours?

500—502. What is the annual gain per cent., if an investment of $875 amounts at simple interest to $1242.50 in 5yr. 3mo.? to $1478.75 in 7yr. 8mo.? to $2000 in 8yr.?

503—505. If 20 per cent. is lost by selling shoes at 75 cents a pair, at what price should they be sold to gain 20 per cent.? 50 per cent.? to lose $33\frac{1}{3}$ per cent.?

506—508. How many changes could be rung on 5 bells? 7 bells? 11 bells?

509—511. A factor purchased 7 bales of cotton, each containing 4cwt. 1qr. 5lb., at $11\frac{1}{2}$ cents a lb. What would be his commission at 3 per cent.? at $2\frac{7}{8}$ per cent.? at $4\frac{2}{3}$ per cent.?

512—514. What number, multiplied by $86\frac{7}{8}$, will give $\frac{2}{3}$? $1.\dot{5}$? $7.3\dot{8}$?

515—517. Divide an estate of $5000 among two heirs, in the proportion of $\frac{1}{2}$ to $\frac{1}{3}$; $\frac{3}{7}$ to $\frac{5}{9}$; $\frac{11}{13}$ to $\frac{2}{5}$.

518—520. Find the sum of £$1\frac{3}{8}$ and £$7\frac{11}{16}$; £$48\frac{2}{3}$ and $16\frac{7}{16}$s.; £$9\frac{5}{12}$ and £$8\frac{13}{120}$.

521—523. At £1 5s. 3d. per yard, what will be the price of $8\frac{1}{2}$yd. of broadcloth? of $3\frac{3}{4}$yd.? $6\frac{5}{8}$yd.?

524—526. How much tea, at 50 cents and 75 cents a pound, will form a mixture worth 60 cents a pound? 70 cents a pound? $62\frac{1}{2}$ct. a pound?

527—529. How much United States Bank

would have been required to pay a debt of $641.50, when the bills were selling at a discount of 12½ per cent.? 16⅔ per cent.? 25 per cent.?

530—532. What is the discount of a note at 90 days for $850, at 6 per cent.? 5½ per cent.? 7 per cent.?

533—535. What is the present worth of $3000 due in 3yr. 5mo. 18dy. at 5 per cent.? 6 per cent.? 7 per cent.?

536—538. Find the prime factors of 38400; 4235; 141151.

539—541. How many inches in 39½ rods? 3.97 furlongs? $8\frac{5}{11}$ miles?

542—544. What number, diminished by ¼ and ⅓ of itself, gives 8⅔? 17¼? 29½?

545—547. Find the value in sterling money of $563.75, when the premium of exchange is 9 per cent.; 8¾ per cent.; 10½ per cent.

548—550. In a certain partnership A. advanced ⅓ of the stock, B. ⅖, and C. the remainder. How should a gain of $5000 be divided between them? $7500? $3580?

551—553. How much tea at 62½ cents, and at $1 per pound, may be mixed with 13lb. at 75 cents and 10lb. at 87½ cents, to make the whole worth 70 cents per lb.? 81ct. per lb.? 94ct. per lb.?

554—556. How many bushels of potatoes must be given in exchange for 387½lb. of beef, at 7 cents a pound, when potatoes are worth 25 cents a bushel? when they are worth 33⅓ct.? 37½ct.?

557—559. Allowing 6 per cent. compound interest, what is the present worth of an annuity of $375, to continue 15 years? 13 years? 27 years?

560—562. Bought 85 yards of broadcloth for $297.50, and sold it at $4.87½ per yard. What was the whole gain? the gain per yard? the gain per cent.?

563—565. A cistern is filled by two pipes, one of which would fill it in 3 hours, and the other in 1 hour and 20 minutes. It has also a discharging pipe, by which it can be emptied in 2¼ hours. In what time will it be filled if the pipes are all open? if the discharging pipe is closed? if the smallest pipe is closed?

566—568. A garrison of 500 men is supplied with provisions for 18 months. How many may be added to the garrison, to consume the whole in 12 months? in 8 months? in 15 months?

569—571. If pure silver is worth $16.50 per lb., how many ounces of alloy must be added to each pound, to make a pound of the mixture worth $16? $15.75? $14.25?

572—574. A. could do a piece of work in 4 days, B. in 3 days, and C. in 5 days. In what time will it be done, if they all work together? In what time would it be done by A. and C.? by B. and C.?

575—577. Divide $600 among three men, so that ⅓ of the first share, shall be equivalent to ⅔ of the second share, or $\frac{4}{13}$ of the third share. Divide in the same proportions $876; $954.

578—580. How much money must be invested to build a wall worth $375, and renew it every 15 years, allowing 6 per cent. compound interest? How much at 7 per cent.? 5 per cent.?

581—583. A difference of 15° in longitude, is equivalent to an hour's difference of time. Then what is the difference of time between two places, whose difference of longitude is 35° 17′? 9° 13′? 28° 41′?

584—586. What is the least number that can be divided by 3, by 4, by 5, and by 6, with no remainder? with 1 remainder? with 2 remainder?

587. How many bricks, each 8 inches long, 4 inches wide, and 2 inches thick, in a garden wall that is 8ft. high, and 1½ft. thick; the outside of the wall measuring 153⅔ feet on each of the two longest sides and 110ft. on each of the two shortest sides?

588. A carpenter offered to build a house for the cost of the twelfth nail that he should drive, estimating the first nail at 1 mill, the second at 4 mills, the third, at 16 mills, and so continuing, quadrupling the price of each nail for the price of the next. What was the amount of his charge?

589. What is the distance between the opposite corners of a room, the length of the longest side being 20ft., and the length of the shortest side 12ft.?

590. A. owns ⅖ of a house, B. owns ¼, and C. owns the rest. The house rents for $500, of which $200 are expended for taxes and repairs. How should the balance be divided among the owners?

591. What would be gained by purchasing $1500 Pennsylvania stock, in London, for £225, and selling it in New York, at 85 per cent., allowing $50 for interest and commissions, and 9 per cent. for the premium of exchange?

592. A laborer agreed to pay $.75 for his board, on every day that he was idle, on condition that he should receive $1.75 for every day he worked; at the end of 40 days he received $45. How many days did he work, and how many was he idle?

593. A person bought 180 apples, at 2 for a cent, and 180 at 3 for a cent. How much did he gain or lose by selling the whole at 5 for 2 cents?

594. If a quantity of provisions serves 1500 men 12 weeks, at the rate of 20oz. a day for each man, how many men will the same provisions maintain for 20 weeks, at the rate of 8oz. a day for each man?

595. A father left his son a fortune, ¼ of which he squandered in 8 months; $\frac{3}{7}$ of the remainder lasted him 12 months longer, after which he had £820 left. What was the original legacy?

596. A daughter received $840, which was just ⅞ of her brother's fortune. What was their father worth?

597. If 20 men perform a piece of work in 12 days, how many men will do three times as much, in two-fifths of the time?

598. Divide $2000 among A. B. and C., so as to give A. $240 more, and B. $190 less than C.

599. What is the square of that number, which, if multiplied by ⅔ of ⅘ of 1½, will produce 1?

600. If I own $\frac{3}{16}$ of a ship which is worth $6000, *what part* shall I have left after selling ⅖ of ⅘ of my *share, and* what will it be worth?

601. A cannon ball flies about 1500 feet per second. If it could continue at this rate, at what time would it reach the sun, supposing the distance to be 95 million miles, and allowing it to start on its journey at noon on the first of January, 1849?

602. A levy of 1240 horses is to be distributed among three regiments in the proportion of 1, $\frac{8}{11}$, and $\frac{7}{9}$. How many will each regiment receive?

603. On a debt of $400, due in 9mo., a payment of $100 is made in 3mo. In what time should the remainder be paid?

604. On a debt of $1100, due Oct., 3, 1848, a payment of $159.20 was made Apl. 3, and another of $300, May 18. When should the remainder be paid?

605. What is the price per bushel, of a mixture of 19 bushels of wheat, at $1, 10bu. of rye, at $.75, 5bu. of corn at $.50, and 10bu. of oats, at $.40?

606. A. and B. can do a piece of work in 24 days, and A. could do it alone, in 28 days. In what time could B. do it?

607. A. and B., at the commencement of their business, had an equal capital; A. lost $\frac{1}{4}$ of his stock, and B. gained $500, when B.'s capital was found to be twice as great as A.'s. What was the original stock of each?

608. A garrison of 800 men, is stocked with 5 months' provisions. How many must leave, in order that the supply may last 16 months?

609. How much per cent. is gained, by selling at $$5\frac{2}{3}$ per yard, cloth that cost $$129.62\frac{1}{2}$ by the piece of $30\frac{1}{2}$ yards?

610. What would be received by discounting at bank, a note for $875, payable in 90 days, allowing money to be worth 7 per cent.?

611. What would it cost to insure a house for $3000, at a premium of $1\frac{1}{4}$ per cent., there being an additional charge of $2 for the policy?

612. The cube of 2.5 is the square root of what number?

613. Find the value in Pennsylvania currency, of 10s. 6d., New England currency.

614. The square of the cube of 3, is how many times the cube of the square root of 9?

615. What is the side of a cubical pile, that contains 256 cords of wood?

616. If 36 men, in $127\frac{1}{2}$ days of $13\frac{1}{2}$ hours, dig a trench $33\frac{3}{4}$yd. long, $10\frac{1}{4}$ft. deep, and $15\frac{3}{5}$ft. wide, how many men, in $7\frac{3}{7}$ days of $12\frac{8}{11}$ hours, will dig a similar trench $82\frac{5}{11}$yd. long, $7\frac{2}{3}$ft. deep, and 10ft. wide?

617. A. B. and C. form a joint stock of $549, by which they gain $117. A.'s money is in trade 4 months, B.'s 5 months, and C.'s 13 months, and their shares of the gain are in proportion to the numbers 2, 3, and 4, respectively. What was each man's stock?

618. A. B. and C. traded in company; A. put in $3000, B. $2000, and C. the remainder. They gained $1500, of which C. took $500. What did C. contribute, and what was A.'s share of the gain?

619. How long will it take 5 persons to arrange themselves at table in every possible order, if they eat 3 meals a day?

620. Sold a quantity of sugar for $150, thereby losing 25 per cent., whereas I expected to gain 15 per cent. What was my total loss?

621. If the wheel of a carriage turns once and a half in going a rod and a half, how often will it turn in 1 mile?

622. A ladder standing upright against a wall, reaches the top, but the foot being removed 12 feet from the wall, it reaches to a point only 6 feet above the ground. Required the height of the wall.

623. A tract of 1500 acres is divided among three persons. B. has as much as A. and C., wanting 10 acres, and the shares of A. and C. are to each other in the proportion of 10 to 3. Required the share of each.

624. A fortune was to be disposed of in the following manner: to A. $\frac{1}{5}$, to B. $\frac{3}{10}$, to C. $\frac{1}{4}$, to D. $\frac{1}{8}$, and the

remainder, which was $1000, was to be paid to C. What was each man's share of the estate?

625. A mechanic hired a certain number of men at $1 per day, $\frac{2}{3}$ as many at 75 cents, and $\frac{3}{4}$ as many at 50 cents as at 75 cents. What was the number of each, the daily wages of the whole being $42?

626. A horse is tied to a stake, so that he can graze an acre of pasture. What is the length of the rope?

627. A., B., and C., agreed to dine together. A. furnished 2 loaves, B. 3 loaves, and C., having no bread, paid 10 cents for his share. How should the money be divided between A. and B.?

628. Bought goods amounting to $2500, on 12 months' credit; at the end of 3 months $400 were paid, at the end of 6 months $500, and at the end of 8 months $200. When will the balance of $1400 become due?

629. Bought a piece of cloth which proved to be but $\frac{3}{4}$ as long as it was marked, but I find that if I sell it at $5.25 per yard, I shall gain 5 per cent. on the prime cost. What did I give per yard?

630. At the commencement of business, A.'s capital was twice that of B., but after some time, B. having gained 50 per cent., and A. having lost $2000, A. has only $\frac{2}{3}$ as much as B. What was the original stock of each?

631. Two pieces of cloth were originally of equal length; but after 17 yards had been sold from one, and 22 yards from the other, the shorter piece contained only $\frac{3}{4}$ as much as the other. What was the original length?

632. What is the least number, which being divided by 2, 3, 4, 5, 6, 7, 8, or 9, leaves a remainder 1 less than the divisor?

633. Two men, who are 81 miles apart, travel towards each other. A. starts 2 hours before B., and travels $17\frac{1}{2}$ miles in 5 hours; B. goes 8 miles in 3 hours. How far will each travel before they meet?

233. Book-keeping.

Farmers, mechanics, and others whose business is small, will usually find it convenient to keep their accounts by Single Entry. For this purpose, two books will be necessary; the Day-Book, in which every transaction is originally entered, and the Ledger, in which the charges are transferred to the account of each individual. The following will serve to illustrate the

DAY-BOOK.

Portland, July 4th, 1848.

	John Wilson,	Dr.		
‖	To 3 tons hay, at 20.00	60.00		
	" 25 bushels of potatoes, at .20	5.00	65	00
	William Appleton,	Cr.		
‖	By amount of his bill for pasturing,		28	00
	July 5th.			
	George Jones,	Cr.		
	By 10 days' labor, at 1.50	15.00		
‖	" 15 cords of wood at 3.00	45.00		
	" 97lb. of beef, at 6¼ct.	6.06	66	06
	William Appleton,	Dr.		
	To services of my son George, at haying, 16 days, at .75,	12.00		
‖	" use of my team, 5 days,	7.50		
	" cash to balance his account,	8.50	28	00
	July 6th.			
	Charles Williams,	Dr.		
‖	To 25 bushels of oats, at .30,	7.50		
	" 19 " " corn, at .50,	9.50	17	00

	July 7th.			
‖	John Mason,	Cr.		
	By 15 days labor, at repairing barn,		22	50
‖	George Jones,	Dr.		
	To cash in full of his account,		66	06
‖	John Wilson,	Cr.		
	By 5 yards of broadcloth, at 4.00,	20.00		
	" 30lb sugar, at 8cts.	2.40	22	40
	July 8th.			
‖	Henry Thomas,	Cr.		
	By 1 months' time in my service,		15	00
‖	Henry Thomas,	Dr.		
	To 1 pair of boots,	2.00		
	" 1 barrel of flour sent to his mother,	6.00		
	" Cash.	7.00	15	00
‖	John Wilson,	Cr.		
	By cash to balance his account,		42	60

POSTING.

In the Ledger, either a whole page or part of a page, is reserved for each individual, and all the amounts are transferred at the owner's convenience, from the Day-Book to the proper page of the Ledger. When the accounts are thus transferred, they are said to be POSTED, and some mark should be made against every entry in the Day-Book, (as ||), to show that it is posted.

The debits and credits of each individual, are either placed on opposite pages, or on opposite sides of the same page. The difference between the *Dr.* side and the *Cr.* side, is called the *balance.* In opening a new set of books, or in transferring an account to a new page of the Ledger, the balance is debited in one place, and credited in another. For example, if I wish to transfer Charles Williams' account from page 1 to page 15 of the Ledger, I would make the following entry to his credit, on page 1, "By balance transferred to page 15, $17.00," and open the new account on page 15, by the following entry: "To balance of former account $17.00," (or, "To balance from page 1, $17.00.")

A narrow column is usually ruled in the Ledger, (in addition to the columns for the date and amounts,) for inserting the page of the Day-Book on which the charge was first entered.

The Ledger should be provided with an index, alphabetically arranged, in which the name of each individual may be entered, with the page on which his account is posted.

The several charges given in our example of a Day-Book, are posted in the following

LEDGER.

Dr. John Wilson. Cr.

1848.					1848.				
July 4.	To sundries,	1	$65	00	July 7.	By sundries,	2	22	40
					" 8.	" cash,	2	42	60
			$65	00				$65	00

Dr. William Appleton. Cr.

1848.					1848.				
July 5.	To sundries,	1	$28	00	July 4.	By pasturing,	1	$28	00

Dr. George Jones. Cr.

1848.					1848.				
July 7.	To cash in full,	2	$66	06	July 5.	By sundries,	1	$66	06

Dr. Charles Williams. Cr.

1848.									
July 6.	To sundries,	1	$17	00					

Dr. John Mason. Cr.

					1848.				
					July 7.	By labor,	2	$22	50

Dr. Henry Thomas. Cr.

1848.					1848.				
July 8.	*To* sundries,	2	$15	00	July 8.	By labor,	2	$15	00

EXERCISES FOR THE PUPIL.

Make a Ledger and Day-Book of your own, and make the entries already given. Copy the following entries also into the Day-Book, and post them into the Ledger.

July 10th.

Martin Upham, Dr.			
To 9 bushels of beans, at 1.50,		13	50
Charles Williams, Dr.			
To order for groceries, on James Jones,		5	00
Charles Williams, Cr.			
By his note in full of account,		22	00
July 11th.			
Amos Roberts, Dr.			
To 56lb. butter, at .20,	11.20		
" 128lb. cheese, at .08,	10.24	21	44
John Mason, Dr.			
To cash on account,		10	00
July 12th.			
George Jones, Dr.			
To 1 barrel of flour,	6.00		
" 3 barrels of apples, at 1.00,	3 00	9	00
William Appleton, Cr.			
By 40 loads of gravel, at .50,		20	00

FORMS OF NOTES, RECEIPTS, AND BONDS.

1. *Business Note.*

$80 00. Augusta, Sept. 12, 1848.

Four months after date, I promise to pay Messrs. Williamson & Eckley, or order, eighty dollars without defalcation. Value received.

Due, Jan. 12—15. Thomas Ellis.

2. *Note on Interest.*

$100 00. Cincinnati, June 7, 1848.

For value received, I promise to pay James Thomas, or order, one hundred dollars, on demand, with interest from this date.

William K

3. *Note with Security.*

$200 00. Lexington, March 4, 1848.

We jointly and severally promise to pay to Lewis Appleton, or order, on the fifth day of August next, two hundred dollars, with lawful interest, for value received.

George Erwin.
Charles Jones.

4. *Receipt in full.*

Received, Concord, Aug. 18th, 1848, from Benjamin May, the sum of two hundred and fifty dollars, in full of all demands.

$250 00. Samuel Worth.

5. *Receipt on Account.*

Received, Providence, June 11, 1848, from Philip Parsons, the sum of one hundred dollars, on account.

$100 00. Asa Plympton.

6. *Specific Receipt.*

Received, New Haven, Jan. 5, 1848, from Benjamin Harris, the sum of seventy-five dollars, in full for six tons of hay.

$75 00. Joseph Bake.

7. *Bond, common Form.*

Know all men by these presents, that I, Andrew Brown, of the city of Worcester, and State of Massachusetts, merchant, am held and firmly bound unto Stephen Mason, of the said city, gentleman, in the sum of one thousand dollars, good and lawful money of the United States, to be paid to the said Stephen Mason, or his certain attorney, executors, administrators, or assigns; to which payment well and truly to be made, I do bind myself, my heirs, executors, and administrators, and every of them firmly by these presents. Sealed with my seal, dated the first day of June, A. D. one thousand eight hundred and forty-eight.

The condition of this obligation is such, that if the above bound Andrew Brown, his heirs, executors, administrators, or any of them, shall and do well and truly pay, or cause to be paid, unto the above named Stephen Mason, his executors, administrators, or assigns, the just and full sum of five hundred dollars, lawful money aforesaid, with legal interest for the same, on or before the first day of July, in the year of our Lord one thousand eight hundred and fifty, without fraud or further delay, then this obligation to be void and of no effect, or else to remain and be in full force and virtue in law. Andrew Brown, [*Seal.*]

Signed, sealed, and delivered in presence of
James Newton. George Earle.

END OF ARITHMETIC.

Stereotyped by S. DOUGLAS WYETH, No. 7 Pear St. Philadelphia.

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KEY

TO THE

COMMON-SCHOOL ARITHMETIC.

By PLINY E. CHASE, A. M.

WORCESTER, MASS:
PUBLISHED BY A. HUTCHINSON & CO.

Entered according to the Act of Congress, in the year 1848, by
A. HUTCHINSON & CO.,
in the Clerk's Office of the District Court of the United States,
for the District of Massachusetts.

ADVERTISEMENT TO THE KEY.

THE attempt to prevent the use, by scholars, of any work which is designed "for teachers only," nearly always fails, and the author has known of so many instances in which a habit of deceit has been strengthened by such an attempt, that he thinks it best to place the Key in the hands of every pupil. But in order that the learner may find some necessity for self-reliance, the answers to many of the questions have been purposely omitted. It may be well for every teacher to enter all the deficient answers in a blank book, (which can be done with very little labor,) and he can then easily assure himself whether his pupils work understandingly, or are in the habit of *forcing* their results.

Great pains have been taken to make the Key free from error. In Federal Money, the answers are usually given to the nearest cent; and in Sterling Money, to the nearest farthing.

The sign + following a decimal, shows that the decimal may be still further continued, if necessary.

⁂ The Arithmetic will be furnished at the same price without the Key, to such teachers as may prefer to have the answers omitted.

KEY.

The answers which are not given in the Key, are either contained in the body of the work, or may be easily determined by the teacher.

I. NUMERATION.

§ **13**. (119—130.) 40 mill. 117 thous. and 20; 8 mill. 40 thous. and 3; 100 mill. and 900; 1 thous. and 9; 8 mill. 807 thous.; 500 mill.; 400 mill. and 16; 7 bill. 3 mill. 605; ten; 28 mill. 400 thous. 127; 4 trill. 15 bill. 90 mill. 600 thous.; 7 quad. 200 bill. 50 thous. and 7; 19; 5 trill. 649 bill. 380 mill. 75 thous. 109; 50 quint. 34 quad. 700 bill.

§ **17**. (181—190.) 120 mill. 7 thous. and 95; 12 mill. 709, and 5 *tenths;* 1 mill. 200 thous. and 70, and 95 *hundredths;* 120 thous. and 7, and 95 *thousandths;* 12 thous., and 7 thous. 95 *ten-thousandths;* 1 thous. 200, and 7 thous. 95 *hundred-thousandths;* 120, and 7 thous. 95 *millionths;* 12, and 7 thous. 95 *ten-millionths;* 1, and 20 mill. 7 thous. 95 *hundred-millionths;* 120 mill. 7 thous. 95 *billionths.*

§ **19**. (217—222.) 5000000; 900000000; 2000002, 90060004; 200000060; 807009000. (223 — 228.) 8000000000; 2000007015; 10000000020; 1070000000; 200000041000; 1000001010. (229–233.) 7000000000000; 60003000000000; 2000000200000; 508000023000050; 1010000015000. (234 — 240.) 1000000000000100; 2000040000; 6000000000000000006; 7000059; 3004; 200000010; 10000030209000000005000.

§ **21**. (241—250.) .8; .02; .11; .45; .7; 5.2;

.52; .052; 50.0; .009. (251—255.) 70.8; 7.08; .708; 700.008; 700.8. (256—260.) .004; 1.016; 9.0007; .0095; 114.02. (261—268.) 200090; 20009; 2000.9; 200.09; 20.009; 2.0009; .20009; .020009. (269—270.) 704.001005; .0013000063. (271.) .06 & .003; .02 & .007; .04 & .005. (277.) 50; 500; 5000; 50000. (278.) 150; 1500; 15000; 150000; 1500000. (279.) 280; 2800; 28000; 280000; 2800000; 28000000. (280.) 0.2407; 2.407; 24.07; 240.7; 2407; 24070; 240700.

§ 22. (341—345.) $14000003.0047; $1.65; $16509; $26001.005; $105026.00206. (346—350.) $1650000; $ .0165; $5005005.000005; $35019.01001; $23000060001.0000408.

II. ADDITION.

ADDITION TABLE.

1+0= 1	2+0= 2	3+0= 3	4+0= 4	5+0= 5
1+1= 2	2+1= 3	3+1= 4	4+1= 5	5+1= 6
1+2= 3	2+2= 4	3+2= 5	4+2= 6	5+2= 7
1+3= 4	2+3= 5	3+3= 6	4+3= 7	5+3= 8
1+4= 5	2+4= 6	3+4= 7	4+4= 8	5+4= 9
1+5= 6	2+5= 7	3+5= 8	4+5= 9	5+5=10
1+6= 7	2+6= 8	3+6= 9	4+6=10	5+6=11
1+7= 8	2+7= 9	3+7=10	4+7=11	5+7=12
1+8= 9	2+8=10	3+8=11	4+8=12	5+8=13
1+9=10	2+9=11	3+9=12	4+9=13	5+9=14

6+0= 6	7+0= 7	8+0= 8	9+0= 9
6+1= 7	7+1= 8	8+1= 9	9+1=10
6+2= 8	7+2= 9	8+2=10	9+2=11
6+3= 9	7+3=10	8+3=11	9+3=12
6+4=10	7+4=11	8+4=12	9+4=13
6+5=11	7+5=12	8+5=13	9+5=14
6+6=12	7+6=13	8+6=14	9+6=15
6+7=13	7+7=14	8+7=15	9+7=16
6+8=14	7+8=15	8+8=16	9+8=17
6+9=15	7+9=16	8+9=17	9+9=18

§ **25**. (92—100.) 100; 93; 94; 85; 91; 98; 95; 88; 82.

§ **27**. (101—105.) 99; 118; 666; 799; 1099. (106—110.) 148; 148; 541; 761; 1652. (111—115.) 8155; 10719; 6797; 98509; 61501. (116—120.) 176180; 63676; 24360; 19875; 243962. (121—125.) 2980; 3751; 15914; 2152; 2516. (221—225.) 769.83; 7.1734; 1134.579; 32.15; 103.57. (226—230.) 523.877; 7.3699; .01231; 3402.55; 201.4753; (231—235.) 52.8296; 2.048; 1000; 7723.05; 1429.495. (236—240.) $451.05; $977; $1100; $108.412; $529.862.

§ **28**. (241—250.) A. D. 1799; 2091; $852; 440091; 1190160; 446m.; 2300ct.; $15500; 366dy.; 156 strokes. (251—260.) 384A.; $5856; $29.375; $27.75; $10022.875; 189.375yd.; 901bbl. for $4206.625; $26159; $29.875; 10 pieces,—252.25yd.,—$1185.125.

III. SUBTRACTION.

SUBTRACTION TABLE.

1—1=0	2—2=0	3—3=0	4—4=0	5—5=0
2—1=1	3—2=1	4—3=1	5—4=1	6—5=1
3—1=2	4—2=2	5—3=2	6—4=2	7—5=2
4—1=3	5—2=3	6—3=3	7—4=3	8—5=3
5—1=4	6—2=4	7—3=4	8—4=4	9—5=4
6—1=5	7—2=5	8—3=5	9—4=5	10—5=5
7—1=6	8—2=6	9—3=6	10—4=6	11—5=6
8—1=7	9—2=7	10—3=7	11—4=7	12—5=7
9—1=8	10—2=8	11—3=8	12—4=8	13—5=8
10—1=9	11—2=9	12—3=9	13—4=9	14—5=9

6—6=0	7—7=0	8—8=0	9—9=0
7—6=1	8—7=1	9—8=1	10—9=1
8—6=2	9—7=2	10—8=2	11—9=2
9—6=3	10—7=3	11—8=3	12—9=3
10—6=4	11—7=4	12—8=4	13—9=4
11—6=5	12—7=5	13—8=5	14—9=5
12—6=6	13—7=6	14—8=6	15—9=6
13—6=7	14—7=7	15—8=7	16—9=7
14—6=8	15—7=8	16—8=8	17—9=8
15—6=9	16—7=9	17—8=9	18—9=9

§ **30**. (81—90.) 171; 565; 783; 1121; 70; 1000; 3040; 8632; 1350; 80404. (91—100.) 3; 206; 332; 460; 35; 93200; 7000; 6060; 4101; 7674.

§ **32**. (111—120.) 273; 228; 111; 60; 147; 192; 291; 289; 16; 163. (121—130.) 84; 538; 606; 195; 868; 389; 681; 6; 367; 385. (131—140.) 7774; 7149; 6998; 7421; 95; 1682; 7499; 6016; 6294; 2147. (141—150.) 612; 1903; 3897; 692; 8008; 1601; 438; 794; 296; 8086. (151—160.) 389; 899; 118; 1866; 19; 8642; 9713; 10999; 9999; 7097. (161—170.) 113889; 114929; 103080; 31247; 9033; 99999; 99034; 79936; 17728; 68612. (171—180.) 839; 95; 3087; 1925; 23993; 3739; 2284; 3986; 8094; 1898. (181—190.) 56.492; 297.09; .00988; 3608.9; 1.6248; .48077; 50.09; 48.902; 97.62; .6961. (191—200.) 8.036; .06012; .0375; $12.625; 142.75; $ .81; $9.034; $5.11; 4.929; $27.182. (201 — 205.) $9.37; $29.50; 194.06; 12.3456; $26.625. (206 — 215.) $980.375; $69; $620.90; $58.125; $969.67; $528.95; $932.991; $997.94375; $999.999; $166.975. (216—220.) $7.995; $2.875; $90.13; $94.10; $9.96125. (221 — 225.) 591.9827; 15.872; 779.1001; $919897.9375; $6621853.2247.

§ **33**. (229—240.) 7; 8 and 507 over; 19yr.; 1706; —; $3775; —; $275; 213; 11008; 54205; 8. (241 — 250.) 8; 10bu. and $ .25; $231.625; 97.75m.; $224.375; $92.19; $2162.625; $90.75; $5416.125; $852.64.

§ **34**. (301—310.) 2122; 16681; 11303; 217356; 1; 938; 808; 182; 29yr.; 37m. (311—325.) 112; 1703; 9812; 19081; $41; $442.495; $74.72; 23.97338121; 8.0341014; $2.27; 8yd.; $6063.45; 47, —28; 871.75gal.; 8h.

IV. MULTIPLICATION.

§ **38**. (144—152.) 412; 618; 824; 1030; 1236; 1442; 1648; 1854; 0. Sum, 9064. (153—160.) 984; 1476; 1968; 2460; 2952; 3444; 3936; 4428. Sum, *21648*. (161—170.) 0; 6820; 3410; 11935; 1705; *5115;* 10230; 13640; 8525; 15345. Sum, 76725.

MULTIPLICATION TABLE.

2× 0= 0	3× 0= 0	4× 0= 0	5× 0= 0
2× 1= 2	3× 1= 3	4× 1= 4	5× 1= 5
2× 2= 4	3× 2= 6	4× 2= 8	5× 2=10
2× 3= 6	3× 3= 9	4× 3=12	5× 3=15
2× 4= 8	3× 4=12	4× 4=16	5× 4=20
2× 5=10	3× 5=15	4× 5=20	5× 5=25
2× 6=12	3× 6=18	4× 6=24	5× 6=30
2× 7=14	3× 7=21	4× 7=28	5× 7=35
2× 8=16	3× 8=24	4× 8=32	5× 8=40
2× 9=18	3× 9=27	4× 9=36	5× 9=45
2×10=20	3×10=30	4×10=40	5×10=50
2×11=22	3×11=33	4×11=44	5×11=55
2×12=24	3×12=36	4×12=48	5×12=60

6× 0= 0	7× 0= 0	8× 0= 0	9× 0= 0
6× 1= 6	7× 1= 7	8× 1= 8	9× 1= 9
6× 2= 12	7× 2= 14	8× 2= 16	9× 2= 18
6× 3= 18	7× 3= 21	8× 3= 24	9× 3= 27
6× 4= 24	7× 4= 28	8× 4= 32	9× 4= 36
6× 5= 30	7× 5= 35	8× 5= 40	9× 5= 45
6× 6= 36	7× 6= 42	8× 6= 48	9× 6= 54
6× 7= 42	7× 7= 49	8× 7= 56	9× 7= 63
6× 8= 48	7× 8= 56	8× 8= 64	9× 8= 72
6× 9= 54	7× 9= 63	8× 9= 72	9× 9= 81
6×10= 60	7×10= 70	8×10= 80	9×10= 90
6×11= 66	7×11= 77	8×11= 88	9×11= 99
6×12= 72	7×12= 84	8×12= 96	9×12=108

10× 0= 0	11× 0= 0	12× 0= 0
10× 1= 10	11× 1= 11	12× 1= 12
10× 2= 20	11× 2= 22	12× 2= 24
10× 3= 30	11× 3= 33	12× 3= 36
10× 4= 40	11× 4= 44	12× 4= 48
10× 5= 50	11× 5= 55	12× 5= 60
10× 6= 60	11× 6= 66	12× 6= 72
10× 7= 70	11× 7= 77	12× 7= 84
10× 8= 80	11× 8= 88	12× 8= 96
10× 9= 90	11× 9= 99	12× 9=108
10×10=100	11×10=110	12×10=120
10×11=110	11×11=121	12×11=132
10×12=120	11×12=132	12×12=144

(171—180.) 18360; 45900; 0; 9180; 73440; 27540; 64260; 55080; 82620; 36720. Sum, 413100. (181—190.) 241419; 80473; 482838; 724257; 321892; 0; 160946; 643784; 563311; 402365. Sum, 3621285. (191 — 200.) 430556; 123016; 307540; 553572; 184524; 61508; 0; 246032; 492064; 369048. Sum, 2767860. (201—205.) 908590; 763200; 448760; 300590; 270000. (206 — 210.) 341180; 656080; 1592600; 2500000; 1743080. (211—215.) 381000; 916230; 504870; 673050; 2994000. (216 — 220.) 477200; 897400; 762840; 1465680; 2395040. (221—225.) 1030750; 4706050; 1937250; 3415000; 4529350. (226—240.) 1806900; 1388400; 2849580; 5404920; 3461340; 2108050; 1619800; 3324510; 6305740; 4033230; 2409200; 1851200; 3799440; 7206560; 4615120. (241 — 250.) 36630; 60300; 74790; 82350; 543960; 333810; 5041620; 8488890; 7361910; 2067750. (251 — 294.) 479800&c.; 655000&c.; 2754000&c.; 3229600&c.; 4658500&c.; 10920000&c.; 51464000&c.; 84087200&c.; 8980200000&c. (295–300.) 2136240000; 186921000000; 24032700000; 1602180000000; 8010900000000; 1335150000. (301–310.) 18285300000; 12190200000000; 24380400000000; 487608000000; 3657060000000; 30475500000; 426657000000000; 548559000000; 48760800000; 3047550000000.

§ **40**. (311 — 320.) 56378; 64432; 72486; 88594; 96648; 112756; 128864; 140945; 60405; 84567. (321 — 330.) 159381; 194799; 212508; 247926; 259732; 265635; 283344; 289247; 318762; 324665. (331 — 340.) 527408; 593334; 602752; 621588; 678096; 725186; 762858; 791112; 828784; 904128. (341—350.) 2717.05; 4192.02; 4968.32; 7685.37; 8384.04; 8539.30; 6288.03; 9393.23; 10247.16; 11178.72. (351—360.) 2.2869; 3.5574; 2.3716; 4.7432; 2.7104; 4.0656; 6.0984; 4.5738; 2.7951; 7.4536. (361—370.) 61.18; 15.295; 29.4975; 17.48; 68.8275; 38.2375; 69.92; 45.885; 30.59; 117.99. (371 — 380.) 16077.6; 14162.4; 263.76; 152.712; 14.8008; 296.184; 4782.96; 1.6632; 72660; 146.496. (381—390.) 74.9; 68250; 1272.25; 7980; *1711.5*; 8.7325; 364.175; 878775; 16.5725; 193900.

(391 — 400.) 50746.5; 64638; 370440; 4488.75; 79.4745; 649.4985; 52825.5; 7069545; 1936.305; 33660.9.

§ 42. (401 — 410.) 5457; 4173; 6099; 7383; 9309; 9951; 11877; 15087; 18939; 19581. (411—420.) 79871; 111518; 58773; 123574; 91927; 135630; 131109; 143165; 58773; 167277. (421—430.) 487426; 979678; 2446782; 4362704; 2934208; 593598; 3112770; 4242054; 1930400; 3387852. (431—440.) 2258520; 3961980; 6775560; 5445330; 9168060; 4134240; 7703850; 8507730; 3416490; 9560430. (441 — 450.) 54752079; 31513712; 22534401; 65566195; 8305301; 20647173; 28458200; 74800132; 57485564; 32914155. (451 — 460.) 30664280000; 332209240000; 368048600000; 3095006800000; 343718000000; 293627860000; 196583524000; 309303718000; 10574156000; 3858138000000. (461 — 470.) 47237785920; 36609168960; 24090091200; 44317195200; 5830790400; 34861939200; 16247512800; 7288842240; 41380545600; 394859520000.

§ 44. (521—530.) .162; .0392; .6084; .008649; 75.739; 169.88; 1581; .1505; 5.544; 49. (531—540.) 1.5246; .03585; .06183; .032364; .0018438; .00217856; .000084568; .1029; .001296; .93304. (541—550.) $1.677; $ .41925; $2.09625; $1.25775; $5.031; $6.708; $1.048125; $ .005240625; $26.3289; $49.27585. (551 — 560.) 4761; 37.41; $5.625; $2.856; $7.453125; $3.90625; 41400; .00079898; $2967.30; $ .003515625. (561 — 570.) $ .4662; $22.31; $91.806; $8.722; $35.9375; $456.484; $134.995; $ .335353; 48790.44; 135.1224.

§ 45. (571—580.) 4459; 156585; 233600; 256, —125440; 525600; 42560; 336; 1813dy.; 3441; 19, —323. (581 — 590.) 888; 254198; 64428 times; 335699ct.; 1219420866400; $410.625; $39.50; $290.64; $142.625; $738.61. (591 — 600.) 2610; $2979.20; 17.04,—1.1718; $163.5875; $37.734375; $43.134; $42.2875; 180,—3,—7.008,—21; $396.68; $79.89.

§ 46. (601 — 610.) 262; 26280; 700, — 993; 3669; 237; 123oz.,—1977dr.; 2637min.,—158220sec.; 1209; 10092225; 1782862942. (611 — 620.) 384; 11919lb.; $23.92; $18.90; $899; 37m.; 5206916; 3217gall.; 70297; 63998. (621—629.) 792; 447066; 293880; 20377; 583; $231.25; $894.375; $82.25; $2719. (631—639.) 1st share, $125,—2d, $375,—3d, $500, — 4th, $1000, — whole estate, $2000; $73.9375; $136.25; I lose $9.10; $414.4375; $340.20; $1021.875; 3459600; 356400829. (640—645.) 1330.90713856; 381078.125; 537824; 4[illegible]1; .00001801088541; 94.206436. (646 — [illegible]1.) 3486784401; 10000000000000000; [illegible]2057; 1.41158161, 225163944; 832884lb.

V. DIVISION.

DIVISION TABLE.

2÷2=1
4÷2=2
6÷2=3
8÷2=4
10÷2=5
12÷2=6
14÷2=7
16÷2=8
18÷2=9

3÷3=1
6÷3=2
9÷3=3
12÷3=4
15÷3=5
18÷3=6
21÷3=7
24÷3=8
27÷3=9

4÷4=1
8÷4=2
12÷4=3
16÷4=4
20÷4=5
24÷4=6
28÷4=7
32÷4=8
36÷4=9

5÷5=1
10÷5=2
15÷5=3
20÷5=4
25÷5=5
30÷5=6
35÷5=7
40÷5=8
45÷5=9

6÷6=1
12÷6=2
18÷6=3
24÷6=4
30÷6=5
36÷6=6
42÷6=7
48÷6=8
54÷6=9

7÷7=1
14÷7=2
21÷7=3
28÷7=4
35÷7=5
42÷7=6
49÷7=7
56÷7=8
63÷7=9

8÷8=1
16÷8=2
24÷8=3
32÷8=4
40÷8=5
48÷8=6
56÷8=7
64÷8=8
72÷8=9

9÷9=1
18÷9=2
27÷9=3
36÷9=4
45÷9=5
54÷9=6
63÷9=7
72÷9=8
81÷9=9

§ 50. (221—230.) 854; 4057; 3609; 678; 4875; 1687; 2968; 1395; 2269; 4989. (231—240.) 753; 475; 2379; 1378; 1674; 2775; 2339; 744; 2936; 985. (241—250.) 259; 1277; 1756; 2333; 685; 2499; 2192; 1358; 864; 1973. (251—260.) 358; 1643; 1889; 756; 3864; 937; 6541; 17791; 1974; 1359. (261—270.) 1454 + 5; 239 + 1; 983 + 3; 7013 + 2; 12253 + 1; 7458 + 2; 11795; 14958 + 2; 1666 + 3; 7187 + 3. (271—280.) 747 + 6; 1227 + 1; 1394 + 5; 617 + 6; 1712 + 6; 3683 + 5; 2642; 969 + 6; 863 + 3; 7581 + 5. (281—290.) 3138 + 5; 7875; 1125 + 7; 465 + 5; 653 + 3; 582 + 3; 886 + 2; 9149 + 3; 4166 + 5; 8838 + 5. (291—300.) 274 + 2; 1261 + 1; 972 + 1; 2177 + 6; 9333 + 3; 9739 + 3; 3566 + 7; 5417 + 4; 676 + 3; 779 + 4.

§ 52. (301 — 320.) 43520; $29013\frac{1}{3}$; 21760; 17408; $14506\frac{4}{6}$; $12434\frac{2}{7}$; 10880; $9671\frac{1}{9}$; 8704; $7912\frac{8}{11}$; $7253\frac{4}{12}$; $6695\frac{5}{13}$; $6217\frac{2}{14}$; $5802\frac{10}{15}$; 5440; 5120; $4835\frac{10}{18}$; $4581\frac{1}{19}$; 4352; $4144\frac{16}{21}$. (321—330.) $8822\frac{1}{22}$; $8438\frac{11}{23}$; $8086\frac{21}{24}$; $7763\frac{10}{25}$; $7464\frac{21}{26}$; $7188\frac{9}{27}$; $6931\frac{17}{28}$; $6692\frac{17}{29}$; $6469\frac{15}{30}$; $6260\frac{25}{31}$. (331 — 340.) $9941\frac{69}{91}$; $9833\frac{64}{92}$; $9727\frac{89}{93}$; $9624\frac{44}{94}$; $9523\frac{15}{95}$; $9423\frac{92}{96}$; $9326\frac{78}{97}$; $9231\frac{62}{98}$; $9138\frac{38}{99}$; 9047. (341—350.) 9944; $9745\frac{6}{50}$; $9554\frac{2}{51}$; $9370\frac{16}{52}$; $9193\frac{27}{53}$; $9023\frac{14}{54}$; $8859\frac{11}{55}$; $8522\frac{34}{56}$; $8548\frac{20}{57}$; $8400\frac{56}{58}$. (351—360.) $1013\frac{40}{360}$; $1010\frac{110}{361}$; $1007\frac{186}{362}$; $1004\frac{268}{363}$; $1001\frac{356}{364}$; $999\frac{85}{365}$; $996\frac{184}{366}$; $993\frac{289}{367}$, $991\frac{32}{368}$; $988\frac{148}{369}$. (361 — 365.) $1563\frac{139}{158}$; $664\frac{85}{372}$; $405\frac{448}{609}$; $1016\frac{206}{243}$; $1000\frac{93}{247}$. (366—370.) $10022\frac{130}{139}$; $1678\frac{703}{931}$; $2696\frac{292}{391}$; $1846\frac{530}{863}$; $3706\frac{48}{430}$. (371—375.) $50615\frac{501}{1640}$; $22422\frac{2857}{3702}$; $10004\frac{5913}{8297}$; $11019\frac{2974}{7533}$; $20705\frac{2766}{4009}$. (376 — 380.) $19092\frac{1720}{2931}$; $9978\frac{3748}{5608}$; $10021\frac{3108}{3584}$; $5972\frac{2732}{9370}$; $6865\frac{3757}{8151}$. (381 — 385.) $59587\frac{6}{1190}$; $16163\frac{1455}{4387}$; $8403\frac{4022}{8438}$; $7628\frac{8276}{9295}$; $9409\frac{2312}{7536}$. (386 — 390.) $9998\frac{2279}{6701}$; $7968\frac{3933}{8408}$; $8421\frac{1401}{7956}$; $15476\frac{3273}{4329}$; $7091\frac{3108}{9448}$. (391 — 395.)

7496[illegible]; 15299[illegible]; 10019[illegible]; 3428[illegible]; 32267[illegible]. (396—400.) 4023[illegible]; 7747[illegible]; 93627[illegible]; 53979[illegible]; 175122[illegible].

§ 54. (401—410.) 18; 45; 225; 300; 150; 500; 125; 25; 600; 240. (411—420.) 315; 1260; 1575; 2100; 9000; 70000; 5625; 4375; 50000; 840. (421—430.) 35; 14; 55; 220; 50; 275; 87.5; 437.5; 60.15625; 343.75. (431—440.) 48.5; 242.5; 1.2125; 60.625; 3.03125; 7.76; 1.552; 51$\frac{1}{15}$; 570[illegible]; 6.339 +. (441—450.) .007; .0003; .000042; .00168; .00065625; .13125; .0002625; .0056; .000024; .0000125. (451—460.) $60; $3; $ .008; $ .075; $ .50; .5; 3.75; $ .0125; $ .000279 +; .0625. (461—470.) $ .11; 1.1; .55; .05; $ .015625; $ .78125; $195.3125; 550; $ .000859375; .000694.

§ 55. (521—530.) 27m.; 475ct.; 190; 125; 29dy.; 407; 875; 193; 87bu.; 361. (531—540.) 978; 596 + 15; 1097; 1930; 19762p. 3f.; 46507s. 9p.; 22178pd. 7s.; 1183145p.,—98595s.,—4929pd.; 14748min.,—245h.,—10dy.; 47000ft.,—15666yd. (541—550.) 87dy.; 297; 41$\frac{132}{365}$; 3270; 549; 698; 80891; 329pd. 10s. 4p.; $1.75; 79. (551—562.) $439.19; 194; $1.125; $ .94; 27dy.; $1.05; $1.09; 37; 265$\frac{14}{25}$; $ .125; 61$\frac{1}{3}$gal.; $1.61.

§ 56. (563—575.) 738$\frac{26}{113}$; 3963; 17; 52560; 604800; $37.50; 47; 1811$\frac{3}{12}$; $1194.80; $187; 4 and 76; 984ct. lost; $208. (576—590.) $17; 774; 2ct.; 39; 218; 619; 57; 1825; 94; 1369; $65; 39yd.; $3[illegible]; 19; 27 girls, 54 boys. (591—605.) $23; 56; 50 weeks; $19.375; $ .125; 4pd. 10s. 5p.; 588; 79; 5850; 35ct.; 80ct.; $26.25; 32lb.; 56lb.; 15w. 5w. (606—626) 9dy.; 3w.; 160w.; $726.75; $45.75; $128.50; $968.50; $925.45; $70.20; $18; 7; 20; 49dy.; 50bu.; $48; 60225lb.; $18.50; $ .25; $11.50; $853.125; 768. (627—650.) .5; .25; .5; .75; .2; .4; .6; .8; .5; .333 +; .375; .4375; .28125; .44; .666 +; .6363 +; .888 +; .0101 +; .515625; .056; .0703125; .714285 +; .9090 +; .1538 +. (651—675.) 14; 18; 8; 17.[illegible] .8333 +; .888 +;

19.4 ; 4.625 ; .3636 + ; .5833 + ; 2.666 + ; 9.71428 + ; 28.666 + ; .875 ; 5.9375 ; .6875 ; 4.75 ; .2105 + ; 9.588 + ; $3\frac{9}{13}$; .5625 ; $274\frac{2}{19}$; 684 ; 3716.

VI. PRIME AND COMPOSITE NUMBERS.

§ **59**. (21—40.) $2 \times 3 \times 17$; $2 \times 3 \times 5 \times 17$; $2^4 \times 3 \times 17$; $2^2 \times 7 \times 11$; $2^3 \times 3 \times 17$; $3^3 \times 19$; 7^3; $2 \times 3^2 \times 23$; $2^3 \times 89$; $2^5 \times 29$; $2^3 \times 3^4$; $3^3 \times 37$; $2^6 \times 3^2$; $2^2 \times 3 \times 47$; 17^2; $2 \times 3 \times 7^2$; $2^2 \times 193$; $2 \times 7 \times 11$; $2^3 \times 23$; $3 \times 17 \times 19$. (41—60.) $2 \times 13 \times 23$; $2^2 \times 3 \times 23$; $2 \times 3 \times 7 \times 11$; $3 \times 13 \times 17$; $2 \times 13 \times 37$; $2^3 \times 3 \times 5 \times 7$; $2 \times 3 \times 137$; $2^2 \times 3 \times 7 \times 11$; 13×19; $2^2 \times 3^3 \times 5$; $2 \times 3 \times 5 \times 17$; $3 \times 5 \times 61$; $7 \times 11 \times 13$; $2^3 \times 3 \times 7 \times 11$; $2 \times 7^2 \times 19$; $2^2 \times 5 \times 59$; $2^2 \times 3^2 \times 37$; $2^5 \times 3 \times 17$; $2^3 \times 13 \times 19$; $2^2 \times 3^5 \times 7$.

§ **61**. (61—80.) 227 ; 229 ; 233 ; 239 ; 241 ; 251 ; 257 ; 263 ; 269 ; 271 ; 277 ; 281 ; 283 ; 293 ; 307 ; 311 ; 313 ; 317 ; 331 ; 337. (81—90.) 347 ; 349 ; 353 ; 401 ; 409 ; 439 ; 953. (91—100.) 359×1; $2^2 \times 7 \times 13$; 367×1; 373×1; $2 \times 3^3 \times 7$; 379×1; 383×1; $2 \times 7 \times 191$; $2 \times 3^2 \times 241$; $2 \times 11 \times 13 \times 31$.

§ **64**. (101—110.) 6 ; 12 ; 30 ; 30 ; 42 ; 42 ; 24 ; 36 ; 105 ; 120. (111—120.) 60 ; 72 ; 84 ; 54 ; 98 ; 245 ; 231 ; 297 ; 75 ; 385. (121—125.) 72 ; 36 ; 616 ; 840 ; 2145. (126—130.) 120 ; 1980 ; 3276 ; 3059 ; 180. (131—135.) 96 ; 864 ; 420 ; 2052 ; 660. (136—140.) 2520 ; 1386 ; 72 ; 44100 ; 210.

§ **67**. (141—150.) 2 ; 3 ; 13 ; 4 ; 5 ; 9 ; 27 ; 2 ; 7 ; 6. (151—160.) 2 ; 3 ; 5 ; 7 ; 6 ; 10 ; 14 ; 15 ; 12 ; 21. (161—170.) 1 ; 4 ; 9 ; 6 ; 3 ; 14 ; 16 ; 24 ; 42 ; 11. (171—180.) 12 ; 77 ; 81 ; 1 ; 32 ; 107 ; 327 ; 716 ; 96 ; 1. (181—190.) 2 ; 46 ; 25 ; 125 ; 233 ; 74 ; 48 ; 58 ; 106 ; 421. (191—195.) 107 ; 647 ; 421 ; 1 ; 409. (196—200.) 1 ; 1 ; 449 ; 323 ; 339.

§ **69**. (201—210.) 2; $\frac{4}{3}$; 7; 7; $\frac{4}{5}$; $\frac{3}{4}$; $\frac{7}{5}$; 2 : 5; 5 : 1; 4 : 7. (211—220.) $\frac{4}{5}$; 2; $\frac{11}{10}$; $\frac{7}{8}$; $\frac{1}{3}$; $\frac{2}{7}$; 7 : 5; 1; 1; $\frac{1}{3}$. (221—230.) 3 ÷ 2; 3 : 5; $\frac{13}{5}$; $\frac{5}{7}$; 3 : 2; 2 ÷ 5; $\frac{2}{5}$; $\frac{14}{3}$; 30; $\frac{1}{40}$. (231—240.) $\frac{2}{5}$; $\frac{9}{11}$; 7 : 5; $\frac{3}{5}$; $\frac{1}{2}$; $\frac{4}{3}$; $\frac{3}{11}$; $\frac{1}{2}$; $\frac{7}{19}$; $\frac{11}{40}$. (241—250.) $\frac{8}{21}$; $\frac{1}{103}$; 273; $\frac{4}{5}$; $\frac{1}{7}$; $\frac{4}{3}$; $\frac{3}{5}$; $\frac{4}{5}$; $\frac{13}{3}$; $\frac{4}{5}$. (266—270.) 3948; 12900; 21420; 1872; 49049. (271—275.) 806; 1496; 44; 26; 366. (276—295.) 72; 48; 144; 216; 96; 108; 27; 54; 48; 32; 288; 432; 18; 16; 12; 9; 8; 6; 4; 3. (296—315.) 144; 36; 54; 216; 162; 432; 108; 81; 48; 24; 27; 12; 9; 6; 16; 8; 3; 4; 324; 648. (316—340.) 20; 80; 84; 21; 168; 240; 280; 210; 7; 6; 3; 30; 60; 120; 15; 14; 28; 56; 112; 8; 16; 35; 24; 48; 12. (341—360.) 270; 405; 486; 162; 90; 135; 810; 45; 54; 30; 81; 15; 10; 5; 9; 27; 243; 18; 6; 3. (361—390.) 72; 150; 300; 450; 600; 120; 360; 225; 12; 6; 8; 4; 2; 3; 9; 18; 36; 180; 90; 45; 15; 30; 60; 20; 5; 50; 10; 100; 200; 25.

VII. FRACTIONS.

§ **71**. (1—20.) 2; 3; 5; 2; 6; 4; 3; 2; 4; 3; 9; 6; 3; 2; 7; 3; 7; 5; 11; 3. (21—40.) $4\frac{1}{2}$; $2\frac{3}{4}$; $3\frac{2}{3}$; $2\frac{1}{5}$; $2\frac{2}{7}$; $2\frac{6}{8}$; $3\frac{4}{9}$; $4\frac{7}{10}$; $8\frac{10}{11}$; $8\frac{4}{12}$; $6\frac{7}{13}$; $7\frac{13}{14}$; $8\frac{9}{15}$; $30\frac{7}{16}$; $19\frac{6}{17}$; $28\frac{15}{18}$; $44\frac{6}{19}$; $17\frac{13}{20}$; $19\frac{3}{31}$; $14\frac{17}{45}$. (41—60.) .5; .25; .5; .75; .333 +; .666 +; .6; .166 +; .333 +; .5; .666 +; .833 +; .125; .25; .375; .5; .875; .111 +; .222 +; .777 +. (61—80.) .333 +; .444 +; .555 +; .666 +; .888 +; $5\frac{2}{5}$; 13; 21; $44\frac{3}{11}$; $10\frac{29}{97}$; $25\frac{1}{35}$; $35\frac{2}{17}$; 20; 23; 10; $43\frac{9}{44}$; $69\frac{65}{93}$; $75\frac{12}{13}$; 48; 38. (81—100.) .533 +; $3\frac{2}{3}$; .2727 +; $1\frac{2}{3}$; .75; .625; $19\frac{1}{5}$; $14\frac{18}{19}$; .21875; $96\frac{67}{101}$; 16; .6; .214 +; $19\frac{2}{3}$; $30\frac{8}{20}$; .104; .01066 +; $93\frac{3}{4}$; $9\frac{8}{13}$; $771\frac{28}{85}$.

§ **72**. (101—120.) $\frac{36}{2}$; $\frac{54}{3}$; $\frac{72}{4}$; $\frac{90}{5}$; $\frac{108}{6}$; $\frac{126}{7}$; $\frac{144}{8}$; $\frac{162}{9}$; $\frac{180}{10}$; $\frac{198}{11}$; $\frac{216}{12}$; $\frac{360}{20}$; $\frac{450}{25}$; $\frac{540}{30}$; $\frac{720}{40}$; $\frac{900}{50}$; $\frac{1080}{60}$; $\frac{1350}{75}$; $\frac{1800}{100}$; $\frac{18000}{1000}$. (121—140.) $\frac{7}{4}$; $\frac{21}{8}$; $\frac{29}{9}$; $\frac{29}{7}$; $\frac{81}{11}$; $\frac{55}{6}$; $\frac{61}{7}$; $\frac{101}{12}$; $\frac{60}{13}$; $\frac{116}{7}$; $\frac{71}{4}$; $\frac{37}{2}$; $\frac{59}{2}$; $\frac{97}{5}$; $\frac{47}{5}$;

$\frac{118}{4}$; $\frac{339}{4}$; $\frac{128}{13}$; $\frac{63}{29}$; $\frac{22}{75}$. (141—160.) $\frac{1.3}{10}$; $\frac{13}{100}$; $\frac{130}{1000}$; $\frac{1300}{10000}$; $\frac{1.97}{10}$; $\frac{19.7}{100}$; $\frac{197}{1000}$; $\frac{197000}{10000000}$; $\frac{8}{10}$; $\frac{80}{100}$; $\frac{800}{1000}$; $\frac{800000}{10000000}$; $\frac{215}{10}$; $\frac{2150}{100}$; $\frac{21500}{1000}$; $\frac{21500000}{10000000}$; $\frac{.9}{10}$; $\frac{9}{100}$; $\frac{90}{1000}$; $\frac{90000}{10000000}$. (161—180.) $\frac{9}{10}$; $\frac{7}{10}$; $\frac{16}{100}$; $\frac{4}{100}$; $\frac{247}{1000}$; $\frac{13}{1000}$; $\frac{5}{1000}$; $\frac{84}{100}$; $\frac{184}{100}$; $\frac{206}{100}$; $\frac{593}{10}$; $\frac{87}{10}$; $\frac{19}{10000}$; $\frac{802}{10}$; $\frac{802}{100}$; $\frac{3}{10}$; $\frac{62}{100}$; $\frac{119}{100}$; $\frac{4}{100}$; $\frac{83}{1000}$.

§ 73. (201—220.) $\frac{1}{6}$; $\frac{1}{8}$; $\frac{1}{10}$; $\frac{1}{12}$; $\frac{1}{14}$; $\frac{3}{16}$; $\frac{7}{18}$; $\frac{5}{22}$; $\frac{2}{9}$; $\frac{2}{15}$; $\frac{5}{12}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{10}{33}$; $\frac{16}{45}$; $\frac{1}{8}$; $\frac{1}{4}$; $\frac{1}{2}$; $\frac{15}{38}$. (221—230.) $\frac{1}{24}$; $\frac{1}{4}$; $\frac{1}{7}$; $\frac{7}{30}$; $\frac{2}{5}$; $\frac{4}{105}$; $\frac{1}{54}$; $\frac{1}{30}$; $\frac{5}{77}$; $\frac{21}{440}$. (231—240.) $1\frac{5}{18}$; $\frac{7}{40}$; 1; $\frac{5}{8}$; $\frac{5}{6}$; 1; $\frac{5}{7}$; 1; $\frac{3}{14}$; $\frac{1}{28}$. (241—250.) $1\frac{4}{11}$; $3\frac{1}{9}$; $6\frac{3}{7}$; $\frac{20}{143}$; $\frac{1}{8}$; $4\frac{1}{12}$; $\frac{3}{4}$; $1\frac{9}{20}$; $\frac{11}{65}$; $\frac{23}{36}$.

§ 74. (291—310.) $\frac{9}{10}$; $\frac{3}{4}$; $\frac{3}{7}$; $\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{11}$; $\frac{18}{25}$; $\frac{12}{17}$; $\frac{9}{13}$; $\frac{2}{3}$; $\frac{6}{11}$; $\frac{4}{7}$; $\frac{3}{7}$; $\frac{1}{2}$; $\frac{2}{5}$; $\frac{1}{3}$; $\frac{16}{19}$; $\frac{8}{11}$; $\frac{4}{5}$; $\frac{2}{7}$. (311—325.) $\frac{20}{29}$; $\frac{10}{13}$; $\frac{5}{11}$; $\frac{4}{5}$; $\frac{2}{3}$; $\frac{36}{47}$; $\frac{24}{31}$; $\frac{18}{23}$; $\frac{12}{17}$; $\frac{9}{11}$; $\frac{8}{11}$; $\frac{6}{11}$; $\frac{4}{5}$; $\frac{3}{5}$; $\frac{2}{7}$.

§ 75. (326—345.) $\frac{3}{4}$, $\frac{1}{4}$; $\frac{3}{6}$, $\frac{2}{6}$; $\frac{5}{10}$, $\frac{8}{10}$; $\frac{3}{6}$, $\frac{5}{6}$; $\frac{8}{12}$, $\frac{9}{12}$; $\frac{4}{6}$, $\frac{5}{6}$; $\frac{3}{9}$, $\frac{5}{9}$; $\frac{9}{12}$, $\frac{2}{12}$; $\frac{15}{20}$, $\frac{14}{20}$; $\frac{12}{15}$, $\frac{7}{15}$; $\frac{16}{36}$, $\frac{21}{36}$; $\frac{10}{14}$, $\frac{2}{14}$; $\frac{1}{3}$, $\frac{1}{3}$; $\frac{3}{9}$, $\frac{5}{9}$; $\frac{16}{24}$, $\frac{21}{24}$; $\frac{15}{36}$, $\frac{26}{36}$; $\frac{15}{50}$, $\frac{36}{50}$; $\frac{3}{21}$, $\frac{5}{21}$; $\frac{10}{25}$, $\frac{7}{25}$; $\frac{16}{36}$, $\frac{15}{36}$. (346—360.) $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$; $\frac{10}{30}$, $\frac{15}{30}$, $\frac{12}{30}$; $\frac{6}{12}$, $\frac{9}{12}$, $\frac{2}{12}$; $\frac{4}{8}$, $\frac{2}{8}$, $\frac{7}{8}$; $\frac{16}{24}$, $\frac{6}{24}$, $\frac{19}{24}$; $\frac{15}{35}$, $\frac{14}{35}$, $\frac{10}{35}$; $\frac{20}{36}$, $\frac{27}{36}$, $\frac{33}{36}$; $\frac{6}{36}$, $\frac{9}{36}$, $\frac{16}{36}$; $\frac{14}{36}$, $\frac{21}{36}$, $\frac{24}{36}$; $\frac{10}{36}$, $\frac{28}{36}$, $\frac{27}{36}$; $\frac{6}{45}$, $\frac{40}{45}$, $\frac{18}{45}$; $\frac{20}{28}$, $\frac{14}{28}$, $\frac{7}{28}$, $\frac{18}{28}$; $\frac{20}{35}$, $\frac{15}{35}$, $\frac{21}{35}$; $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$; $\frac{21}{24}$, $\frac{6}{24}$, $\frac{16}{24}$. (361—375.) $\frac{33}{44}$, $\frac{12}{44}$; $\frac{20}{36}$, $\frac{15}{36}$; $\frac{21}{24}$, $\frac{14}{24}$; $\frac{25}{30}$, $\frac{9}{30}$, $\frac{16}{30}$; $\frac{7}{42}$, $\frac{27}{42}$, $\frac{16}{42}$; $\frac{49}{70}$, $\frac{40}{70}$, $\frac{18}{70}$; $\frac{10}{20}$, $\frac{15}{20}$, $\frac{18}{20}$; $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{36}{60}$, $\frac{50}{60}$; $\frac{49}{105}$, $\frac{27}{105}$; $\frac{88}{231}$, $\frac{28}{231}$; $\frac{12}{90}$, $\frac{55}{90}$, $\frac{51}{90}$; $\frac{3}{6}$, $\frac{4}{6}$, $\frac{1}{6}$; $\frac{26}{91}$, $\frac{63}{91}$; $\frac{35}{105}$, $\frac{63}{105}$, $\frac{75}{105}$; $\frac{35}{40}$, $\frac{32}{40}$, $\frac{30}{40}$.

§ 76. (376—395.) $\frac{2}{5}$; $\frac{3}{8}$; $1\frac{1}{15}$; $4\frac{4}{5}$; $\frac{3}{4}$; $\frac{1}{3}$; $\frac{1}{12}$; $1\frac{1}{2}$, $\frac{22}{27}$; $\frac{11}{15}$; $1\frac{1}{27}$; $\frac{10}{21}$; $\frac{14}{45}$; $19\frac{3}{5}$; $\frac{1}{33}$; $1\frac{5}{7}$; $2\frac{3}{4}$; $\frac{7}{8}$; $1\frac{8}{15}$; $1\frac{1}{14}$. (396—410.) $\frac{5}{18}$; $\frac{4}{5}$; $\frac{5}{11}$; $\frac{6}{11}$; $\frac{1}{2}$; $\frac{28}{81}$; $1\frac{1}{3}$; 32; 49; 3; $\frac{1}{12}$; $\frac{1}{14}$; $\frac{16}{43}$; 4; $1\frac{1}{5}$. (411—425.) $1\frac{1}{8}$; $\frac{21}{80}$; $\frac{32}{35}$; $\frac{27}{32}$; $\frac{29}{42}$; $\frac{24}{29}$; $1\frac{1}{7}$; $\frac{29}{31}$; $\frac{15}{17}$; $\frac{1}{2}$; $\frac{1}{3}$; 8; $\frac{15}{22}$; $\frac{2}{27}$; $11\frac{23}{24}$.

§ 77. [illegible]

§ 78. (11—20.) $2\frac{1}{12}$; $1\frac{1}{17}$; [illegible]; [illegible]; [illegible]; [illegible]; [illegible]; [illegible]; [illegible]; $18\frac{1}{4}$. (21—30.) 10; [illegible]; [illegible]; [illegible]; [illegible]; [illegible]; [illegible]; [illegible]; [illegible]; 9. (31—40.) [illegible]; [illegible]; [illegible]; $1\frac{4}{15}$; $1\frac{17}{60}$; $2\frac{13}{60}$; $1\frac{3}{14}$; $1\frac{7}{18}$; $1\frac{5}{18}$; $1\frac{3}{28}$. (41—50.) $14\frac{1}{4}$; $4\frac{3}{4}$; $7\frac{1}{2}$; 5; $46\frac{11}{12}$; $20\frac{1}{3}$; $1\frac{17}{21}$; $7\frac{5}{36}$; $15\frac{19}{36}$; $12\frac{11}{18}$; (51—60.) $10\frac{2}{3}$; $12\frac{13}{24}$; $7\frac{7}{24}$; $6\frac{34}{77}$; $2\frac{29}{63}$; $2\frac{13}{60}$; $7\frac{7}{22}$; $15\frac{4}{5}$; $17\frac{5}{18}$; $3\frac{18}{35}$.

§ 79. (71—80.) $5\frac{1}{4}$; $6\frac{3}{4}$; $2\frac{2}{3}$; $4\frac{2}{3}$; $3\frac{4}{5}$; $4\frac{3}{5}$; $\frac{3}{4}$; $1\frac{1}{4}$; $\frac{5}{8}$; $7\frac{7}{8}$. (81—90.) $1\frac{1}{6}$; $7\frac{1}{4}$; $8\frac{1}{3}$; $4\frac{1}{12}$; 6; $7\frac{5}{6}$; $8\frac{3}{4}$; $9\frac{11}{12}$; $5\frac{2}{3}$; $4\frac{7}{12}$. (91—110.) $1\frac{5}{18}$; $4\frac{2}{9}$; $6\frac{1}{6}$; $3\frac{1}{9}$; $3\frac{1}{18}$; $\frac{17}{18}$; $5\frac{1}{3}$; $6\frac{5}{6}$; $8\frac{7}{9}$; $4\frac{13}{18}$; $7\frac{2}{3}$; $5\frac{11}{18}$; $3\frac{5}{9}$; $4\frac{1}{2}$; $8\frac{4}{9}$; $\frac{7}{18}$; $5\frac{5}{12}$; $7\frac{3}{4}$; $4\frac{4}{5}$; $2\frac{13}{15}$. (111—120.) $3\frac{6}{7}$; $\frac{11}{14}$; $4\frac{5}{14}$; $8\frac{3}{8}$; $\frac{13}{30}$; $5\frac{7}{18}$; $\frac{5}{12}$; $7\frac{5}{12}$; $1\frac{17}{20}$; $1\frac{4}{5}$.

§ 80. (131—140.) $\frac{3}{28}$; 1; $2\frac{1}{5}$; $8\frac{17}{18}$; $26\frac{2}{3}$; $5\frac{5}{8}$; $10\frac{1}{2}$; $1\frac{1}{6}$; $31\frac{1}{6}$; $3\frac{2}{3}$. (141—150.) 1; $\frac{2}{3}$; 1; 2; 2; 6; $1\frac{8}{21}$; $15\frac{2}{3}$; $\frac{49}{64}$; $4\frac{5}{8}$. (151—160.) $12\frac{5}{6}$; $9\frac{1}{3}$; 35; 7; 43; 23; 23; 11; 57; $\frac{7}{22}$; $2\frac{1}{2}$. (161—170.) 55; 45; 129; 1; 35; 29; 37; 23; 57; 2. (171—180.) 3; 1; 7; 1; 5; 1; *16*$\frac{1}{3}$; $38\frac{17}{28}$; $3\frac{22}{24}$; $\frac{20}{37}$.

§ 81. (191—200.) 23; $7\frac{2}{3}$; $3\frac{2}{7}$; $2\frac{1}{11}$; $1\frac{5}{18}$; $1\frac{1}{22}$; $1\frac{11}{12}$; $\frac{115}{152}$; $\frac{7}{8}$; $\frac{23}{40}$. (201—210.) $\frac{13}{19}$; $\frac{11}{16}$; $\frac{4}{35}$; $\frac{21}{34}$; $\frac{34}{35}$; $\frac{39}{43}$; $\frac{10}{19}$; $\frac{9}{16}$; $\frac{47}{54}$; $\frac{27}{38}$. (211—220.) $\frac{1}{4}$; $\frac{13}{21}$; $\frac{1}{3}$; 4; $2\frac{4}{35}$; $\frac{5}{7}$; $\frac{17}{32}$; $\frac{2}{21}$; 9; $\frac{32}{61}$. (221—230.) 5; $\frac{410}{623}$; $\frac{24}{55}$; $\frac{19}{56}$; 2; $\frac{1}{3}$; $\frac{1}{3}$; $\frac{105}{248}$; $\frac{27}{50}$; $\frac{9}{11}$. (231—240.) $\frac{5}{168}$; $\frac{169}{288}$; $1\frac{17}{63}$; 208; $\frac{2}{5}$; $\frac{74}{143}$; $\frac{259}{754}$; $1\frac{9}{20}$; $\frac{11}{360}$; $\frac{1253}{9136}$.

§ 82. (241—250.) $.\dot{1}4285\dot{7}$; $.\dot{2}8571\dot{4}$; $.\dot{4}2857\dot{1}$; $.\dot{5}7142\dot{8}$; $.\dot{7}1428\dot{5}$; $.\dot{8}5714\dot{2}$; $.08\dot{3}$; $.41\dot{6}$; $.58\dot{3}$; $.91\dot{6}$. (251—270.) $.1\dot{6}$; $.8\dot{3}$; $.\dot{0}\dot{9}$; $.\dot{2}\dot{7}$; $.\dot{6}\dot{3}$; $.\dot{8}\dot{1}$; $.\dot{9}\dot{0}$; $.0\dot{5}$; $.2\dot{7}$; $3\dot{8}$; $.6\dot{1}$; $.7\dot{2}$; $.9\dot{4}$; $.041\dot{6}$; $.208\dot{3}$; $.291\dot{6}$; $.458\dot{3}$; $.541\dot{6}$; $.708\dot{3}$; $.791\dot{6}$. (271—280.) $\frac{7}{45}$; $\frac{38}{165}$; $5\frac{1}{6}$; $\frac{1}{300}$; $1\frac{3}{110}$; $\frac{41}{220}$; $\frac{2707}{9990}$; $4\frac{34}{45}$; $4\frac{25}{33}$; $\frac{5}{6}$. (281—290.) $7\frac{2}{3}$; $41\frac{1}{3}$; $\frac{83}{100}$; 16; $4\frac{1}{11}$; $5\frac{2}{55}$; $\frac{8}{495}$; $7\frac{88}{99}$; $\frac{388}{4950}$; $\frac{907}{9000}$. (291—300.) $2\frac{10}{11}$; $2\frac{221}{1303}$; $3\frac{88}{705}$; 3; $7\frac{242}{405}$; $\frac{2401}{4455}$; $\frac{17}{8100}$; $35\frac{1}{12}$; $1\frac{13}{27}$; $\frac{9}{770}$.

§ 83. (301—310.) 43; 3; 15; 11; 13; 19; 21; 29; 22; 80. (311—320.) $\frac{112}{4}$; $\frac{196}{7}$; $\frac{308}{11}$; $\frac{420}{15}$; $\frac{616}{22}$; $\frac{1092}{39}$; $\frac{1764}{63}$; $\frac{2240}{80}$; $\frac{2800}{100}$; $\frac{7000}{250}$. (321—330.) $\frac{5}{18}$; $\frac{7}{9}$; 12; $5\frac{8}{11}$; $3\frac{1}{4}$; 1; $2\frac{3}{4}$; $29\frac{5}{7}$; $1\frac{1}{9}$; $33\frac{3}{4}$. (331—340.) $\frac{1}{2}$; $\frac{10}{43}$; $\frac{2}{19}$; $\frac{7}{19}$; $\frac{1}{21}$; $4\frac{1}{2}$; $1\frac{13}{20}$; $\frac{47}{64}$; $\frac{5}{14}$; $13\frac{1}{2}$. (341—350.) $6\frac{1}{8}$; $4\frac{1}{2}$; $10\frac{4}{9}$; $10\frac{7}{8}$; $15\frac{5}{12}$; $8\frac{3}{4}$; $7\frac{7}{9}$; $7\frac{11}{36}$; $1\frac{31}{36}$; $18\frac{1}{15}$. (351—360.) $\frac{27}{55}$; $\frac{6}{7}$; $\frac{25}{44}$; $\frac{1}{20}$; $\frac{33}{196}$; $1\frac{29}{63}$; $2\frac{17}{24}$; $\frac{19}{85}$; $9\frac{1}{3}$; $6\frac{14}{27}$. (361—370.) $5\frac{7}{11}$; $7\frac{4}{13}$; $7\frac{7}{19}$; $10\frac{7}{8}$; $14\frac{14}{15}$; $9\frac{1}{17}$; $42\frac{14}{25}$; $34\frac{13}{29}$; $55\frac{1}{2}$; $32\frac{9}{31}$. (371—380.) $\frac{39}{5}$; $\frac{94}{7}$; $\frac{113}{9}$; $\frac{92}{11}$; $\frac{71}{13}$; $\frac{113}{6}$; $\frac{149}{10}$; $\frac{403}{17}$; $\frac{234}{19}$; $\frac{6051}{56}$. (381—390.) $\frac{2}{3}$; $\frac{9}{26}$; $\frac{3}{4}$; $\frac{7}{11}$; $\frac{7}{19}$; $\frac{87}{401}$; $\frac{339}{524}$; $\frac{5}{18}$; $\frac{109}{181}$; $\frac{41}{247}$. (391—400.) 10; $4\frac{2}{3}$; $23\frac{5}{6}$; $\frac{2}{15}$; $\frac{3}{4}$; $120\frac{1}{3}$; $40\frac{1}{4}$; $\frac{16}{59}$; $\frac{11}{31}$; 21. (401 410.) $1\frac{1}{18}$; $6\frac{4}{9}$; $9\frac{13}{36}$; $\frac{7}{80}$; $4\frac{25}{28}$; $5\frac{7}{10}$; $1\frac{16}{45}$; $2\frac{9}{14}$; $3\frac{19}{60}$; $43\frac{38}{105}$. (411—420.) $\frac{2}{15}$; $2\frac{7}{90}$; $\frac{84}{99}$; $7\frac{62}{99}$; $13\frac{21}{22}$; $\frac{397}{495}$; $\frac{331}{900}$; $1\frac{1}{2}$; $\frac{31}{100}$; $\frac{241}{275}$. (421—430.) $.8\dot{3}$; .2105 +; .2890625; .6923 +; $.4\dot{6}$; $.9\dot{6}$; $.80208\dot{3}$; .0875; .1; $.\dot{0}6\dot{3}$. (431—440.) 9.84, 98.4, 984, 9840; 6.84; 192.3, 1923; 4820; 7.5; 7500. (441—450.) $\frac{42}{105}$, $\frac{50}{105}$; $\frac{32}{616}$, $\frac{91}{616}$; $\frac{70}{231}$, $\frac{66}{231}$; $\frac{147}{42}$, $\frac{36}{42}$, $\frac{33}{42}$, $\frac{16}{42}$; $\frac{45}{99}$, $\frac{88}{99}$; $\frac{52}{63}$, $\frac{40}{63}$; $\frac{21}{70}$, $\frac{5}{70}$, $\frac{28}{70}$; $\frac{143}{187}$, $\frac{99}{187}$; $\frac{66}{117}$, $\frac{45}{117}$; $\frac{60}{105}$, $\frac{70}{105}$, $\frac{42}{105}$. (451—460.) $2\frac{1}{2}$; $2\frac{4}{7}$; $\frac{6}{17}$; $2\frac{2}{3}$; $8\frac{4}{5}$; 24; $\frac{4}{17}$; $4\frac{1}{8}$; $3\frac{4}{7}$; $129\frac{6}{11}$. (461 470.) $22\frac{2}{3}$; $11\frac{2}{3}$; $8\frac{4}{11}$; 32; 59; 1143; 56.4; $38\frac{7}{34}$; 23; $2\frac{7}{13}$. (472—480.) $7\frac{5}{6}$bu.; $2\frac{14}{15}$bu.; 5bu.; 8bu.; $4.75; $$12\frac{2}{3}$; $16.625; $52.25; $27.90625. (481—490.) $5.50; $4.80; $$5\frac{1}{3}$; $2.75; $4; $\frac{8}{15}$h.; $\frac{4}{5}$h.; $\frac{19}{35}$h.; $\frac{16}{25}$h.; $\frac{44}{75}$h. (491—500.) $13\frac{1}{3}$mo.; $11\frac{1}{3}$mo.; 20mo.; $10\frac{1}{3}$mo.; 34mo.; $3\frac{1}{4}$m.; 10m.; $13\frac{3}{4}$m.; $21\frac{2}{3}$m.; $19\frac{1}{3}$m. (501—510.) $$4\frac{5}{16}$; $34-50; $69; $$32\frac{11}{32}$; $40.25; $ $.18\frac{3}{4}$; $ .25; $ $.37\frac{1}{2}$; $ $.31\frac{1}{4}$; $ .20. (511—520.) 4bu.; $2\frac{2}{15}$bu.; $\frac{1}{3}$bu.; $\frac{8}{15}$bu.; $17\frac{7}{8}$bu.; 6ct.; $4\frac{1}{2}$ct.; $5\frac{1}{4}$ct.; $6\frac{3}{4}$ct.; $10\frac{1}{2}$ct. (521—530.) $2\frac{2}{3}$r.; 20r.; 10r.; $17\frac{13}{27}$r.; $21\frac{1}{3}$r.; 1r.; $1\frac{7}{9}$r.; 14r.; $35\frac{5}{9}$r.; $7\frac{11}{27}$r. (531—540.) $36; $234; $171; $310.50; $552; $345; $172.80; $409.50; $204; $235.50.

VIII. COMPOUND NUMBERS.

§ **98.** (161—170.) 300℈; 100ʒ; $12\frac{1}{2}$ʒ; $1\frac{1}{24}$℔; $233\frac{1}{3}$ʒ; $29\frac{1}{6}$ʒ; $2\frac{31}{72}$℔; $62\frac{1}{2}$ʒ; $5\frac{5}{24}$℔; $39\frac{1}{6}$℔. (191—200.) 128; 256; 304; 377; 586; 1280; 1568; 1020; 1911; 2304. (201—210.) 5oz. 10dr.; 12oz. 8dr.; 1lb. 14oz.; 2lb. 4oz.; 3lb. 3oz. 14dr.; 2lb. 13oz. 9dr.; 2lb. 2oz. 9dr.; 3lb. 6oz. 8dr.; 3lb. 10oz.; 3lb. 8oz. 2dr. (211—220.) 3qr. 13lb.; 2qr. 16lb.; 3qr.; 1cwt. 3qr.; 2cwt. 1qr. 18lb.; 2cwt. 3qr.; 5cwt.; 6cwt. 3qr.; 2T. 3cwt. 3qr. 5lb.; 3T. 8cwt. 16lb. (271—280.) 3200; 4480; 1920; 1419; 5105; 2560; 4340; 8320; 25640; 50560. (281—290.) 121; 242; 605; $2389\frac{3}{4}$; $2571\frac{1}{4}$; 9680; 1936; 2904; 6050; 9680. (291—300.) 3; 6; 5; $6\frac{1}{2}$; $10\frac{5}{12}$; 12; $18\frac{23}{24}$; 24; $68\frac{7}{12}$; $60\frac{25}{88}$. (301—310.) 320r., 8R., 2A.; 420r., $10\frac{1}{2}$R., $2\frac{5}{8}$A.; 360r., 9R., $2\frac{1}{4}$A.; 423r., $10\frac{23}{40}$R., $2\frac{103}{160}$A.; 550r., $13\frac{3}{4}$R., $3\frac{7}{16}$A.; 1600r., 40R., 10A.; 2610r., $65\frac{1}{4}$R., $16\frac{5}{16}$A.; 1332r., 33.3R., $8\frac{13}{40}$A.; 1320r., 33R., $8\frac{1}{4}$A.;

490r., $12\frac{1}{4}$R., $3\frac{1}{16}$A. (311—320.) 81; 135; 189; 351; 370; 188; 560; 233; 133; 345. (321—330.) 3456, 8640; 13824; 5324; 32832; 8632; 51840; 138240; 150278; 186624. (331—340.) $1\frac{31}{54}$ft., $\frac{85}{1458}$yd.; $3\frac{403}{864}$ft., $\frac{2995}{23328}$yd.; $3\frac{1}{8}$ft., $\frac{25}{216}$yd.; $31\frac{1}{4}$ft., $1\frac{17}{108}$yd.; $44\frac{307}{432}$ft., $1\frac{7651}{11664}$yd.; $55\frac{155}{864}$ft., $2\frac{10819}{23328}$yd.; $51\frac{1637}{1728}$ft., $1\frac{43109}{46656}$yd.; $53\frac{845}{864}$ft., $1\frac{23309}{23328}$yd.; $72\frac{73}{216}$ft., $2\frac{3961}{5832}$yd.; $51\frac{1647}{1728}$ft., $1\frac{43119}{46656}$yd. (371—380.) 1pk. 1qt. 1pt.; 1bu.; 3bu.; 6bu.; 5bu. 3pk. 1qt.; 2bu. 3pk. 2qt. 1pt.; 8bu. 7qt. 1pt.; 12bu. 3pk. 7qt. 1pt.; 19bu. 2pk. 7qt. 1pt.; 46bu. 1pk. 6qt. (421–430.) 64qt., 16gal.; 370qt., $92\frac{1}{2}$gal., $1\frac{59}{126}$hhd.; 417qt., $104\frac{1}{4}$gal., $1\frac{55}{84}$hhd.; 558qt., $139\frac{1}{2}$gal., $2\frac{3}{14}$hhd.; 690qt., $172\frac{1}{2}$ gal., $2\frac{31}{42}$hhd.; $798\frac{1}{2}$qt., $199\frac{5}{8}$gal., $3\frac{85}{504}$hhd.; 2160qt., 540gal., $8\frac{4}{7}$hhd.; 750qt., $187\frac{1}{2}$gal., $2\frac{41}{42}$hhd.; 1260qt., 315gal., 5hhd.; 2268qt., 567gal., 9hhd. (481—490.) 1825; 4380; 2555; 6935; 9125; 4745; 1820; 3200; 1200; 4800. (491—500.) 2820; 2280; 10800; 61200; 172800; 1036800; 460800; 28740; 119820; 126144000.

§ **100**. (501—510.) 93; 147; $103\frac{1}{2}$; 285; 488; 1368; 2.28; 432; 1715; 3780. (511—520.) 1oz. 12dwt. 1gr.; 8lb. 11oz.; 37lb. 5oz.; 3lb. 3oz. 3dwt.; 6lb. 5oz.; 1lb. 8oz. 9dwt. 11gr.; 8lb. 3oz. 3dwt.; 35lb. 9dwt.; 41lb. 7oz. 19dwt.; 16lb. 8oz. 3dwt. 11gr. (521—530.) 10℥ 4ʒ 2℈ $2\frac{6}{17}$gr.; 7ʒ 12gr.; 18cwt. 3qr. 8lb. 3oz. $12\frac{4}{17}$dr.; 3qr. 20lb. 2oz. 8.96dr.; 24lb. 5oz. 12.16dr.; 2fur. 4r. 1yd. $5\frac{13}{19}$in.; 23r. 2ft. $7\frac{5}{19}$in.; 3qr. $1\frac{5}{7}$na.; 355A. 2R. 8r. 26yd. 8ft.; 13r. 22yd. 8.91ft. (531—540.) $\frac{217}{360}$; $\frac{29}{60}$; $\frac{253}{360}$; $\frac{67}{72}$; $\frac{19}{24}$; $\frac{3}{40}$; $\frac{1}{576}$; $\frac{7}{270}$; $\frac{31}{36}$; $\frac{23}{60}$. (541—550.) .046875; .21875; .828125; .4375; .625; .171875; .265625; .84375; .515625; .703125. (551—560.) 88; 164; 27; 6048; 1890; 261; 12064, 8088; 1764; 4038. (561—570.) 2yd. 2ft. 832in.; 20yd. 3ft.; 5cords 5 cord ft. 9c. ft.; 55tons 25ft.; 235tons 30ft.; 2° 25′ 20″; 156° 40′; 13circ. 116°; 1dy. 3h. 28min. 50sec.; 5y. 46dy. 16h. (571—580.) 10h. 54min. $32\frac{8}{11}$sec.; 34min. $17\frac{1}{7}$sec.; 55′ 48″; 2pk. 6.4qt.; 5gal. 2qt. 1pt. 1.44gi.; 35c. ft.; 1R. 5r. 21yd. 5ft. $66\frac{6}{7}$in.; 23yd. 2ft. $60\frac{12}{13}$in.; 1fur. 16r. 2yd. $1\frac{13}{17}$ft.; 13s. 4d. (581—590.) .3; .075; .15; .525; .6; .95; $.291\dot{6}$; $.\dot{6}$; .8; $.40\dot{5}$. (591—600.) $\frac{127}{320}$; $\frac{1}{384}$; $\frac{7}{3200}$; $\frac{157}{1760}$; $\frac{95}{63360}$; $\frac{151}{63360}$; $\frac{111}{440}$; $\frac{323}{8640}$; $\frac{1}{352}$; $\frac{313}{320}$.

§ **102**. (602—610.) £9 15s.; £7 12s. 4d.; £7 15s. 11d.; £10 18s. 9d.; 9lb. 8oz. 2dwt.; 11lb. 11oz. 16dwt. 17gr.; 1lb. 3oz. 14dwt. 12gr.; 6℔ 11℥ 3ʒ 1℈; 1℔ 6℥ 5ʒ 1℈ 16gr. (611—620.) 3T. 5cwt. 2qr. 6oz.; 1T. 2qr. 13lb. 11oz.; 1qr. 1lb. 11oz. 13dr.; 1T. 1qr. 1lb.; 2m. 4fur. 16r.; 1m. 39r. 4yd.; 7yd. 1ft. 4in.; 22yd. 1qr. 2na.; 1yd. 3qr. 2na.; 8yd. 2qr. 1na. (621—630.) 6m. 72A. 13r.; 60yd. 4ft. 73in.; 7m. 86A. 1R.; 20yd. 14ft.; 32cords; 17bu. 2pk. 2qt.; 1bu. 2pk. 4qt.; 5pk. 5qt.; 1circ. 41° 18′ 26″; 3circ. 152° 35″. (631—640.) 6hhd. 58gal. 1qt. 1pt.; 1hhd. 1pt. 1gi.; 11gal. 2gi.; 1hhd. 45gal. 3qt.; 1hhd. 59gal. 1qt.; 9y. 242dy. 10h.; 1dy. 12h. 17sec.; 13y. 257dy.; 1dy. 18h. 4min.; 7y. 26dy. 9h.

§ **104**. (642—650.) £2 6s. 3d.; 3d. 2qr.; 7s. 4d. 2qr.; £1 19s. 8d.; £2 19s. 11d. 2qr.; 1lb. 8dwt.; 3lb. 11oz. 19dwt. 6gr.; 1lb. 5oz.; 3oz. 8dwt. 6gr. (651—660.) 9℥; 1℔ 10℥ 1ʒ; 1℥ 2ʒ 2℈ 5gr.; 1qr.; 1cwt. 1qr. 1lb.; 3qr. 18lb. 8oz. 1dr.; 19cwt. 3qr. 23lb. 2oz. 11dr.; 1T. 16cwt. 2qr.; 1m. 8r.; 7fur. 39r. 4yd. 2ft. 6in. (661—670.) 5yd. 1in.; 2m. 2fur. 29r.; 1yd. 3qr. 3na.; 2yd. 3qr. 1na.; 5yd. 2qr. 2na.; 4sq. m. 639A. 3R. 35r.; 636A. 3R.; 23yd. 6ft. 41in.; 26ft. 88in.; 1bu. 5qt. 1pt. (671—680.) 3pk. 1pt.; 1pt.; 1qt.; 8gal. 2qt. 1pt.; 300° 31′ 33″; 82y. 8mo. 3dy.; 4y. 10mo. 13dy.; 6y. 9mo. 28dy.; 111y. 2mo. 11dy.; 282y. 6mo. 8dy.

§ **109**. (771—780.) 127yd. 1qr. 1na.; £250 4s. $6\frac{1}{4}$d.; 12cwt. 3qr. 21lb. 11oz.; 92bu. 6qt. 1pt.; 12m. 3fur. 38r. 2yd. 2ft.; 12T. 11cwt. 1qr. 14lb. 13oz.; 86bu. 2pk. 1qt. 1pt.; 176m. 6fur. 39r.; 152lb. 5oz. 7dwt. 18gr.; 3yd. 2qr. $1\frac{1}{5}$na. (781—790.) 2oz. 16dwt. $18\frac{1}{6}$gr.; 23m. 26r. 3yd. 2ft.; 1bu. $3\frac{3}{13}$qt.; $4\frac{156}{1543}$; 4lb. 1oz. 3dwt. 23gr.; 516gal. 1qt. 1pt.; 12yd. 1na.; £49 16s. 11d. 1qr.; 2cwt. 11lb. $5\frac{5}{47}$oz.; 4179gal. 2gi. (791—800.) $1180\frac{17}{32}$; £115 4s. $8\frac{1}{4}$d.; 5℔ 9℥ 1ʒ 9gr.; 1℔ 1℥ 6ʒ 2℈ 13gr.; 5A. 3R. 10r.,—143A. 3R. 30r.; 15 cords,–1cd. 7cd. ft.,–13cd. 1cd. ft.; 107 gal. 2qt. 1pt.; 12s. 8d. $2\frac{2}{31}$qr.; 35 cords 8cub. ft.; 1A. 1R. $2\frac{14}{33}$r. (801–815.) In 334dy.; 47T. 11ft. 864in.; 44m. 1r.; *14y. 8mo.* 1dy.; 28y. 1mo. 13dy.; 3y. 5mo. 15dy.; *789.12 ; 1T.* $6\frac{5}{19}$c. ft.; 1cwt. 3qr. = 196lb.; $\frac{7}{8}$; [illegible];

$\frac{3}{8}$; $\frac{15}{16}$; $\frac{7}{15}$. (816—830.) $.46458\dot{3}$; .375; $.\dot{1}3\dot{6}$; $\frac{7}{8}$; .84375; 1qr. 21lb.; 9s. 7.44d.; 11mo. $2\frac{44}{53}$dy.; 17gal. $1\frac{5}{11}$pt.; 5h. 20min.; \$ .375; 5fur. 3r. 3yd. 1ft. 6in.; 10oz. $5\frac{5}{7}$dwt.; 3qr. 3lb. 11.136oz.; 5mo. 15dy. (831—840.) $93\frac{657}{1792}$; $13\frac{3409}{9000}$; \$7524.66; 1lb. 2oz. $5.085\dot{2}$dr.; £514 10s. 7d.; 456bu.; 96264000; $65\frac{1123}{1472}$ct.; \$98.50; \$14.27. (841—850.) \$371.31$\frac{1}{4}$; 5460; 37; 1s. $1\frac{17}{48}$d.; 22A. 2R. $13\frac{3}{5}$r.; 18yd. 3qr. $1\frac{7}{168}$na.; \$$1\frac{6}{63}$; \$1.33$\frac{1}{3}$; $88\frac{23}{45}$; \$2744.625.

IX. PERCENTAGE.

§ **111**. (11—20.) 9; 33.75; .1431; .00405; 6.3; 2.9115; 442.35; 540; .3915; 6.885. (21—30.) .2475; .7425; 2.2; 8.25; 19.25; .0026125; 26.125; 55; .91$\frac{2}{3}$; .34375. (31—40.) .24; .36; .0052; .18125; .0096875; 1.25; 2.1875; .134375; $\frac{1}{8}$; .09$\frac{3}{8}$. (41—50.) \$156.25; \$208.33$\frac{1}{3}$; \$312.50; \$416.66$\frac{2}{3}$; \$625; \$833.33$\frac{1}{3}$; \$937.50; \$1666.66$\frac{2}{3}$; \$2187.50; \$2500. (61—70.) 46.3925; \$13.75; \$9.35; \$ .031625; 22.3575; .0026125; 129.8; \$ .462; 247.5; 1.98. (71—90.) 50; $62\frac{1}{2}$; $83\frac{1}{3}$; $41\frac{2}{3}$; $6\frac{1}{4}$; $18\frac{3}{4}$; $31\frac{1}{4}$; $43\frac{3}{4}$; $56\frac{1}{4}$; $68\frac{3}{4}$; $81\frac{1}{4}$; $93\frac{3}{4}$; 28; $\frac{8}{10}$; 20; $\frac{4}{25}$; 92; $77\frac{7}{9}$; $42\frac{6}{7}$; $88\frac{4}{17}$. (91—100.) 10.88; 47; 12.5; 85; 11.57; 2.1875; 15; .1075; 1.224; 40.48.

§ **112—117**. (101—110.) \$88.36; \$1875.14; \$9231.67; \$130.955; £3 16s. $8\frac{3}{8}$d.; £2 17s. $9\frac{6}{25}$d.; \$215.49; \$20.25; £90 10s. 8.7d.; \$153.41. (111—121.) \$71; \$125.835; \$40.375; \$60; \$24.28; \$3145.50; \$2813.25; \$4025; \$11569.69; \$11875, \$7663$\frac{1}{3}$, \$4750; \$121.50. (122—127.) \$17.29; \$35.25; \$5.62; \$137.50; \$157.61; \$52.42. (128—140.) 20; \$11.25, or 20 per cent.; 15; \$ .08$\frac{1}{3}$; $10\frac{5}{7}$; $10\frac{5}{7}$; \$22.915 = $15\frac{1}{2}$ per cent. *nearly*; 25; 25; $20\frac{5}{8}$; 25; $34\frac{2}{5}$; $14\frac{2}{7}$.

§ **118—120**. (181—190.) \$32.83; \$65.66; \$229.79; \$147.72; \$218.85; \$111.25; \$14.92; \$20.52; \$192.86; \$8.66. (191—200.) \$53.66; \$120.73; \$404.66; \$40.24; \$50.68; \$304.05; \$203.89; \$218.20; \$460.55; \$347.27. (201—210.) \$18; \$33; \$5.95;

$43.50 ; $ .75 ; $14.25 ; $3.66⅔ ; $78 ; $3.50 ; $30.08⅓. (211—220.) $28.50 ; $53.04 ; $36.10 ; $15.83 ; $95 ; $308.75 ; $468.35 ; $151.21 ; $47.50 ; $516.80. (221—230.) $4 ; $9 ; $27 ; $54⅓ ; $410 ; $210 ; $80 ; $75 ; $743 ; $590. (231—240.) $3.75 ; $2.34 ; $9.375 ; $79.69 ; $318.75 ; $375 ; $156.61 ; $17.19 ; $62.34 ; $150. (241—250.) $252 ; $15 ; $450 ; $29.40 ; $130.50 ; $105 ; $31 ; $14.70 ; $225 ; $192.30. (251—260.) $2.75 ; $5.25 ; $7.75 ; $10.25 ; $7.50 ; $12.50 ; $17.50 ; $27.50 ; $16.50 ; $12.25. (261—270.) $13.30 ; $16.625 ; $47.25 ; $75.25 ; $27.30 ; $11.90 ; $19.95 ; $94.50 ; $3.675 ; $16.22. (271—280.) $1.83 ; $1.24 ; $2.41 ; $3.50 ; $1.75 ; $11.08 ; $23.92 ; $15.38 ; $2.625 ; $4.63. (281—290.) $1.30 ; $ .89 ; $1.72 ; $2.50 ; $1.25 ; $7.92 ; $17.08 ; $10.98 ; $1.875 ; $3.31. (291—300.) $3.67 ; $7 ; $10.33 ; $13.67 ; $10 ; $16.67 ; $23.33 ; $36.67 ; $22 ; $16.33.

§ 122—127. (301—310.) $202.17 ; $105.47 ; $814.12 ; $223.70 ; $167.78 ; $419.44 ; $559.26 ; $41.66 ; $104.58 ; $720.32. (311—320.) $189.11 ; $203.65 ; $211.88 ; $196.26 ; $189.04 ; $107.75 ; $85.56 ; $209.26 ; $110.85 ; $69.18. (321—330.) 5 ; 4½ ; 6 ; 5 ; 4 ; 4½ ; 7 ; 8 ; 5⅓ ; 6¼. (331—340.) 4y. ; 2y. 8mo. ; 4mo. 24dy. ; 11mo. 10dy. ; 16⅔ ; 25 ; 20 ; $14\frac{2}{7}$; $18\frac{2}{11}$; $14\frac{22}{27}$. (341—350.) $1666⅔ ; $27500 ; $19875 ; $26250 ; 8727\frac{3}{11}$; 7272\frac{8}{11}$; 13090\frac{10}{11}$; 29090\frac{10}{11}$; 5133\frac{129}{187}$; 7480\frac{40}{77}$. (351—360.) $500 ; $1200 ; $2475 ; $4923 ; $1834.86 ; $1785.71 ; $1673.64 ; $1913.88 ; $1888.57 ; $1837.95. (361—370.) $85.52 ; $114.49 ; $343.75 ; $38.22 ; $160.71 ; $64.59 ; $123.85 ; $61.84 ; $244.77 ; $111.11. (371—391.) $1989 ; $492.25 ; $372.94 ; $185.53 ; $734.625 ; $455.58 ; $493.54 ; $373.28 ; $185.86 ; $737.19 ; $456.77 ; $490.96 ; $372.59 ; $185.20 ; $732.06 ; $454.38 ; $489.67 ; $372.25 ; $184.87 ; $729.50 ; $453.19. (392—400.) $8705.89 ; $1190.025 ; $962.72 ; $105.22 ; £63 4s. 3½d. ; $ .05 ; $365⅚ ; $465.42 ; $571.29.

X. ANALYSIS AND RATIO.

§ **129.** (1—10.) 30w.; 27w.; 45w.; 54w.; 9w.; 10w.; 6w.; 5w.; 3w.; 2w. (11—20.) 210; 180; 42; 90; 84; 60; 126; 45; 30; 20. (21—30.) 24; 9; 30; 6; 18; 27; 12; 21; 180; 42. (31—40.) \$58.50; \$121.50; \$378; \$238.50; \$223$\frac{7}{8}$; \$30$\frac{15}{16}$; \$445.50; \$168.75; \$15$\frac{3}{16}$; \$4.22. (41—50.) 18dy.; 39dy.; 13dy.; 9dy.; 6dy.; 6$\frac{1}{2}$dy.; 9$\frac{3}{4}$dy.; 4$\frac{1}{3}$dy.; 3dy.; 2dy. (51—60.) 4; 6; 2; 2.4; 1$\frac{1}{2}$; 2$\frac{10}{19}$; 12.8; 3$\frac{3}{31}$; 2$\frac{2}{11}$; 2$\frac{2}{3}$. (61—70.) 9$\frac{21}{23}$; 8$\frac{1}{7}$; 7$\frac{1}{8}$; 14$\frac{1}{4}$; 8$\frac{10}{13}$; 7$\frac{5}{21}$; 17$\frac{7}{18}$; 12$\frac{3}{14}$; 6$\frac{1}{3}$; 7$\frac{3}{5}$. (71—80.) \$18.75; \$43.75; \$34.375; \$39$\frac{1}{16}$; \$73$\frac{7}{16}$; \$118.75; \$593.75; \$111.875; \$51$\frac{9}{16}$; \$5$\frac{55}{64}$. (81—90.) 15$\frac{1}{5}$; 19; 38; 12$\frac{2}{3}$; 6$\frac{1}{3}$; 8$\frac{4}{5}$; 25$\frac{1}{3}$; 10$\frac{6}{7}$; 7.6; 6. (91—100.) 15ft.; 22$\frac{1}{2}$ft.; 11$\frac{1}{4}$ft.; 3$\frac{3}{4}$ft.; 8$\frac{3}{4}$ft.; 11$\frac{1}{8}$ft.; 6$\frac{3}{32}$ft.; 13$\frac{3}{4}$ft.; 33$\frac{3}{4}$ft.; 20$\frac{5}{6}$ft.

§ **131.** (101—110.) 7 : 9; 15 : 3; &c. (111—120.) $\frac{3}{5}$; $\frac{11}{2}$; &c. (131—140.) 1 : 2; 1 : 5; 3 : 4; 3 : 5; 3 : 1; 9 : 1; 5 : 3; 3 : 4; 11 : 6; 5 : 9. (141—150.) $\frac{3}{100}$; $\frac{3}{8}$; $\frac{1}{10}$; $\frac{72}{7}$; $\frac{1}{10}$; $\frac{99}{16}$; $\frac{18}{1}$; $\frac{1}{8}$; $\frac{1}{6}$; $\frac{2}{1}$. (151—160.) 167 : 690; 2220 : 1; 27 : 400; 500 : 67; 2 : 21; 19 : 3360; 16 : 3; 12 : 1297; 32 : 5; 451 : 14. (171—180.) $\frac{39}{5561}$; $\frac{114}{5561}$; $\frac{576}{5561}$; $\frac{3252}{5561}$; $\frac{17}{55610}$; $\frac{8640}{5561}$; $\frac{5400}{5561}$; $\frac{126}{5561}$; $\frac{20448}{5561}$; $\frac{1638}{5561}$.

§ **133.** (181—190.) 200m.; 800m.; 40m.; 220m.; 190m.; 174m.; 155m.; 241m.; 17$\frac{1}{2}$m.; 18$\frac{3}{4}$m. (191—200.) 48; 72; 32; 144; 96; 24; 16; 18; 4; 8. (201—210.) \$5.60; \$4$\frac{2}{3}$; \$12$\frac{4}{9}$; \$28; \$11$\frac{2}{3}$; \$6; \$10; \$16; \$4.59; \$11.45. (211—220.) \$28.89; \$23.33; \$87.78; \$212.72; \$16.39; \$45.56; \$36.28; \$2.40; \$360; \$7.09. (221—230.) £22 10s.; £19 13s. 9d.; £8 2s.; £6 8s. 10$\frac{1}{2}$d.; £100; £22 5$\frac{1}{4}$d.; £78 15s.; £140 12s. 6d.; £22 2s. 5$\frac{3}{4}$d.; £8 8s. 9d. (231—240.) 356bu. 1pk.; 337bu. 2pk.; 393bu. 3pk.; 365bu. 2pk. 4qt.; 440bu. 2pk. 4qt.; 403bu. 4qt.; 450bu.; 412bu. 2pk.; 431bu. 1pk.; 421bu. 3pk. 4qt. (241—250.) 123ft. 9in.; 82ft. 6in.; 165ft.; 132ft.; 103$\frac{1}{8}$ft.; 156ft. 9in.; 138$\frac{3}{16}$ft.; 198ft.; 144$\frac{63}{80}$ft.; 163.35ft.

§ **135.** (251—260.) £28 2s. 6d.; £31 5s.; £3

\$250 ; A.'s \$58.50, B.'s \$45.50, C.'s \$65 ; \$156, \$312, \$468 ; 70, 42, 56, 28 ; A. \$24, B. \$12, C. \$36 ; A. \$56.19, B. \$233.33, C. \$210.48 ; $\$\frac{53}{78}$; A. \$160, B. \$80, C. \$40 ; Widow \$400, Eldest Son \$480, Youngest Son \$400, Daughter \$320 ; 150, 100, 225, 175. (221—230.) A. \$222.22, B. \$277.78 ; A. \$100, B. \$96, C. \$80 ; A. \$78, B. \$93.60, C. \$62.40 ; A. \$195 ; B. \$130 ; C. \$234 ; A. \$585.60, B. \$549, C. \$457.50 ; A. \$52.50, B. \$17.50, C. \$21, D. \$63, E. \$140 ; A. \$10.91, B. \$13.64, C. \$15.45 ; A. \$1.80, B. \$1.92, C. \$2 ; A. \$45, B. \$50, C. \$67.50 ; A. $\$9\frac{3}{8}$, B. $\$11\frac{1}{4}$, C. $\$6\frac{7}{8}$, D. $\$13\frac{1}{4}$.

XII. AVERAGE.

§ 165—171. (31—40.) $2\frac{5}{16}$; $11\frac{7}{12}$; $6\frac{5}{8}$; $4\frac{3}{8}$; $12\frac{5}{16}$; 8.53 ; $35\frac{413}{600}$; $1\frac{77}{240}$; 265.5 ; $56\frac{1}{4}$. (41—45.) $31\frac{3}{4}$yd. ; $173\frac{3}{4}$lb. ; 61 tons of hay, 35bu. of grain ; $\$1.17\frac{1}{2}$ per bu. ; 12yr. 7mo. 6dy. (46—50.) \$ .30 ; $22\frac{3}{8}$ carats ; 10oz. ; $71\frac{5}{12}$° ; 7ct. (51—60.) 4mo. 5dy. ; 80dy. ; 18mo. 4dy. ; 49dy. ; March 10 ; Apl. 17 ; \$59.03 ; \$42 ; Oct. 26 ; June 26 ; In 19dy. (61—68.) May 29 ; Jan. 24 ; Jan. 20 ; Feb. 18 ; May 22 ; Sept. 4 ; Apl. 5 ; May 12. (69—80.) 1 at 5ct., 2 at 8ct. ; 2 at 5ct., 1 at 8ct. ; 5 at 5ct., 1 at 8ct. ; 1 at 5ct., 1 at 8ct. ; 1 at 5ct., 5 at 8ct. ; 5 at 5ct., 7 at 8ct. ; 1 at 5ct., 3 at 8ct. ; 2 at 70ct., 1 at \$1 ; 1 at .70, 2 at \$1 ; 5 at .70, 1 at \$1 ; 1 at .70, 5 at \$1 ; 1 of each. (81—90.) 2bu. at .40, 1bu. at 70 ; 5 at .40, 1 at .70 ; 1 at .40, 2 at .70 ; 1 at .40, 5 at .70 ; 1 of each ; 5lb. at .30, 4lb. at 75 ; 5 at .30, 13 at .75 ; 5 at .30, 1 at .75 ; 2 at .30, 1 at .75 ; 7 at .30, 2 at .75. (102—110.) 20gal. ; $13\frac{7}{11}$gal. ; $24\frac{2}{3}$gal. ; $37\frac{1}{2}$gal. ; $13\frac{1}{3}$gal. ; 40gal. ; 120gal. ; 20gal. ; $26\frac{2}{3}$gal. (126—130.) 60gal. ; 120gal. ; 15gal. ; 6gal. ; $34\frac{2}{7}$gal.

XIII. INVOLUTION AND EVOLUTION.

§ 173—179. (1—10.) 256 ; 64 ; 16 ; 1 ; .125 ; 1.5625 ; .00004096 ; 19.683 ; 130321 ; 13.0321. (81—100.) 11 ; 15 ; 20 ; 19 ; 12 ; 17 ; 14 ; 16 ; 13 ; 18 ; 25 ; 30 ; 21 ; 27 ; 22 ; 29 ; 23 ; 24 ; 26 ; 28. (101—

120.) 3.1; .32; 3.7; 40; 50; 3.3; .39; 36; 3.4; 38; $4\frac{1}{2}$; 41; 4.9; .43; 4.7; 42; 44; 4.6; 4.8; $3\frac{1}{2}$. (121—140.) $\frac{2}{3}$; $\frac{4}{5}$; $\frac{1}{2}$; $\frac{8}{7}$; $\frac{8}{3}$; $2\frac{2}{3}$; $2\frac{1}{2}$; $\frac{9}{12}$; $3\frac{2}{3}$; $3\frac{1}{4}$; $2\frac{2}{5}$; $3\frac{1}{6}$; $1\frac{5}{7}$; $2\frac{1}{8}$; $8\frac{1}{3}$; $15\frac{1}{2}$; $5\frac{5}{7}$; $\frac{1}{17}$; $\frac{9}{48}$; $12\frac{6}{7}$. (141—160.) 1.41; 2.64; 3.60; 5.38; 2.40; 7.93; 11.83; 2.76; 9.97; 24.49; 4.89; 1.03; 21.93; 6.14; 19.41; .61; 3.31; 6.63; 13.26; 5.47. (161—170.) 12r.; 13r.; 9r.; 41r.; 4.8r.; 14r.; $3\frac{1}{2}$r.; $8\frac{1}{3}$r.; $15\frac{1}{2}$r.; 38r. (171–180.) 1.49; 2.449; .945; 269.91; 50.7; 106.67; 999; 9.949; 1230; 8072. (181—200.) .547; .447; .894; 1.095; 1.341; 1.643; .353; .707; 1.060; .176; .530; .117; .235; .471; .935; .589; .942; 1.178; .883; .824.

§ **183**. (201—220.) 11; 20; 30; 12; 19; 40; 13; 18; 50; 14; 17; 21; 16; 22; 15; 23; 29; 24; 31; 28. (221—240.) 25; 33; 3.7; 41; 26; 35; 4.4; 46; 27; .48; 4.2; 39; 32; 4.9; 36; .45; 38; .47; 4.3; .34. (241—260.) $\frac{2}{3}$; $\frac{1}{4}$; $\frac{5}{6}$; $3\frac{1}{2}$; $2\frac{2}{3}$; $\frac{5}{9}$; $\frac{1}{10}$; $2\frac{3}{4}$; $\frac{9}{20}$; $2\frac{2}{5}$; $\frac{2}{30}$; $1\frac{1}{5}$; $4\frac{1}{2}$; $5\frac{1}{2}$; $4\frac{1}{3}$; $2\frac{4}{5}$; $6\frac{2}{3}$; $10\frac{1}{2}$; $9\frac{1}{2}$; $4\frac{1}{5}$. (261—280.) 1.25; 1.91; 2.35; 3.07; 1.79; 3.97; 5.19; 1.97; 4.63; 8.43; 2.88; 1.02; 7.83; 3.35; 7.22; .72; 2.22; 4.44; 8.89; 3.10. (281—290.) 12in.; 11in.; 19in.; $10\frac{1}{2}$in.; $4\frac{1}{2}$in.; 2.8in.; 43in.; 49in.; $4\frac{1}{5}$in.; 51in. (291 — 300.) 13.2; 2.3513; 8300; 9.27; 3.6593; 4.0207; .4344; .9026; 10.09; 7.806; 9.84. (301—320.) .952; .793; .634; .476; .317; .158; .119; .111; .170; .341; .512; .683; .854; .5; .793; 1.040; 1.5; 2; $2\frac{1}{2}$; 35.

XIV. PROGRESSION, OR SERIES.

§ **198 — 206**. (131 — 140.) 320; $177.147; 743008370688 grains; $\frac{256}{4782969}$; $265.72; 19682; $13\frac{121}{243}$; 8; $6\frac{2}{3}$; $20\frac{68}{81}$. (141—159.) $12.76; $4.26; $27.66; $510.51; $718.93, $750.34, $771.78; $857.34, $947.29, $1011.64; $1116.48, $1343.74, $1518.20; $808.65, $876.81, $924.84; $1002.22, $1164.90, $1286.30. (160—174.) $242.95, $184.56, $140.56; $87.20, $54.29, $33.95; $154.67, $107.62, $75.14; $436.30, $371.36, $316.57; $559.50, $499.80, $44

(175—189.) 23yr. 5mo. 11dy., 14yr. 2mo. 13dy., 11yr. 10mo. 21dy.; 37yr. 1mo. 29dy., 22yr. 6mo. 4dy., 18yr. 10mo. 6dy.; 14yr. 5mo. 15dy., 8yr. 9mo. 2dy., 7yr. 3mo. 29dy.; 48yr. 3dy., 29yr. 1mo., 24yr. 4mo. 5dy.; 6yr. 2mo., 3yr. 8mo. 24dy., 3yr. 1mo. 15dy. (190—200.) 6; 5; 4; 7; 4½; 3; 7; 4½; 6; 4; 5. (201—210.) $7905.82; $1318.08; $7363.98; $2327.60; $15476.20; $29033.59; $13590.42; $1888.21; $6370.58; $21274.35. (211—220.) $3663.90; $1418.625; $6393.05; $14879.68; $9710.35; $12659.57; $7970.84; $2486.53; $4961.95; $51342.76. (221—230.) $21561; $7233.53; $30509.78; $63381.04; $11265.93; $55329.32; $77196.67; $23621.85; $16000.95; $28089.84. (231—240.) $4196.92; $7485.82; $145167.95; $39529.09; $36819.90; $31852.88; $55717.39; $42400.84; $23497.91; $93753.79.

§ 208—215. (241—250.) $3030.64; $4524.25; $5814.49; $8723.55; $4050.46; $5405.21; $1906.28; $7265.76; $3552.98; $4975.47. (251—253.) $28.41; $40.46; $21.35. (261—270.) $20453.38; $11776.14; $4276.82; $17324.16; $18351.87; $9935.66; $15539.60; $22023.73; $12618.99; $18822.53. (271—280.) $3797.79; $4126.05; $3506.88; $3956.90; $3648; $3248.28; $7026.31; $5018.80; $3011.28; $6524.43. (281—290.) 3333\frac{1}{3}$; 7111\frac{1}{9}$; 1111\frac{1}{9}$; 1388\frac{8}{9}$; 2083\frac{1}{3}$; 4444\frac{4}{9}$; 22222\frac{2}{9}$; 666\frac{2}{3}$; 3888\frac{8}{9}$; 2777\frac{7}{9}$. (291 — 300.) $5625; $11250; 6428\frac{4}{7}$; 8181\frac{9}{11}$; $5000; $3600; $7500; $9473.68; $10000; $4500. (301 — 310.) $12500; 10714\frac{2}{7}$; $9375; 8333\frac{1}{3}$; $15000; 13636\frac{4}{11}$; $10000; 16666\frac{2}{3}$; $18750; $11538.46. (311—313.) £12500; £3750; £5625. (321—330.) $5213.86; $2606.93; $1564.16; $782.08; $1303.46; $881.14; $443.18; $2241.96; $2487.01; $62.57. (331—340.) $2145.97; $1992.96; $1147.45; $870.30; $656.72; $285.83; $1380.16; $147.08; $1633.43; $2118.83. (341—350.) $5442.18; $4701.16; $4061.04; $4264.09; $2886.10; $3508.08; $3867.65; $5183.03; $4477.29; $2617.78. (351—360.) $3209.70; $3853.69; $3547.97; $2641.82; $2331.13; $2191.07; $2602.06; $2393.59; $2224.57; $5627.60. (361—370.) $543.69; $364.53; $271.59; $425.57;

$1059.20 ; $252.98 ; $129.23 ; $157.15 ; $235.84 ; $137.88. (371—380.) $1134.09 ; $785.72 ; $601.71 ; $905.13 ; $2117.09 ; $564.43 ; $310.43 ; $368.99 ; $529.95 ; $328.67. (381–390.) 120 ; 40320 ; 3628800 ; 720 ; 362880 ; 39916800 ; 5040 ; 87178291200 ; 479001600 ; 6227020800. (396.) 403291461126605635584000000.

XV. MENSURATION.

§ **216—229**. (1—12.) 6A. 3R. ; 7A. 2R. 25r. ; 27r. ; 55r. ; 207r. ; 23r. ; 25ft. ; 2A. 1r. ; 125.664r., 1256.64sq.r., 35.45r. ; 47.747r., 1791r., 42.32r. ; 25.23r., 79.26r. ; 52.32r., 164.37r. (13—22.) 44.72ft., 400sq. ft. ; 85.85ft., 1159sq. ft. ; 95.39ft., 1431sq. ft. ; 55.9ft. ; 160.07m. ; 91.53ft. ; 198.94sq. in. ; 452.39sq. ft. ; 201062400sq. m. ; 47.206ft., 148.296ft. (23 — 32.) 502.656ft. ; 1176.5292ft. ; 31.334 gallons ; 15.83in. ; 6.12gal. ; 8.57in. ; 17$\frac{4}{7}$sq. in. ; 7.86in. ; 12960c. in. ; 5068.448c. in. (33—42.) 84ft. 1309in. ; 27.9gal. ; 577.5c. in. ; 452.39in. ; 223.48gal. ; 136.16gal. ; 230.26T. ; 824.43T. ; 365.92T. ; 586.7T.

MISCELLANEOUS EXAMPLES.

§ **232**. (1—12.) $\frac{4}{9}$; $\frac{10}{49}$; $\frac{11}{75}$; $\frac{2}{7}$; $\frac{11}{92}$; $\frac{360}{1673}$; 12.8 ; 15$\frac{53}{88}$; 1$\frac{1257}{1300}$; 14$\frac{5}{6}$; 24$\frac{63}{110}$; 18$\frac{95}{153}$. (13—24.) 7$\frac{3}{4}$, 1$\frac{3}{4}$; 13$\frac{2}{9}$, 1$\frac{5}{9}$; 18$\frac{139}{396}$, 9$\frac{109}{396}$; $\frac{8}{81}$, 1$\frac{41}{71}$, 5$\frac{1}{3}$; 53$\frac{11}{72}$, 58$\frac{1}{9}$, 45$\frac{65}{72}$; $\frac{27}{28}$, 16$\frac{1}{70}$, 30$\frac{5660}{6993}$. (25—35.) 5, 7$\frac{11}{12}$, 21$\frac{1}{9}$; 11$\frac{1}{12}$, $4.375, £5 11s. 5d. ; 2112, 744, 4696, 4800, 368. (41—55.) 19.45, 8.05, 78.375 ; 16$\frac{3}{8}$, 1$\frac{5}{8}$, 66$\frac{3}{8}$; 5$\frac{1}{6}$, 3$\frac{1}{2}$, 3$\frac{1}{18}$; 1$\frac{5}{8}$, $\frac{1}{8}$, $\frac{21}{32}$; 2$\frac{1}{12}$, $\frac{7}{12}$, 1. (56—60.) 50r. ; 80r. ; 350r. ; 17$\frac{7}{9}$r. ; 17$\frac{7}{9}$r. (61—63.) 80lb. ; 180lb. ; 60lb. (71—73.) $51.01 ; $43.725 ; $61.94. (81—83.) 35A. ; 35A. ; 330A. (91—93.) $322.58 ; $294.12 ; $291.26.

(101—103.) 23,—$92 ; $115 ; $103.50. (111—113.) A. $55, B. $62.50, C. $32.50 ; A. $27.50, B. $31.25, C. $16.25 ; A. $82.50, B. $93.75, C. $48.75. (121—123.) 126bu. ; 100.8bu. ; 168bu. (131—133.) In 4$\frac{4}{9}$dy. ; 5$\frac{5}{11}$dy. ; 5dy. (141—143.) $936 ; $748.80 ; $585. (151—153.) $40.50 ; $23.625 ; $20.25. (161—163.)

$17\frac{47}{48}$m.; $89\frac{11}{48}$m.; $880\frac{47}{48}$m. (171—173.) 900; 108; 100. (181—183.) 18; 24; 30. (191—193.) 640 368; 468.

(201—203.) 7200; 2880; 1296. (211—213.) \$310.50; \$258.75; \$414. (221—223.) \$1250; \$2250; \$900. (231—233.) 74ft. 3in.; 31ft. 6in.; 63.225ft. (241—243.) \$365; \$182.50; \$292. (251—253.) \$337.50; \$312; \$157.50. (261—263.) 33yd.; 44yd.; $16\frac{1}{2}$yd. (271—273.) \$4.75; \$$4\frac{7}{8}$; \$4.85. (281—283.) \$60; \$15; \$120. (291—293.) \$15; \$5.25; \$14.25.

(301—303.) 2; 8; 3. (311—313.) \$ .50; \$ .25; \$ .37½. (321—323.) \$14.49; \$16.10; \$9.66. (331—333.) 20oz.; $19\frac{1}{5}$oz., 16oz.; 32oz., $26\frac{2}{3}$oz. (341—343.) $2\frac{2}{3}$ft.; $10\frac{2}{3}$ft.; $1\frac{1}{3}$ft. (351—353.) $21\frac{3}{8}$lb.; $56\frac{1}{3}$lb.; $8\frac{2}{3}$lb. (361—363.) \$68.75; \$192.50; \$4620. (371—373.) 33088; 24816; 57904. (381—383.) 135; 90; 73½ (391—393.) 12; 120; 36.

(401—403.) 100; 150; 450. (411—412.) \$ .01; \$ .02. (421, 422.) \$ .001; \$ .002. (431—433.) 5min. $27\frac{3}{11}$sec. past 1; 16min. $21\frac{9}{11}$sec. past 3; 21min. $49\frac{1}{11}$sec. before 8. (441—444.) \$5434716; \$3584834; \$2871689; \$2537772. (451, 452.) \$435.92; $13\frac{1}{3}$yd. (455, 458, 461.) \$$190\frac{5}{8}$; \$$311\frac{1}{9}$; \$393.75. (464, 467, 470.) \$447.75; £1 1s. $3\frac{3}{4}$d.; $\frac{41}{784}$. (473, 476, 479.) $87\frac{28}{31}$lb.; $4\frac{7}{10}$, $\frac{4}{10}$; \$700. (482, 485, 488.) 13ct.; \$31.41; \$671.12. (491, 494, 497.) \$529.81; 6; 25.

(500, 503, 506.) 8 per cent.; \$1.12½; 120. (509, 512, 515.) \$11.62; $\frac{16}{2085}$; \$3000, \$2000. (518, 521, 524.) £$9\frac{1}{16}$; £10 14s. 7½d.; 3lb. at \$ .50, 2lb. at \$ .75. (527, 530, 533.) \$733.14; \$13.18; \$2556.82. (536, 539, 542.) $2^9 \times 3 \times 5^2$; 7821; 52. (545, 548, 554.) £116 7s. [illegible]; A. \$$1666\frac{2}{3}$, B. \$2000, C. \$$1333\frac{1}{3}$; 108½. (557, 560, 563.) \$3642.09; \$$116\frac{7}{8}$; $1\frac{13}{23}$h. (566, 569, 572.) 250; ½oz.; $1\frac{13}{47}$dy. (575, 578, 581, 584.) \$$194\frac{22}{37}$, \$$162\frac{6}{37}$, \$$243\frac{9}{37}$; \$643.52; 2h. 21min. 8sec.; 60. (587—*600.*) 168912; \$4194.304; 23.32; A. \$150, B. \$100, *C. \$50*; *\$135*; He worked 30 days,—idle 10dy.; Lost

6ct.; 2250; £1913 6s. 8d.; \$1920; 150; A. \$890, B. \$460, C. \$650; 1$\frac{9}{16}$; $\frac{37}{240}$, worth \$925.

(601.) Aug. 7, 1859, at 8h. 53min. 20sec. P. M. Observe, that 1852 and 1856 are Leap Years. (602—610.) 495, 360, 385; 11mo.; Jan. 21, 1849; \$ .75; 168dy.; \$1000; 550 men; 33$\frac{1}{3}$. per cent; \$859.18. (611—616.) \$39.50; 244.140625; 13s. 1$\frac{1}{2}$d.; 27; 32ft.; 740. (617.) Since the gain is proportioned to the stock *multiplied* by the time, the stocks must be proportioned to the gain *divided* by the time; therefore dividing \$549 into three parts, in the proportion of $\frac{2}{4}$, $\frac{3}{5}$, and $\frac{4}{13}$, we find A.'s share \$195; B.'s \$234; C.'s \$120. (618.) As C. took $\frac{1}{3}$ of the gain, he must have contributed $\frac{1}{3}$ of the stock, or $\frac{1}{2}$ as much as A. and B. C.'s stock \$2500; A.'s gain \$600. (619.) 40 days. (620.) \$150 is .75 of the first cost, which was therefore \$200. .40 of \$200 = \$80, my entire loss. (621.) 320 times. (622.) 13.4ft. (623.) If B. had 10 acres more, (making 1510A. in all,) the whole could be divided in the proportions of 10, 13, and 3. A.'s share was 580$\frac{10}{13}$ acres; B.'s 745 acres; C.'s 174$\frac{3}{13}$ acres. (624.) A.'s share \$1600, B.'s \$2400, C.'s \$3000, D.'s \$1000.

(625.) 6 at \$1, 4 at \$ .75, and 3 at \$ .50, would have received \$10.50. The actual number was therefore 24 at \$1, 16 at \$ .75, and 12 at \$ .50. (626.) 7.14 rods. (627.) The money should be divided in proportion to the quantity of bread furnished to C. by each. A should have 2 cents, B. 8 cents. (628.) In 17$\frac{2}{7}$mo. from the time of purchase. (629.) If 3 yards were sold at the price given for 4yd., there would be neither gain nor loss. \$15.75 is equivalent to 5 per cent advance on the price of 4yd., or to the price of 4.2yd. The cloth therefore cost \$3.75 per yard. (630.) \$2000 is the difference between $\frac{4}{5}$ of 150 per cent. of B.'s original capital, and twice his original capital. B.'s was therefore \$2000, A.'s \$4000. (631.) 5yd is $\frac{1}{4}$ of what was left after cutting off 17yd. The original length was 37yd. (632.) 2519, which is 1 less than the least common multiple. (633.) A. 49 miles, B. [illegible] miles.

END OF THE KEY.

Stereotyped by S. DOUGLAS WYETH, No. 7 Pear St., Philada.

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